

$$(e) \tan^2 \theta = \sqrt{3}$$

$$\tan \theta = \pm \sqrt{3}$$

$$\tan \theta = \sqrt{3} \quad \text{and}$$

$$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$$

$$\therefore \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$(f) \cos^2 \theta - 5 \sin \theta + 6 = 0$$

$$(\cos \theta - 2)(\cos \theta - 3) = 0$$

$$\cos \theta - 2 = 0 \quad \text{and}$$

$$\cos \theta = 2$$

$\theta$  has no solution

$\therefore \theta$  has no solutions

$$\tan \theta = -\sqrt{3}$$

$$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$$

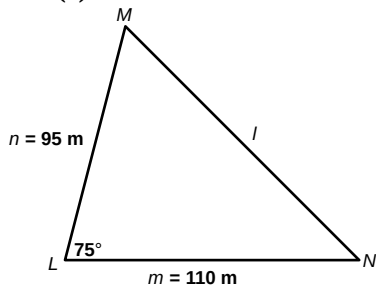
$$\cos \theta - 3 = 0$$

$$\cos \theta = 3$$

$\theta$  has no solution

## Chapter 6 Review Test, page 552

1. (a)



$$l^2 = 95^2 + 110^2 - 2(95)(110)\cos 75^\circ$$

$$l^2 = 15\,715.681\,96$$

$$l = 125.4 \text{ m}$$

$$\frac{\sin M}{110} = \frac{\sin 75^\circ}{125.4}$$

$$\sin M = \frac{110 \sin 75^\circ}{125.4}$$

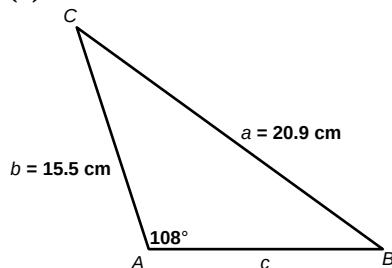
$$\sin M = 0.847\,303\,356\,4$$

$$\angle M \doteq 57.9^\circ$$

$$\angle N = 180^\circ - (75^\circ + 57.9^\circ) = 47.1^\circ$$

$$\therefore l = 125.4 \text{ m}, \angle M = 57.9^\circ, \angle N = 47.1^\circ$$

(b)



$$\frac{\sin B}{15.5} = \frac{\sin 108^\circ}{20.9}$$

$$\sin B = 0.705\,328\,995\,3$$

$$\angle B \doteq 44.9^\circ$$

$$\angle C = 180^\circ - (108^\circ + 44.9^\circ) = 27.1^\circ$$

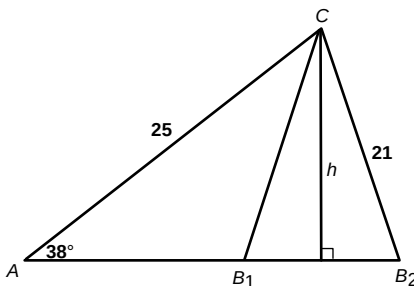
$$\frac{c}{\sin 27.1^\circ} = \frac{20.9}{\sin 108^\circ}$$

$$c = \frac{20.9 \sin 27.1^\circ}{\sin 108^\circ}$$

$$c \doteq 10.0 \text{ cm}$$

$$\therefore \angle B = 44.9^\circ, \angle C = 27.1^\circ, c = 10.0 \text{ cm}$$

2.



$$h = b \sin A$$

$$h = (25)\sin 38^\circ$$

$$h = 15.4 \Rightarrow b \sin A < a \text{ and } b \sin A > b \Rightarrow 2 \text{ solutions}$$

$$\frac{\sin B}{25} = \frac{\sin 38^\circ}{21}$$

$$\sin B = 0.732\,930\,327\,8$$

$$\angle B_2 = 47.1^\circ \text{ and } \angle B_1 = 180^\circ - 47.1^\circ = 132.9^\circ$$

$$\angle C_2 = 180^\circ - (47.1^\circ + 38^\circ) = 94.9^\circ$$

$$\angle C_1 = 180^\circ - (132.9^\circ + 38^\circ) = 9.1^\circ$$

$$\frac{c_2}{\sin 94.9^\circ} = \frac{21}{\sin 38^\circ}$$

$$c_2 = 34.0 \text{ cm}$$

$$\frac{c_1}{\sin 9.1^\circ} = \frac{21}{\sin 38^\circ}$$

$$c_1 = 5.4 \text{ cm}$$

$$\therefore \text{one triangle: } \angle B = 47.1^\circ, \angle C = 94.9^\circ, c = 34 \text{ cm}$$

$$\therefore \text{other triangle: } \angle B = 132.9^\circ, \angle C = 9.1^\circ, c = 5.4 \text{ cm}$$

$$3. (a) \sin \frac{\pi}{6} = \frac{1}{2}$$

$$(b) \cos 135^\circ = \frac{-1}{\sqrt{2}}$$

$$(c) \tan \frac{-5\pi}{6} = \frac{1}{\sqrt{3}}$$

$$(d) \sin 300^\circ = \frac{-\sqrt{3}}{2}$$

$$4. (a) 2 \cos x = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = 30^\circ \text{ and } 330^\circ$$

$$(b) \sin^2 x = \frac{1}{4}$$

$$\sin x = \frac{\pm 1}{2}$$

$$\sin x = \frac{1}{2} \quad \text{and}$$

$$x = 30^\circ, 150^\circ$$

$$\therefore x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$$

$$\sin x = \frac{-1}{2}$$

$$x = 210^\circ, 330^\circ$$

5. (a)

$$\begin{aligned} \text{LS} &= \tan^2 \beta & \text{RS} &= \frac{(1 - \cos \beta)(1 + \cos \beta)}{(1 - \sin \beta)(1 + \sin \beta)} \\ & & &= \frac{1 - \cos^2 \beta}{1 - \sin^2 \beta} \\ & & &= \frac{\sin^2 \beta}{\cos^2 \beta} \\ & & &= \tan^2 \beta \\ \therefore \text{LS} &= \text{RS} \therefore \tan^2 \beta = \frac{(1 - \cos \beta)(1 + \cos \beta)}{(1 - \sin \beta)(1 + \sin \beta)}. \end{aligned}$$

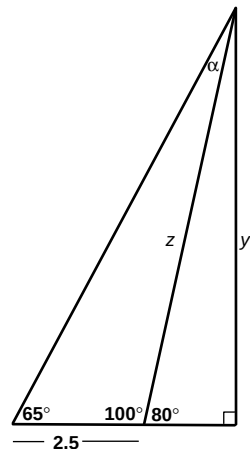
(b)

$$\begin{aligned} \text{LS} &= \sin \theta + \tan \theta & \text{RS} &= \tan \theta(1 + \cos \theta) \\ & & &= \tan \theta + \tan \theta \cos \theta \\ & & &= \tan \theta + \left( \frac{\sin \theta}{\cos \theta} \right) \cos \theta \\ & & &= \tan \theta + \sin \theta \\ \therefore \text{LS} &= \text{RS}, \therefore \sin \theta + \tan \theta = \tan \theta(1 + \cos \theta) \end{aligned}$$

$$\begin{aligned} 6. \quad 2 \cos^2 x - 13 \cos x &= -6 \\ 2 \cos^2 x - 13 \cos x + 6 &= 0 \\ (2 \cos x - 1)(\cos x - 6) &= 0 \\ 2 \cos x - 1 &= 0 & \text{and} & \cos x - 6 = 0 \\ \cos x &= \frac{1}{2} & & \cos x = 6 \\ x &= 60^\circ, 300^\circ & & x \text{ has no solutions} \\ \therefore x &= 60^\circ, 300^\circ \end{aligned}$$

$$\begin{aligned} 7. \quad 1 - 2 \sin^2 \theta &= -\cos \theta \\ 1 - 2(1 - \cos^2 \theta) &= -\cos \theta \\ 1 - 2 + 2 \cos^2 \theta + \cos \theta &= 0 \\ 2 \cos^2 \theta + \cos \theta - 1 &= 0 \\ (2 \cos \theta - 1)(\cos \theta + 1) &= 0 \\ 2 \cos \theta - 1 &= 0 & \text{and} & \cos \theta + 1 = 0 \\ \cos \theta &= \frac{1}{2} & & \cos \theta = -1 \\ \theta &= \frac{\pi}{3}, \frac{5\pi}{3} & & \theta = \pi \\ \therefore \theta &= \frac{\pi}{3}, \pi, \frac{5\pi}{3} \end{aligned}$$

8.



$$\alpha = 180^\circ - (65^\circ + 100^\circ) = 15^\circ$$

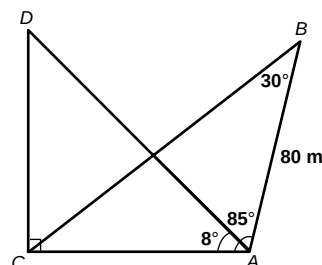
$$\begin{aligned} \frac{z}{\sin 65^\circ} &= \frac{2.5}{\sin 15^\circ} \\ z &= \frac{2.5 \sin 65^\circ}{\sin 15^\circ} \\ z &= 8.75 \text{ m} \end{aligned}$$

$$\begin{aligned} \sin 80^\circ &= \frac{y}{8.75} \\ y &= (8.75) \sin 80^\circ \\ y &= 8.62 \text{ m} \end{aligned}$$

$$\begin{aligned} A &= \frac{b \times h}{2} \\ &= \frac{(2.5)(8.62)}{2} \\ &= 10.8 \text{ m}^2 \end{aligned}$$

$\therefore$  the area of the triangle is  $10.8 \text{ m}^2$ .

9.



$$\angle BCA = 180^\circ - (30^\circ + 85^\circ) = 65^\circ$$

$$\frac{AC}{\sin 30^\circ} = \frac{80}{\sin 65^\circ}$$

$$AC = \frac{(\sin 30^\circ)(80)}{\sin 65^\circ}$$

$$AC = 44.1 \text{ m}$$

$\therefore$  the tree is 6.2 m tall.

$$\begin{aligned} \tan 8^\circ &= \frac{DC}{44.1} \\ DC &= (44.1) \tan 8^\circ \\ DC &= 6.2 \text{ m} \end{aligned}$$

10. If one of the factors of the quadratic trigonometric equation is less than  $-1$  or greater than  $+1$ , the factor will not lead to a solution of the original equation. The functions  $y = \cos x$  and  $y = \sin x$  are only defined for  $-1 \leq y \leq 1$ .

Example:  $\cos^2 x - \cos x - 2 = 0$

$$(\cos x - 2)(\cos x + 1) = 0$$

$$\cos x - 2 = 0 \quad \text{and} \quad \cos x + 1 = 0$$

$$\cos x = 2$$

$$x \text{ has no solutions}$$

$$\cos x = 1$$

$$x = \pi$$

$$\therefore x = \pi$$

$$11. (AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC) \cos B$$

$$\therefore \cos B = \frac{(AC)^2 - (AB)^2 - (BC)^2}{-2(AB)(BC)}$$

$$(XY)^2 = (XB)^2 + (BY)^2 - 2(XB)(BY) \cos B$$

$$\therefore \cos B = \frac{(XY)^2 - (XB)^2 - (BY)^2}{-2(XB)(BY)}$$

$\therefore$  there are two expressions for  $\cos B$  which must be equivalent so equating these two expressions may lead to the desired result

$$\frac{(AC)^2 - (AB)^2 - (BC)^2}{-2(AB)(BC)} = \frac{(XY)^2 - (XB)^2 - (BY)^2}{-2(XB)(BY)}$$

$$\frac{(AC)^2 - (AB)^2 - (BC)^2}{-2(AB)(BC)} = \frac{(XY)^2 - \left(\frac{1}{2}AB\right)^2 - \left(\frac{1}{2}BC\right)^2}{-2\left(\frac{1}{2}AB\right)\left(\frac{1}{2}BC\right)}$$

$$\frac{(AC)^2 - (AB)^2 - (BC)^2}{-2(AB)(BC)} = \frac{(XY)^2 - \frac{1}{4}(AB)^2 - \frac{1}{4}(BC)^2}{-\frac{1}{2}(AB)(BC)}$$

$$\frac{(AC)^2 - (AB)^2 - (BC)^2}{-2(AB)(BC)} = \frac{2(XY)^2 - \frac{1}{2}(AB)^2 - \frac{1}{2}(BC)^2}{-(AB)(BC)}$$

$$(AC)^2 - (AB)^2 - (BC)^2 = 4(XY)^2 - (AB)^2 - (BC)^2$$

$$(AC)^2 = 4(XY)^2$$

$$AC = \pm 2XY$$

But, since length can never be negative,  $AC = 2XY$ .

$\therefore$  the proof is complete.