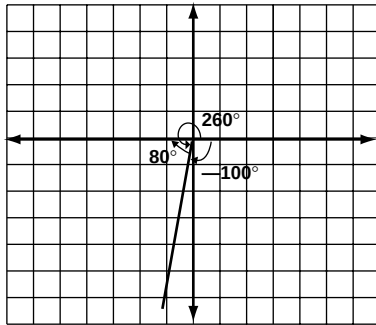


Cumulative Review Test 3

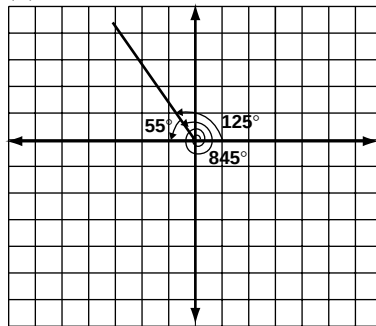
Trigonometric Functions

1.(a)



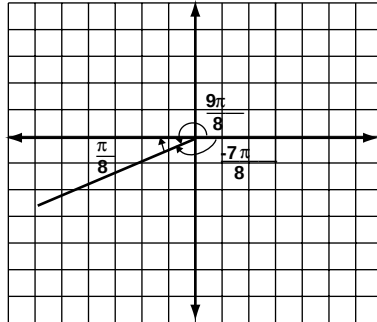
principal angle: 260° , related acute angle: 80°

(b)



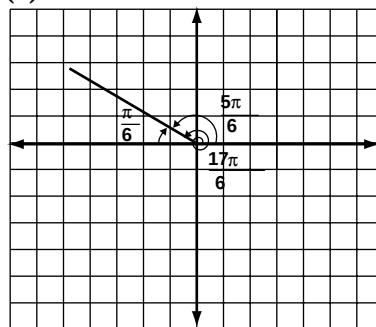
principal angle: 125° , related acute angle: 55°

(c)



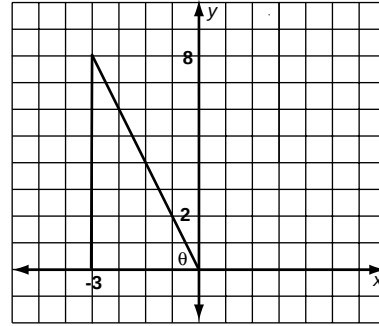
principal angle: $\frac{9\pi}{8}$, related acute angle: $\frac{\pi}{8}$

(d)



principal angle: $\frac{5\pi}{6}$, related acute angle: $\frac{\pi}{6}$

2.(a)



$$(b) \quad r = \sqrt{(-3)^2 + 8^2} = \sqrt{9 + 64} = \sqrt{73}$$

$$\sin \theta = \frac{8}{\sqrt{73}}, \cos \theta = -\frac{3}{\sqrt{73}}, \tan \theta = -\frac{8}{3}$$

$$(c) \quad \theta = \tan^{-1}\left(-\frac{8}{3}\right) = 69^\circ \text{ or } \theta = \cos^{-1}\left(-\frac{3}{\sqrt{73}}\right) = 110^\circ$$

$$3.(a) \quad 15^\circ \times \frac{\pi}{180} = \frac{\pi}{12}$$

$$(b) \quad 120^\circ \times \frac{\pi}{180} = \frac{2\pi}{3}$$

$$(c) \quad -270^\circ \times \frac{\pi}{180} = -\frac{3\pi}{2}$$

$$(d) \quad 216^\circ \times \frac{\pi}{180} = \frac{6\pi}{5}$$

$$4.(a) \quad \frac{13\pi}{6} \times \frac{180}{\pi} = 390^\circ$$

$$(b) \quad \frac{8\pi}{9} \times \frac{180}{\pi} = 160^\circ$$

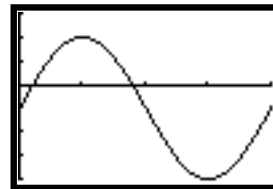
$$(c) \quad \frac{-5\pi}{4} \times \frac{180}{\pi} = -225^\circ$$

$$(d) \quad \frac{13\pi}{16} \times \frac{180}{\pi} = 146.25^\circ$$

$$5.(a) \quad y = 3 \sin \theta - 1$$

i. amplitude = 3, period = 2π , phase shift = none, vertical shift = 1 unit down

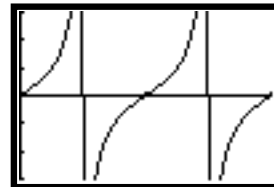
ii.



$$(b) \quad y = \tan \theta$$

i. amplitude = 1, period = 2π , phase shift = none, vertical shift = none

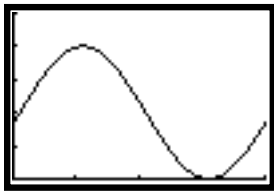
ii.



(c) $y = -2 \cos (\theta - 30^\circ) + 2$

i. amplitude = 2, period = 2π , phase shift = 30° right,
vertical shift = 2 units up

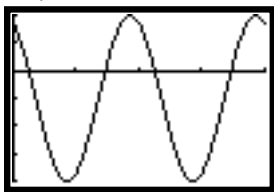
ii.



(d) $y = 3 \sin 2(\theta + 45^\circ) - 1$

i. amplitude = 3, period = π , phase shift = 45° left,
vertical shift = 1 unit down

ii.

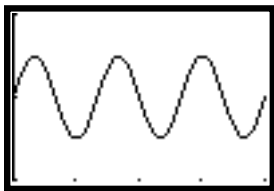


(e) $y = 0.5 \cos 3 \left(\theta - \frac{\pi}{6} \right) + 4$

i. amplitude = 0.5, period = $\frac{2\pi}{3}$,

phase shift = $\frac{\pi}{6}$ right, vertical shift = 4 units up

ii.

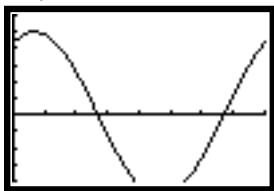


(f) $y = -5 \sin \left(\frac{\theta}{2} - \frac{2\pi}{3} \right) + 1 = -5 \sin \left(\theta - \frac{4\pi}{3} \right) + 1$

i. amplitude = 5, period = 4π ,

phase shift = $\frac{4\pi}{3}$ right, vertical shift = 1 unit up

ii.



6.(a) $y = a \sin b(\theta - c) + d$

$$a = \frac{\max - \min}{2} = \frac{21 - 1}{2} = 10$$

$$d = \frac{\max + \min}{2} = \frac{21 + 1}{2} = 11$$

$$h(t) = 10 \sin 10(t - 9) + 11$$

(b) $10 \sin 10(1) + 11 = 12.74$

(c) $h(t) = 10 + 10 + 1 = 21$ m

$$21 = 10 \sin 10(t - 9) + 11$$

$$10 = 10 \sin 10(t - 9)$$

$$1 = \sin 10(t - 9)$$

$$\sin^{-1}(1) = 10(t - 9)$$

$$t = \frac{\sin^{-1}(1)}{10} + 9$$

$$t = 18 \text{ s}$$

7.(a) $0.5 = \sin \theta$

$$\theta = \sin^{-1}(0.5)$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

(b) $3 = 4 \sin x - 2$

$$4 \sin x = 5$$

$$\sin x = \frac{5}{4}$$

No Solution

(c) $6 = 4 - 5 \cos 3\theta$

$$5 \cos 3 = -2$$

$$\cos 3\theta = \frac{-2}{5}$$

$$3\theta = 113.58^\circ$$

$$\therefore \theta = 37.86^\circ \text{ or } 360^\circ - 37.86^\circ = 322.14^\circ$$

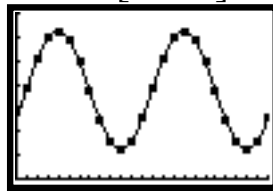
(d) $9 = 4 \tan 2x$

$$\frac{9}{4} = \tan 2x$$

$$2x = 66.04^\circ$$

$$\therefore x = 33.02^\circ \text{ or } 180^\circ + 33.02^\circ = 213.02^\circ$$

8.(a) $y = 2.5 \sin \left[\frac{\pi}{6}(t - 1) \right] + 3.75$



(b) max depth is 6.25 (from graph)

(c) Since the minimum amount of water (depth) is 1.25 m,
then 1 m is okay and a suitable location for their doc.

9.(a) $b^2 = a^2 + c^2 - 2ac \cos B$

$$b^2 = (19.2)^2 + (15.8)^2 - 2(19.2)(15.8) \cos 129^\circ$$

$$b^2 = 368.64 + 249.64 + 381.82$$

$$b = 31.62$$

$$\therefore \angle A = 28.2^\circ \text{ and } \angle C = 22.8^\circ$$

(b) $\frac{18}{\sin 55^\circ} = \frac{21}{\sin C}$

$$21 \sin 55^\circ = 18 \sin C$$

$$\frac{21 \sin 55^\circ}{18} = \sin C$$

$$\angle C = 72.9^\circ$$

$$\therefore \angle A = 180^\circ - 72.9^\circ - 55^\circ = 52.1^\circ$$

$$\frac{18}{\sin 55^\circ} = \frac{a}{\sin 52.1^\circ}$$

$$a \sin 55^\circ = 18 \sin 52.1^\circ$$

$$a = 17.3 \text{ cm}$$

(c) $d^2 = e^2 + f^2 - 2ef \cos D$

$$225 = 324 + 400 - 2(18)(20) \cos D$$

$$225 = 724 - 720 \cos D$$

$$-499 = -720 \cos D$$

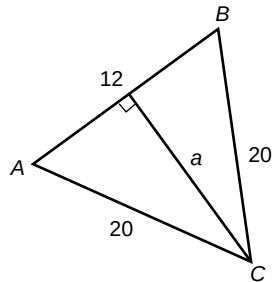
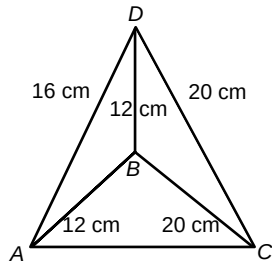
$$0.69 = \cos D$$

$$\angle D = 46.1^\circ$$

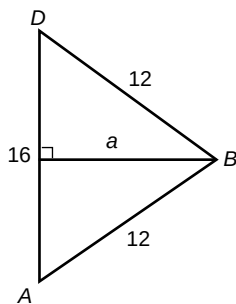
$$\frac{\sin 46.1^\circ}{15} = \frac{\sin E}{18}$$

$$\begin{aligned}\angle E &= 60.1^\circ \text{ and } \angle F = 180^\circ - 60.1^\circ - 46.1^\circ = 73.8^\circ \\ \text{(d) } \angle S &= 180^\circ - 36^\circ - 27^\circ = 117^\circ \\ \frac{q}{\sin 36^\circ} &= \frac{55.4}{\sin 117^\circ} \\ 55.4 \sin 36^\circ &= q \sin 117^\circ \\ \frac{55.4 \sin 36^\circ}{\sin 117^\circ} &= q \\ q &= 36.55 \\ \frac{36.55}{\sin 36^\circ} &= \frac{r}{\sin 27^\circ} \\ \therefore r &= 28.2\end{aligned}$$

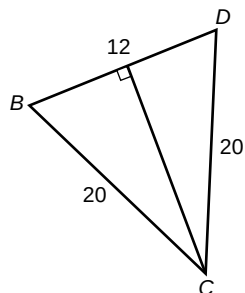
10.



$$\begin{aligned}20^2 &= a^2 + 6^2 \\ a &= 19.079 \text{ cm} \\ \therefore A &= \frac{1}{2}bh = \frac{1}{2}(12)(19.079) = 114.47 \text{ cm}^2\end{aligned}$$

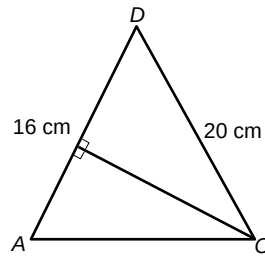


$$\begin{aligned}12^2 &= a^2 + 8^2 \\ a &= 8.94 \text{ cm} \\ \therefore A &= \frac{1}{2}bh = \frac{1}{2}(16)(8.94) = 71.52 \text{ cm}^2\end{aligned}$$



same as $\triangle ABC$

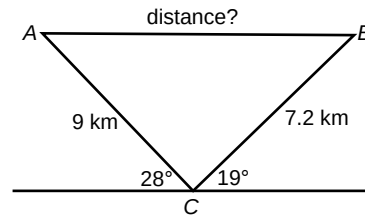
$$a = 19.079 \text{ cm and } A = 114.47 \text{ cm}^2$$



$$\begin{aligned}20^2 &= a^2 + 8^2 \\ a &= 18.33 \text{ cm} \\ \therefore A &= \frac{1}{2}bh = \frac{1}{2}(16)(18.33) = 146.64 \text{ cm}^2\end{aligned}$$

$$\therefore 2(114.47) + 71.52 + 146.64 = 447 \text{ cm}^2$$

11.



$$\begin{aligned}\angle C &= 180^\circ - 19^\circ - 28^\circ = 133^\circ\end{aligned}$$

$$\begin{aligned}c^2 &= a^2 + b^2 - 2ab \cos C \\ &= (7.2)^2 + 9^2 - 2(7.2)(9)\cos 133^\circ \\ &= 51.84 + 81 - 129.6 \cos 133^\circ \\ &= 221.227 \\ \therefore c &= 14.87 \text{ km}\end{aligned}$$

12.(a) $\sin \pi = 0$

$$\text{(b) } \cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\text{(c) } \tan (-225^\circ) = -1$$

$$\text{(d) } \sin 300^\circ = -\frac{\sqrt{3}}{2}$$

$$\text{(e) } \sin 45^\circ \cos 135^\circ + \tan 30^\circ \tan 210^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)$$

$$= -\frac{1}{2} + \frac{1}{3}$$

$$= -\frac{1}{6}$$

$$\text{(f) } \cos \frac{5\pi}{3} \cos \frac{3\pi}{2} - \sin \frac{5\pi}{3} \sin \frac{3\pi}{2}$$

$$= \frac{1}{2}(0) - \left(-\frac{\sqrt{3}}{2}\right)(-1)$$

$$= -\frac{\sqrt{3}}{2}$$

13. (a) $\sin x + \tan x + \cos x \tan x = \tan x (2 \cos x + 1)$

$$\begin{aligned}\text{left side} &= \cos x \tan x + \tan x + \cos x \tan x \\ &= \tan x (\cos x + 1 + \cos x) \\ &= \tan x (2 \cos x + 1) \\ &= \text{right side}\end{aligned}$$

(b) $\sin^2 \theta - \sin^4 \theta = \cos^2 \theta - \cos^4 \theta$

$$\begin{aligned}\text{left side} &= 1 - \cos^2 \theta - (1 - \cos^2 \theta)(1 - \cos^2 \theta) \\ &= 1 - \cos^2 \theta - (1 - \cos^2 \theta - \cos^2 \theta + \cos^4 \theta) \\ &= 1 - \cos^2 \theta - 1 + \cos^2 \theta + \cos^2 \theta - \cos^4 \theta \\ &= \cos^2 \theta - \cos^4 \theta \\ &= \text{right side}\end{aligned}$$

(c) $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = \frac{2}{\cos x}$

$$\begin{aligned}\text{left side} &= \frac{(1 + \sin x)(1 + \sin x) + \cos^2 x}{\cos x(1 + \sin x)} \\ &= \frac{1 + 2 \sin x + 1}{\cos x(1 + \sin x)} \\ &= \frac{2 + 2 \sin x}{\cos x(1 + \sin x)} \\ &= \frac{2(1 + \sin x)}{\cos x(1 + \sin x)} \\ &= \frac{2}{\cos x} \\ &= \text{right side}\end{aligned}$$

(d) $\frac{\tan^2 \alpha}{\tan^2 \alpha + 1} = \sin^2 \alpha$

$$\begin{aligned}\text{left side} &= \frac{\frac{\sin^2 \alpha}{\cos^2 \alpha}}{\left(\frac{\sin^2 \alpha}{\cos^2 \alpha}\right) + 1} \\ &= \frac{\frac{\sin^2 \alpha}{\cos^2 \alpha}}{\frac{\sin^2 \alpha + \cos^2 \alpha}{\cos^2 \alpha}} \\ &= \frac{\sin^2 \alpha}{\cos^2 \alpha} \times \frac{\cos^2 \alpha}{1} \\ &= \sin^2 \alpha \\ &= \text{right side}\end{aligned}$$

14. (a) $2 \sin^2 x \tan x - \tan x = 0$

$$\tan x (2 \sin^2 x - 1) = 0$$

$$\tan x = 0 \quad \text{or}$$

$$\sin x = \frac{1}{\sqrt{2}}$$

$$x = \pi, 2\pi$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

(b) $15 \sin^2 x - 7 \sin x = 2$

$$15 \sin^2 x - 7 \sin x - 2 = 0$$

$$(5 \sin x + 1)(3 \sin x - 2) = 0$$

$$\sin x = -\frac{1}{5} \quad \text{or} \quad \sin x = \frac{2}{3}$$

$$x = 191.5^\circ, 348.5^\circ$$

$$x = 41.8^\circ, 138.2^\circ$$

(c) $\tan^2 x - 2 \tan x + 1 = 0$

$$(\tan x - 1)(\tan x - 1) = 0$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

(d) $4 \cos^2 x - 3 = 0$

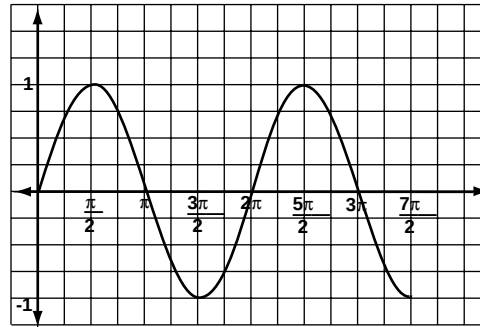
$$4 \cos^2 x = 3$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

15.



$$y = 5 + 3 \sin \frac{\pi}{6} t$$

High Tide:

$$\frac{\pi}{6} t = \frac{\pi}{2}$$

$$t = \left(\frac{\pi}{2}\right)\left(\frac{6}{\pi}\right) = 3 \text{ a.m.}$$

and

$$\frac{\pi}{6} t = \frac{5\pi}{2}$$

$$t = \left(\frac{5\pi}{2}\right)\left(\frac{6}{\pi}\right) = 15 \text{ (3 p.m.)}$$

Low Tide:

$$\frac{\pi}{6} t = \frac{3\pi}{2}$$

$$t = \left(\frac{3\pi}{2}\right)\left(\frac{6}{\pi}\right) = 9 \text{ a.m.}$$

and

$$\frac{\pi}{6} t = \frac{7\pi}{2}$$

$$t = \left(\frac{7\pi}{2}\right)\left(\frac{6}{\pi}\right) = 21 \text{ (9 p.m.)}$$

$\therefore$ The high tides are at 3 a.m and 3 p.m.

The low tides are at 9 a.m and 9 p.m.