

$$3 \cos \theta - 1 = 0 \quad \text{and}$$

$$\cos \theta = \frac{1}{3}$$

$$\theta = 1.23, 5.05$$

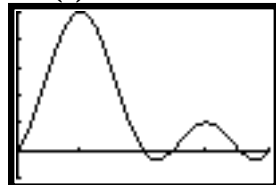
$$\therefore \theta = 1.23, 1.98, 4.30, 5.05$$

$$5 \cos \theta + 2 = 0$$

$$\cos \theta = \frac{-2}{5}$$

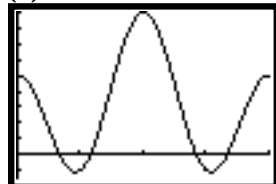
$$\theta = 1.98, 4.30$$

13. (a)



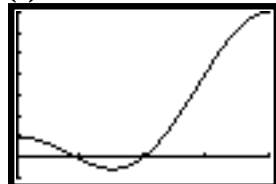
$$\theta = 0^\circ, 180^\circ, 222^\circ, 318^\circ, 360^\circ$$

(b)



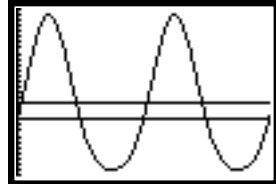
$$\theta = 60^\circ, 104^\circ, 256^\circ, 300^\circ$$

(c)



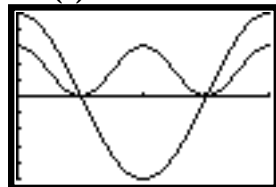
$$\theta = 83^\circ, 180^\circ$$

(d)



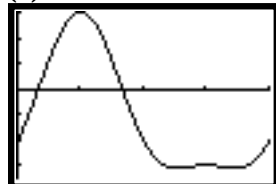
$$\theta = 6^\circ, 84^\circ, 186^\circ, 264^\circ$$

14. (a)



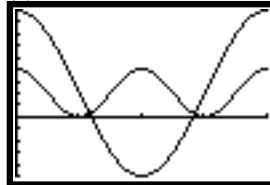
$$x = 1.57, 4.71$$

(b)



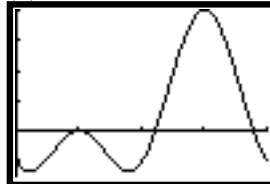
$$x = 0.52, 2.62$$

(c)



$$x = 1.82, 4.46$$

(d)



$$x = 1.57, 3.48, 5.94$$

15. (a) Example: $2 \cos^2 x = \cos x$ OR $\sin^2 x - 4 \sin x + 4 = 0$

(b) No, it not possible to factor all quadratic trigonometric equations. Some equations have irrational roots.

(c) The two factors that play a role when determining the number of solutions of a quadratic trigonometric equation are period and domain.

16. (a) $3 \tan^2 2x = 1$

$$\tan^2 2x = \frac{1}{3}$$

$$\tan 2x = \pm \frac{1}{\sqrt{3}}$$

$$\tan 2x = \frac{1}{\sqrt{3}} \quad \text{and} \quad \tan 2x = \frac{-1}{\sqrt{3}}$$

$$2x = 30^\circ \quad 2x = 150^\circ$$

$$x = 15^\circ, 105^\circ, 195^\circ, 285^\circ \quad x = 75^\circ, 165^\circ, 255^\circ, 345^\circ$$

$$\therefore x = 15^\circ, 75^\circ, 105^\circ, 165^\circ, 195^\circ, 255^\circ, 285^\circ, 345^\circ$$

$$(b) (\sqrt{2} \sin \theta)^2 = (\sqrt{3} - \cos \theta)^2$$

$$2 \sin^2 \theta = 3 - 2\sqrt{3} \cos \theta + \cos^2 \theta$$

$$2(1 - \cos^2 \theta) = 3 - 2\sqrt{3} \cos \theta + \cos^2 \theta$$

$$2 - 2 \cos^2 \theta = 3 - 2\sqrt{3} \cos \theta + \cos^2 \theta$$

$$3 \cos^2 \theta - 2\sqrt{3} \cos \theta + 1 = 0$$

$$\cos \theta = \frac{\sqrt{3}}{3}$$

$$\theta = 0.96$$

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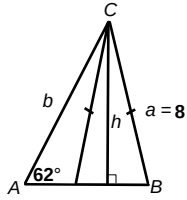
1.

Given Information	What Can Be Found	Law Required
two angles and any side (AAS or ASA)	side	Sine Law
two sides and the contained angle (SAS)	side	Cosine Law
three sides (SSS)	angle	Cosine Law
two sides and an angle opposite one of them (SSA)	angle	Sine Law

2. (a) $\frac{\sin L}{l} = \frac{\sin N}{n}$ or $\frac{\sin L}{l} = \frac{\sin M}{m}$

(b) $m^2 = n^2 + l^2 - 2(n)(l) \cos M$

3. (a)



For two possible angles for B,

$$h = b \sin A < a < b$$

$$8 < b < \frac{8}{\sin 62^\circ}$$

$$8 < b < 9.06$$

(b) For one possible angle for B,

$$a = b \sin A \quad \text{OR} \quad a \geq b$$

$$8 = b \sin 62^\circ \quad 0 < b < 8$$

$$\frac{8}{\sin 62^\circ} = b$$

$$9.06 = b$$

$$\therefore b = 9.06 \text{ or } 0 < b < 8$$

(c) For no possible value for B,

$$a < b \sin A$$

$$b > \frac{a}{\sin A}$$

$$b > \frac{8}{\sin 62^\circ}$$

$$b > 9.06$$

4. (a)

$$\frac{x}{\sin 36^\circ} = \frac{42}{\sin 118^\circ}$$

$$x = \frac{42 \sin 36^\circ}{\sin 118^\circ}$$

$$x = 28.0 \text{ cm}$$

(b)

$$\frac{\sin \theta}{15} = \frac{\sin 20^\circ}{10}$$

$$\sin \theta = \frac{15 \sin 20^\circ}{10}$$

$$\theta = \sin^{-1}(0.513030215)$$

$$\theta = 30.9^\circ$$

But, θ is obtuse $\therefore \theta = 180^\circ - 30.9^\circ = 149.1^\circ$

(c)

$$7^2 = 4^2 + 3^2 - 2(4)(3) \cos \theta$$

$$\frac{7^2 - 4^2 - 3^2}{-2(4)(3)} = \cos \theta$$

$$\theta = \cos^{-1}(-1)$$

$$\theta = 180^\circ$$

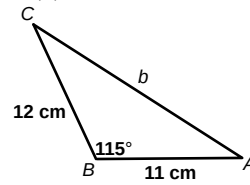
(d)

$$x^2 = 8^2 + 6^2 - 2(8)(6) \cos 135^\circ$$

$$x^2 = 167.882251$$

$$x = 13.0 \text{ cm}$$

5.(a)



$$b^2 = 12^2 + 11^2 - 2(12)(11) \cos 115^\circ$$

$$b^2 = 376.5712211$$

$$b \doteq 19.4 \text{ cm}$$

$$\frac{19.4}{\sin 115^\circ} = \frac{12}{\sin A}$$

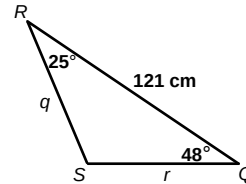
$$\sin A = \frac{12 \sin 115^\circ}{19.4}$$

$$\angle A = \sin^{-1}(0.5606027549)$$

$$\angle A \doteq 34.1^\circ$$

$$\angle C = 180^\circ - (34.1^\circ + 115^\circ) = 30.9^\circ$$

(b)



$$\angle S = 180^\circ - (25^\circ + 48^\circ) = 107^\circ$$

$$\frac{q}{\sin 48^\circ} = \frac{121}{\sin 107^\circ}$$

$$q = \frac{121 \sin 48^\circ}{\sin 107^\circ}$$

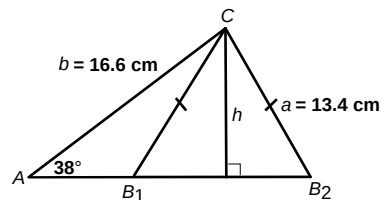
$$q = 94.0 \text{ cm}$$

$$\frac{r}{\sin 25^\circ} = \frac{121}{\sin 107^\circ}$$

$$r = \frac{121 \sin 25^\circ}{\sin 107^\circ}$$

$$r = 53.5 \text{ cm}$$

6.



$$h = b \sin A$$

$$h = 16.6 \sin A$$

$$h = 10.22$$

$$\therefore b \sin A < a < b \Rightarrow 2 \text{ solutions}$$

$$\frac{\sin B}{16.6} = \frac{\sin 38^\circ}{13.4}$$

$$\sin B = \frac{16.6 \sin 38^\circ}{13.4}$$

$$B = \sin^{-1}(0.7626851112)$$

$$\angle B_2 = 49.7^\circ \text{ and } \angle B_1 = 180^\circ - 49.7^\circ = 130.3^\circ$$

$$\angle C_1 = 180^\circ - (38^\circ + 130.3^\circ) = 11.7^\circ$$

$$\angle C_2 = 180^\circ - (38^\circ + 49.7^\circ) = 92.3^\circ$$

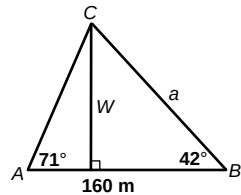
$$\frac{c_1}{\sin 11.7^\circ} = \frac{13.4}{\sin 38^\circ} \quad \frac{c_2}{\sin 92.3^\circ} = \frac{13.4}{\sin 38^\circ}$$

$$c_1 = 4.4 \text{ cm} \quad c_2 = 21.7 \text{ cm}$$

$\therefore$ one triangle: $\angle B = 49.7^\circ$, $\angle C = 92.3^\circ$, and $c = 21.7 \text{ cm}$
 $\therefore$ second triangle $\angle B = 130.3^\circ$, $\angle C = 11.7^\circ$, and $c = 4.4 \text{ cm}$

7. (1) Does the problem involve right, acute, or obtuse angle triangles? (2) Does the problem involve two or three dimensions? (3) Is the sine law or cosine law used? A diagram should be part of every solution.

8.



$$\angle C = 180^\circ - (71^\circ + 42^\circ) = 67^\circ$$

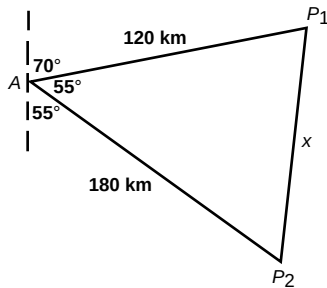
$$\frac{a}{\sin 71^\circ} = \frac{160}{\sin 67^\circ} \quad \sin 42^\circ = \frac{w}{164.3}$$

$$a = \frac{160 \sin 71^\circ}{\sin 67^\circ} \quad w = (164.3) \sin 42^\circ$$

$$a = 164.3 \text{ m} \quad w \doteq 109.9 \text{ m}$$

$\therefore$ the river is approximately 110 m wide.

9.



$$x^2 = 120^2 + 180^2 - 2(120)(180) \cos 55^\circ$$

$$x^2 = 22\,021.497\,95$$

$$x = 148.4 \text{ km}$$

$\therefore$ the two planes are approximately 148.4 km apart.

10.

$$(BC)^2 = 283^2 + 350^2 - 2(283)(350) \cos 29^\circ$$

$$(BC)^2 = 29\,326.836\,02$$

$$BC = 171.2508 \text{ m}$$

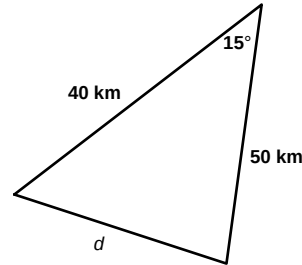
$$\tan 49^\circ = \frac{DC}{171.2508}$$

$$DC = (171.2508) \tan 49^\circ$$

$$DC \doteq 197.0 \text{ m}$$

$\therefore$ the cliff is approximately 197.0 m high.

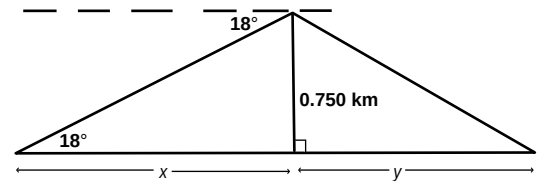
11.



$$d^2 = 40^2 + 50^2 - 2(40)(50) \cos 15^\circ$$

$$d^2 = 236.296\,694\,8$$

$$d = 15.4 \text{ km}$$



$$\tan 18^\circ = \frac{0.750}{x}$$

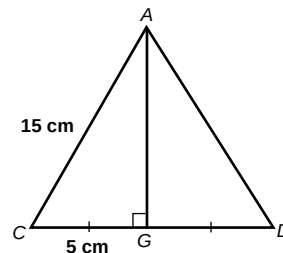
$$x = \frac{0.750}{\tan 18^\circ}$$

$$x = 2.3 \text{ km}$$

$$y = 15.4 - 2.3 = 13.1 \text{ km}$$

$\therefore$ the horizontal distance from the helicopter to the faster car is 13.1 km.

12.



$$AG^2 + 5^2 = 15^2$$

$$AG^2 = 15^2 - 5^2$$

$$AG = \sqrt{200}$$

$$AG = 14.1 \text{ cm}$$

$$(AB)^2 = (AG)^2 + (GB)^2 - 2(AG)(GB) \cos \angle AGB$$

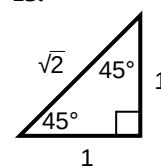
$$25^2 = (14.1)^2 + (14.1)^2 - 2(14.1)(14.1) \cos \angle AGB$$

$$\cos \angle AGB = \frac{25^2 - (14.1)^2 - (14.1)^2}{-2(14.1)(14.1)}$$

$$\angle AGB = \cos^{-1}(-0.571\,852\,522\,5)$$

$$\angle AGB = 124.9^\circ$$

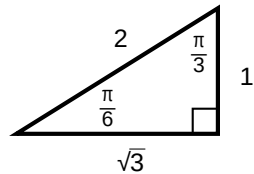
13.



(a) $\cos 45^\circ = \frac{1}{\sqrt{2}}$

(b) $\sin 135^\circ = \frac{1}{\sqrt{2}}$
 (c) $\cos 225^\circ = \frac{-1}{\sqrt{2}}$
 (d) $\tan (-45^\circ) = -1$

14.



(a) $\sin \frac{\pi}{6} = \frac{1}{2}$
 (b) $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$
 (c) $\tan \frac{5\pi}{6} = \frac{-1}{\sqrt{3}}$
 (d) $\sin \frac{5\pi}{3} = \frac{-\sqrt{3}}{2}$

15. (a) $\sin^2(45^\circ) + \cos^2(60^\circ) = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2$
 $= \frac{1}{2} + \frac{1}{4}$
 $= \frac{3}{4}$
 (b) $\sin\left(\frac{-\pi}{4}\right)\cos\left(\frac{5\pi}{4}\right) + \tan^2\left(\frac{\pi}{4}\right) = \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}}\right) + (1)^2$
 $= \frac{1}{2} + 1$
 $= \frac{3}{2}$
 (c) $\cos(60^\circ)\sin^2(240^\circ) - \sin(-60^\circ) = \left(\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{-\sqrt{3}}{2}\right)$
 $= \left(\frac{1}{2}\right)\left(\frac{3}{4}\right) + \frac{\sqrt{3}}{2}$
 $= \frac{3}{8} + \frac{\sqrt{3}}{2}$
 $= \frac{3+4\sqrt{3}}{8}$
 (d) $3\sin^2\left(\frac{\pi}{3}\right) - 2\cos^2\left(\frac{\pi}{6}\right) = 3\left(\frac{\sqrt{3}}{2}\right)^2 - 2\left(\frac{\sqrt{3}}{2}\right)^2$
 $= 3\left(\frac{3}{4}\right) - 2\left(\frac{3}{4}\right)$
 $= \frac{9}{4} - \frac{6}{4}$
 $= \frac{3}{4}$

16.

$$\begin{aligned} \text{LS} &= \tan\left(\frac{3\pi}{4}\right) + \frac{1}{\tan\left(\frac{3\pi}{4}\right)} & \text{RS} &= \frac{1}{\sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{3\pi}{4}\right)} \\ &= -1 + \frac{1}{-1} & &= \frac{1}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}}\right)} \\ &= -1 + (-1) & &= \frac{1}{\frac{-1}{2}} \\ &= -2 & &= -2 \end{aligned}$$

$\therefore \text{LS} = \text{RS}, \therefore \tan\left(\frac{3\pi}{4}\right) + \frac{1}{\tan\left(\frac{3\pi}{4}\right)} = \frac{1}{\sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{3\pi}{4}\right)}$

17. (a) $\sin x = \frac{\sqrt{3}}{2}$

$\therefore x = 60^\circ$ and $x = 180^\circ - 60^\circ = 120^\circ$

(b) $-2 \cos 2x = 1$

$\cos 2x = \frac{-1}{2}$

$2x = 120^\circ$ and $2x = 240^\circ$

$x = 60^\circ$ and $x = 120^\circ$

Since the period is 180° , $\therefore x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

(c) $\sqrt{3} \tan x = 1$

$\tan x = \frac{1}{\sqrt{3}}$

$\therefore x = 30^\circ$ and $x = 180^\circ + 30^\circ = 210^\circ$

18. Example: $\tan \theta \cos \theta = \sin \theta$.

19. Any identity has an infinite number of solutions. The definition of an **identity** is an equation that is true for all values of the variable in it.

20.

$$\begin{aligned} \text{LS} &= \tan x \sin x + \cos x & \text{RS} &= \frac{1}{\cos x} \\ &= \left(\frac{\sin x}{\cos x}\right) \sin x + \cos x \\ &= \frac{\sin^2 x}{\cos x} + \cos x \\ &= \frac{\sin^2 x + \cos^2 x}{\cos x} \\ &= \frac{1}{\cos x} \end{aligned}$$

$\therefore \text{LS} = \text{RS}, \therefore$ the identity is proven.

21.

$$\begin{aligned} \text{LS} &= \tan x & \text{RS} &= \frac{\sin x + \sin^2 x}{\cos x(1 + \sin x)} \\ &= \frac{\sin x}{\cos x} & &= \frac{\sin x(1 + \sin x)}{\cos x(1 + \sin x)} \\ & & &= \frac{\sin x}{\cos x} \end{aligned}$$

$\therefore \text{LS} = \text{RS}, \therefore \tan x = \frac{\sin x + \sin^2 x}{\cos x(1 + \sin x)}$

22.

$$\begin{aligned}\text{LS} &= \sin^4 x - \cos^4 x & \text{RS} &= \sin^2 x - \cos^2 x \\ &= (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) \\ &= (\sin^2 x - \cos^2 x)(1) \\ &= \sin^2 x - \cos^2 x \\ \therefore \text{LS} &= \text{RS}, \therefore \sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x\end{aligned}$$

23.

$$\begin{aligned}\text{LS} &= \frac{1 - \tan^2 \theta}{\tan \theta - \tan^2 \theta} & \text{RS} &= \frac{\tan \theta + 1}{\tan \theta} \\ &= \frac{(1 - \tan \theta)(1 + \tan \theta)}{\tan \theta(1 - \tan \theta)} \\ &= \frac{\tan \theta + 1}{\tan \theta} \\ \therefore \text{LS} &= \text{RS}, \therefore \frac{1 - \tan^2 \theta}{\tan \theta - \tan^2 \theta} = \frac{\tan \theta + 1}{\tan \theta}\end{aligned}$$

24. In a quadratic trigonometric equation, the term $\sin^2 x$, $\cos^2 x$, or $\tan^2 x$ will be present in the equation. The angle (x) will vary.

25. Sometimes, factoring can be used to solve quadratic trigonometric equations.

26. (a) $\cos x (1 - 2 \sin x) = 0$

$$\begin{aligned}\cos x &= 0 & \text{and} & & 1 - 2 \sin x &= 0 \\ x &= 90^\circ, 270^\circ & & & \sin x &= \frac{1}{2} \\ & & & & x &= 30^\circ, 150^\circ\end{aligned}$$

$$\therefore x = 30^\circ, 90^\circ, 150^\circ, 270^\circ$$

(b) $4 \sin^2 x - 1 = 0$

$$4 \sin^2 x = 1$$

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm \frac{1}{2}$$

$$\sin x = \frac{1}{2} \quad \text{and}$$

$$x = 30^\circ, 150^\circ$$

$$\therefore x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

(c) $2 \cos^2 x + \cos x - 1 = 0$

$$(2 \cos x - 1)(\cos x + 1) = 0$$

$$2 \cos x - 1 = 0 \quad \text{and}$$

$$\cos x = \frac{1}{2}$$

$$x = 60^\circ, 300^\circ$$

$$\therefore x = 60^\circ, 180^\circ, 300^\circ$$

(d) $3 \sin^2 x + 4 \sin x = 4$

$$3 \sin^2 x + 4 \sin x - 4 = 0$$

$$(3 \sin x - 2)(\sin x + 2) = 0$$

$$3 \sin x - 2 = 0 \quad \text{and}$$

$$\sin x = \frac{2}{3}$$

$$x = 42^\circ, 138^\circ$$

$$\therefore x = 42^\circ, 138^\circ$$

$$\cos x + 1 = 0$$

$$\cos x = -1$$

$$x = 180^\circ$$

$$\sin x + 2 = 0$$

$$\sin x = -2$$

$$x \text{ has no solutions}$$

(e) $1 - \cos x = 4 \sin^2 x$

$$1 - \cos x = 4(1 - \cos^2 x)$$

$$1 - \cos x = 4 - 4 \cos^2 x$$

$$4 \cos^2 x - \cos x - 3 = 0$$

$$(\cos x - 1)(4 \cos x + 3) = 0$$

$$\cos x - 1 = 0 \quad \text{and}$$

$$\cos x = 1$$

$$x = 0^\circ, 360^\circ$$

$$\therefore x = 0^\circ, 139^\circ, 221^\circ, 360^\circ$$

(f) $5 \cos^2 x - 3 = 0$

$$\cos^2 x = \frac{3}{5}$$

$$\cos x = \pm \sqrt{\frac{3}{5}}$$

$$x = \cos^{-1}(0.774\,596\,669)$$

$$\therefore x = 39.23^\circ, 140.77^\circ, 219.23^\circ, 320.77^\circ$$

$$4 \cos x + 3 = 0$$

$$\cos x = \frac{-3}{4}$$

$$x = 139^\circ, 221^\circ$$

27. (a) $(\sin \theta + 1)(\cos \theta - 1) = 0$

$$\sin \theta + 1 = 0 \quad \text{and}$$

$$\sin \theta = -1$$

$$\theta = \frac{3\pi}{2}$$

$$\therefore \theta = 0, \frac{3\pi}{2}, 2\pi$$

$$\cos \theta - 1 = 0$$

$$\cos \theta = 1$$

$$\theta = 0, 2\pi$$

(b) $\cos^2 \theta = \frac{1}{4}$

$$\cos \theta = \pm \frac{1}{2}$$

$$\cos \theta = \frac{1}{2} \quad \text{and}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\therefore \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$\cos \theta = \frac{-1}{2}$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

(c) $\cos^2 2\theta = \cos 2\theta$

$$\cos^2 2\theta - \cos 2\theta = 0$$

$$\cos 2\theta(\cos 2\theta - 1) = 0$$

$$\cos 2\theta = 0 \quad \text{and}$$

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$\cos 2\theta - 1 = 0$$

$$2\theta = 0, 2\pi$$

$$\theta = 0, \pi$$

$$\text{But the period is } \pi, \therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$$

(d) $10 \sin^2 \theta + 7 \sin \theta = 6$

$$10 \sin^2 \theta + 7 \sin \theta - 6 = 0$$

$$(5 \sin \theta + 6)(2 \sin \theta - 1) = 0$$

$$5 \sin \theta + 6 = 0 \quad \text{and}$$

$$\sin \theta = \frac{-6}{5}$$

$$\theta \text{ has no solutions}$$

$$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$2 \sin \theta - 1 = 0$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$(e) \tan^2 \theta = \sqrt{3}$$

$$\tan \theta = \pm \sqrt{3}$$

$$\tan \theta = \sqrt{3} \quad \text{and}$$

$$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$$

$$\therefore \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$(f) \cos^2 \theta - 5 \sin \theta + 6 = 0$$

$$(\cos \theta - 2)(\cos \theta - 3) = 0$$

$$\cos \theta - 2 = 0 \quad \text{and}$$

$$\cos \theta = 2$$

θ has no solution

$\therefore \theta$ has no solutions

$$\tan \theta = -\sqrt{3}$$

$$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$$

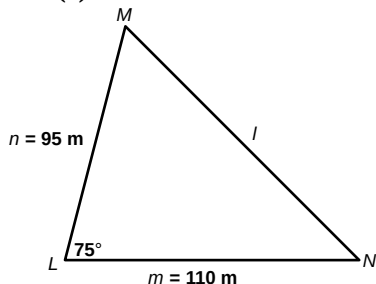
$$\cos \theta - 3 = 0$$

$$\cos \theta = 3$$

θ has no solution

Chapter 6 Review Test, page 552

1. (a)



$$l^2 = 95^2 + 110^2 - 2(95)(110)\cos 75^\circ$$

$$l^2 = 15\,715.681\,96$$

$$l = 125.4 \text{ m}$$

$$\frac{\sin M}{110} = \frac{\sin 75^\circ}{125.4}$$

$$\sin M = \frac{110 \sin 75^\circ}{125.4}$$

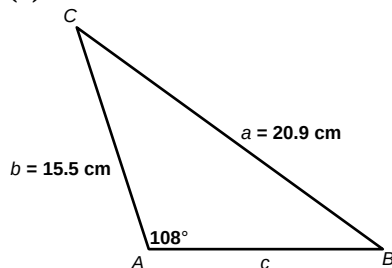
$$\sin M = 0.847\,303\,356\,4$$

$$\angle M \doteq 57.9^\circ$$

$$\angle N = 180^\circ - (75^\circ + 57.9^\circ) = 47.1^\circ$$

$$\therefore l = 125.4 \text{ m}, \angle M = 57.9^\circ, \angle N = 47.1^\circ$$

(b)



$$\frac{\sin B}{15.5} = \frac{\sin 108^\circ}{20.9}$$

$$\sin B = 0.705\,328\,995\,3$$

$$\angle B \doteq 44.9^\circ$$

$$\angle C = 180^\circ - (108^\circ + 44.9^\circ) = 27.1^\circ$$

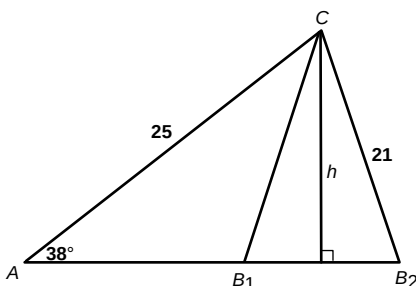
$$\frac{c}{\sin 27.1^\circ} = \frac{20.9}{\sin 108^\circ}$$

$$c = \frac{20.9 \sin 27.1^\circ}{\sin 108^\circ}$$

$$c \doteq 10.0 \text{ cm}$$

$$\therefore \angle B = 44.9^\circ, \angle C = 27.1^\circ, c = 10.0 \text{ cm}$$

2.



$$h = b \sin A$$

$$h = (25)\sin 38^\circ$$

$$h = 15.4 \Rightarrow b \sin A < a \text{ and } b \sin A > b \Rightarrow 2 \text{ solutions}$$

$$\frac{\sin B}{25} = \frac{\sin 38^\circ}{21}$$

$$\sin B = 0.732\,930\,327\,8$$

$$\angle B_2 = 47.1^\circ \text{ and } \angle B_1 = 180^\circ - 47.1^\circ = 132.9^\circ$$

$$\angle C_2 = 180^\circ - (47.1^\circ + 38^\circ) = 94.9^\circ$$

$$\angle C_1 = 180^\circ - (132.9^\circ + 38^\circ) = 9.1^\circ$$

$$\frac{c_2}{\sin 94.9^\circ} = \frac{21}{\sin 38^\circ}$$

$$c_2 = 34.0 \text{ cm}$$

$$\frac{c_1}{\sin 9.1^\circ} = \frac{21}{\sin 38^\circ}$$

$$c_1 = 5.4 \text{ cm}$$

$$\therefore \text{one triangle: } \angle B = 47.1^\circ, \angle C = 94.9^\circ, c = 34 \text{ cm}$$

$$\therefore \text{other triangle: } \angle B = 132.9^\circ, \angle C = 9.1^\circ, c = 5.4 \text{ cm}$$

$$3. (a) \sin \frac{\pi}{6} = \frac{1}{2}$$

$$(b) \cos 135^\circ = \frac{-1}{\sqrt{2}}$$

$$(c) \tan \frac{-5\pi}{6} = \frac{1}{\sqrt{3}}$$

$$(d) \sin 300^\circ = \frac{-\sqrt{3}}{2}$$

$$4. (a) 2 \cos x = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = 30^\circ \text{ and } 330^\circ$$

$$(b) \sin^2 x = \frac{1}{4}$$

$$\sin x = \frac{\pm 1}{2}$$

$$\sin x = \frac{1}{2} \quad \text{and}$$

$$x = 30^\circ, 150^\circ$$

$$\therefore x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

$$\sin x = \frac{-1}{2}$$

$$x = 210^\circ, 330^\circ$$