

25. Samantha: $i = \frac{0.06}{4} = 0.015$, Mark: $i = \frac{0.05}{4} = 0.0125$

$$1000(1.015)^{4n} = 1500(1.0125)^{4n}$$

$$\frac{2}{3} = \left(\frac{1.0125}{1.015} \right)^{4n}$$

$$\ln\left(\frac{2}{3}\right) = (4n)\ln\left(\frac{1.0125}{1.015}\right)$$

$$164.416 \div 4n$$

$$n \doteq 41.1 \text{ years}$$

Chapter 1, Review and Practice, page 98

1. A sequence is an ordered set of numbers. A term is a single element of a sequence. For example, $\{1, 3, 5, 7, \dots\}$ is a sequence, and 5 is the third term.

2. The general term is an expression or formula for all the terms in a sequence. It is a function of a natural number, and using numbers in this formula generates the terms of this sequence.

3. (a) Example: $\{1, 2, 3, \dots, 100\}$

(b) Example: $\{1, 4, 9, 16, \dots\}$

4. (a) $t_n = 4n + 1$, $t_1 = 4(1) + 1 = 5$, $t_2 = 4(2) + 1 = 9$,
 $t_3 = 4(3) + 1 = 13$, $t_4 = 4(4) + 1 = 17$, $t_5 = 4(5) + 1 = 21$

(b) $t_n = 2n^2 - 2n$, $t_1 = 2(1)^2 - 2(1) = 0$, $t_2 = 2(2)^2 - 2(2) = 4$,
 $t_3 = 2(3)^2 - 2(3) = 12$, $t_4 = 2(4)^2 - 2(4) = 24$,
 $t_5 = 2(5)^2 - 2(5) = 40$

(c) $t_n = 3^{n-1}$, $t_1 = 3^{1-1} = 3^0 = 1$, $t_2 = 3^{2-1} = 3^1 = 3$,
 $t_3 = 3^{3-1} = 3^2 = 9$, $t_4 = 3^{4-1} = 3^3 = 27$, $t_5 = 3^{5-1} = 3^4 = 81$

(d) $t_n = -3n + 2$, $t_1 = -3(1) + 2 = -1$, $t_2 = -3(2) + 2 = -4$,
 $t_3 = -3(3) + 2 = -7$, $t_4 = -3(4) + 2 = -10$, $t_5 = -3(5) + 2 = -13$

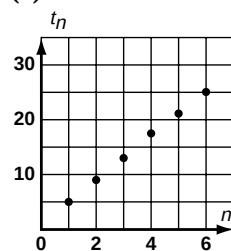
(e) $t_n = (2n + 1)(2n - 1)$, $t_1 = [2(1) + 1][2(1) - 1] = (3)(1) = 3$,
 $t_2 = [2(2) + 1][2(2) - 1] = (5)(3) = 15$,
 $t_3 = [2(3) + 1][2(3) - 1] = (7)(5) = 35$,
 $t_4 = [2(4) + 1][2(4) - 1] = (9)(7) = 63$,
 $t_5 = [2(5) + 1][2(5) - 1] = (11)(9) = 99$

(f) $t_n = \frac{10}{n}$, $t_1 = \frac{10}{1} = 10$, $t_2 = \frac{10}{2} = 5$, $t_3 = \frac{10}{3}$,

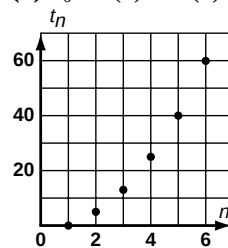
$$t_4 = \frac{10}{4} = \frac{5}{2}, t_5 = \frac{10}{5} = 2$$

5. $t_6 = 4(6) + 1 = 25$

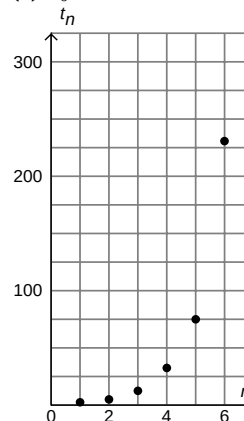
(a)



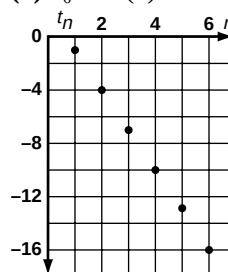
(b) $t_6 = 2(6)^2 - 2(6) = 60$



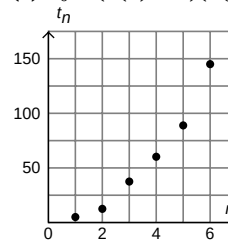
(c) $t_6 = 3^{6-1} = 3^5 = 243$



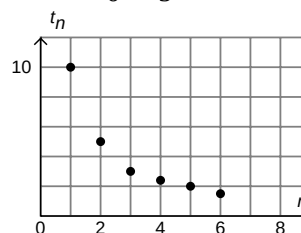
(d) $t_6 = -3(6) + 2 = -16$



(e) $t_6 = (2(6) + 1)(2(6) - 1) = (13)(11) = 143$



(f) $t_6 = \frac{10}{6} = \frac{5}{3}$



6. (a) 2, 7, 12, 17, ...

$$t_n = 2 + (n - 1)(5) = 2 + 5n - 5 = 5n - 3$$

(b) 30, 24, 18, 12, ...

$$t_n = 30 + (n - 1)(-6) = 30 - 6n + 6 = -6n + 36$$

(c) 9, 27, 81, 243, ...

$$t_n = 9(3)^{n-1} = (3^2)(3)^{n-1} = 3^{n+1}$$

$$(d) \frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \frac{5}{16},$$

$$t_n = \frac{5}{2} \left(\frac{1}{2} \right)^{n-1} = 5 \left(\frac{1}{2} \right) \left(\frac{1}{2} \right)^{n-1} = 5 \left(\frac{1}{2} \right)^n = \frac{5}{2^n}$$

$$(e) 2, 5, 10, 17, \dots, t_n = n^2 + 1$$

$$(f) \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, t_n = \frac{n+1}{n+2}$$

$$7. (a) \$24\,000 \times \frac{3}{4} = \$18\,000, \$18\,000 \times \frac{3}{4} = \$13\,500,$$

$$\$13\,500 \times \frac{3}{4} = \$10\,125, \$10\,125 \times \frac{3}{4} = \$7593.75$$

$\therefore$ the sequence is \$18 000, \$13 500, \$10 125, \$7593.75

$$(b) t_n = \$24\,000 \left(\frac{3}{4} \right)^n$$

$$(c) t_7 = \$24\,000 \left(\frac{3}{4} \right)^7 = \$3203.61$$

8. A recursive formula defines each term as a function of one or more previous terms, whereas a general term defines each term as a function of a natural number.

9. A recursive formula consists of a formula that defines each term as a function of previous terms and the base terms. The first term cannot be defined as a function of previous terms since it has none.

$$10. (a) t_n = (t_{n-1}) - 10, t_1 = 7, t_2 = 7 - 10 = -3,$$

$$t_3 = -3 - 10 = -13, t_4 = -13 - 10 = -23, t_5 = -23 - 10 = -33$$

$$(b) t_n = (5t_{n-1}) + 6, t_1 = -3, t_2 = 5(-3) + 6 = -9,$$

$$t_3 = 5(-9) + 6 = -39, t_4 = 5(-39) + 6 = -189,$$

$$t_5 = 5(-189) + 6 = -939$$

$$(c) t_n = (t_{n-1}) \times 2, t_1 = -4, t_2 = (-4)(2) = -8,$$

$$t_3 = (-8)(2) = -16, t_4 = (-16)(2) = -32, t_5 = (-32)(2) = -64$$

$$(d) t_n = 4(t_{n-1})^2 - 2, t_1 = 3, t_2 = 4(3)^2 - 2 = 34,$$

$$t_3 = 4(34)^2 - 2 = 4622, t_4 = 4(4622)^2 - 2 = 8.545 \times 10^7,$$

$$t_5 = 4(8.545 \times 10^7)^2 - 2 = 2.9208 \times 10^{16}$$

$$11. (a) 2, 5, 8, 11, \dots, t_1 = 2, t_n = t_{n-1} + 3$$

$$(b) 18, 11, 4, -3, \dots, t_1 = 18, t_n = t_{n-1} - 7$$

$$(c) 3, 9, 81, 6561, \dots, t_1 = 3, t_n = (t_{n-1})^2$$

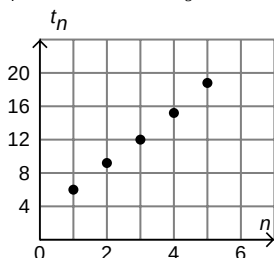
$$(d) 2, 4, 6, 10, \dots, t_1 = 2, t_2 = 4, t_n = t_{n-1} + t_{n-2}$$

$$(e) 1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \dots, t_1 = 1, t_n = \frac{2}{3} t_{n-1}$$

$$(f) 4, 6, 12, 36, \dots, t_1 = 4, t_n = (1 + 2^{n-3}) t_{n-1}$$

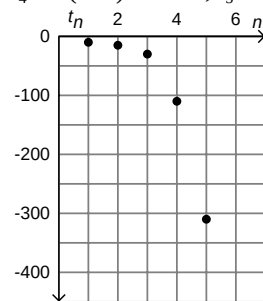
$$12. (a) t_1 = 6, t_2 = 6 + 3 = 9, t_3 = 9 + 3 = 12,$$

$$t_4 = 12 + 3 = 15, t_5 = 15 + 3 = 18$$



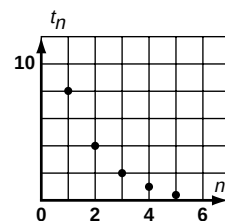
$$(b) t_1 = -4, t_2 = 3(-4) = -12, t_3 = 3(-12) = -36,$$

$$t_4 = 3(-36) = -108, t_5 = 3(-108) = -324$$



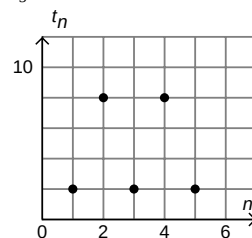
$$(c) t_1 = 8, t_2 = 8 \div 2 = 4, t_3 = 4 \div 2 = 2, t_4 = 2 \div 2 = 1,$$

$$t_5 = 1 \div 2 = \frac{1}{2}$$



$$(d) t_1 = 2, t_2 = 10 - 2 = 8, t_3 = 10 - 8 = 2, t_4 = 10 - 2 = 8,$$

$$t_5 = 10 - 8 = 2$$



$$13. (a) t_1 = 250, t_n = 0.25t_{n-1} + 250.$$

If t_n levels off, then $t_n = t_{n-1}$. So,

$$t_n = 0.25t_{n-1} + 250$$

$$t_n = 0.25t_n + 250$$

$$t_n - 0.25t_n = 250$$

$$0.75t_n = 250$$

$$t_n = \frac{250}{0.75}$$

$$t_n = 333.3 \text{ mg}$$

$\therefore$ the mass of medication will become constant at 333.3 mg.

(b) The sequence slowly increases to the limit. At time $t = 0$, the mass of medication is 250 mg. At $t = 12$ h, the mass in Dennis' body is $250(0.25) + 250 = 312.5$ mg.

at $t = 24$ h, the mass is $312.5(0.25) + 250 = 328.125$ mg

at $t = 36$ h, the mass is $328.125(0.25) + 250 = 332.03$ mg

at $t = 48$ h, the mass is $332.03(0.25) + 250 = 333.00$

at $t = 60$ h, the mass is $333.00(0.25) + 250 = 333.25$ mg

at $t = 72$ h, the mass is $333.25(0.25) + 250 = 333.3$ mg

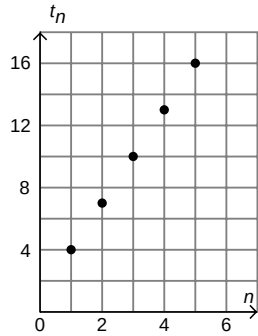
$\therefore$ after 72 h, the mass of medication levels off at 333.33 mg.

14. If the first differences between the terms of a sequence are constant, then the sequence is arithmetic.

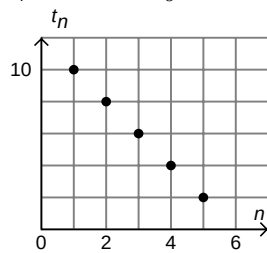
15. (a) $t_n = a + (n - 1)d$

(b) t_n – term, a – first term, n – term index, d – common difference

16. (a) $t_1 = 4$, $t_2 = 4 + 3 = 7$, $t_3 = 7 + 3 = 10$, $t_4 = 10 + 3 = 13$, $t_5 = 13 + 3 = 16$



(b) $t_1 = 10$, $t_2 = 10 - 2 = 8$, $t_3 = 8 - 2 = 6$, $t_4 = 6 - 2 = 4$, $t_5 = 4 - 2 = 2$



17. (a) 6, 12, 18, 24, ..., $a = 6$,

i. $d = 12 - 6 = 6$,

ii. $t_n = 6 + (n - 1)6 = 6n$,

iii. $t_{15} = 6(15) = 90$

(b) -40, -25, -10, 5, ..., $a = -40$,

i. $d = -25 - (-40) = 15$

ii. $t_n = -40 + (n - 1)15 = 15n - 55$

iii. $t_{15} = 15(15) - 55 = 170$

(c) 36, 31, 26, 21, ..., $a = 36$,

i. $d = 31 - 36 = -5$,

ii. $t_n = 36 + (n - 1)(-5) = -5n + 41$,

iii. $t_{15} = -5(15) + 41 = -34$

(d) $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1\frac{1}{4}, \dots$, $a = \frac{1}{4}$,

i. $d = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

ii. $t_n = \frac{1}{4} + (n - 1)\left(\frac{1}{4}\right) = \frac{1}{4}n$,

iii. $t_{15} = \frac{1}{4}(15) = \frac{15}{4} = 3\frac{3}{4}$

(e) 1.2, 1.9, 2.6, 3.3, ..., $a = 1.2$,

i. $d = 1.9 - 1.2 = 0.7$,

ii. $t_n = 1.2 + (n - 1)(0.7) = 0.7n + 0.5$,

iii. $t_{15} = 0.7(15) + 0.5 = 11$

(f) 4, 36, 68, 100, ..., $a = 4$,

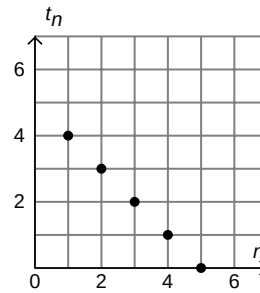
i. $d = 36 - 4 = 32$,

ii. $t_n = 4 + (n - 1)(32) = 32n - 28$,

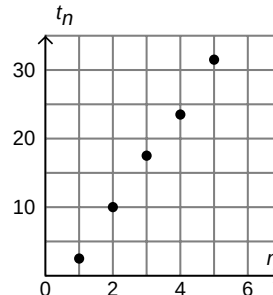
iii. $t_{15} = 32(15) - 28 = 452$

18. (a) $t_1 = -1 + 5 = 4$, $t_2 = -2 + 5 = 3$, $t_3 = -3 + 5 = 2$,

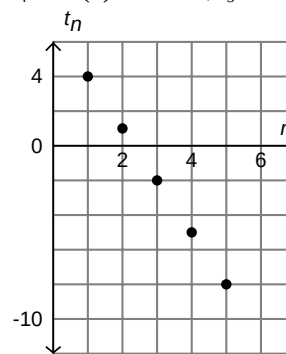
$t_4 = -4 + 5 = 1$, $t_5 = -5 + 5 = 0$



(b) $t_1 = 7(1) - 4 = 3$, $t_2 = 7(2) - 4 = 10$, $t_3 = 7(3) - 4 = 17$, $t_4 = 7(4) - 4 = 24$, $t_5 = 7(5) - 4 = 31$

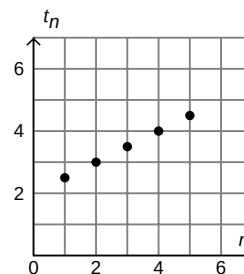


(c) $t_1 = -3(1) + 7 = 4$, $t_2 = -3(2) + 7 = 1$, $t_3 = -3(3) + 7 = -2$, $t_4 = -3(4) + 7 = -5$, $t_5 = -3(5) + 7 = -8$



(d) $t_1 = \frac{1+4}{2} = \frac{5}{2}$, $t_2 = \frac{2+4}{2} = 3$, $t_3 = \frac{3+4}{2} = \frac{7}{2}$,

$t_4 = \frac{4+4}{2} = 4$, $t_5 = \frac{5+4}{2} = \frac{9}{2}$



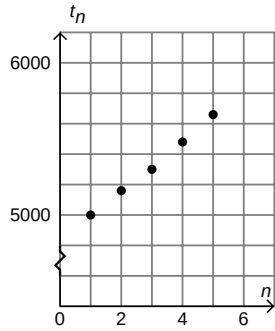
19. (a) $\$5000(0.0325) = \$162.50$

(b) $\$5000$, $\$5162.50$, $\$5325$, $\$5487.50$, $\$5650$

(c) $a = \$5000$, $d = \$162.50$,

$t_n = \$5000 + (n - 1)(\$162.50) = \$162.50n + \$4837.50$

(d)



(e) $t_n = \$5975$

$$t_n = 162.50n + 4837.50$$

$$5975 = 162.50n + 4837.50$$

$$1137.50 = 162.50n$$

$$n = 7$$

$\therefore$ the original deposit has earned interest for 7 years.

20. (a) $t_5 = 24, t_9 = 36$

$$t_9 - t_5 = 36 - 24$$

$$4d = 12$$

$$d = 3$$

$$t_n = 12 + 3(n - 1) = 3n + 9$$

(b) $t_6 = -41, t_{15} = -95$

$$t_{15} - t_6 = -95 + 41$$

$$9d = -54$$

$$d = -6$$

$$t_n = -11 - 6(n - 1) = -6n - 5$$

(c) $t_{11} = 91, t_{20} = 172$

$$t_{20} - t_{11} = 172 - 91$$

$$9d = 81$$

$$d = 9$$

$$t_n = 1 + 9(n - 1) = 9n - 8$$

(d) $t_2 = -8, t_{12} = -158$

$$t_{12} - t_2 = -158 + 8$$

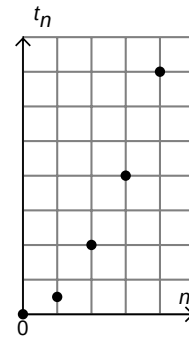
$$10d = -150$$

$$d = -15$$

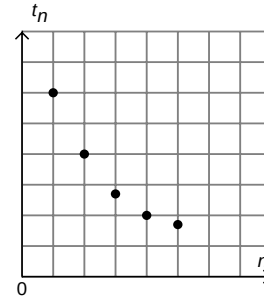
$$t_n = 7 - 15(n - 1) = -15n + 22$$

21. If the ratio between any two successive terms is constant, then the sequence is geometric.

22. (a)



(b)



23. (a) $t_n = ar^{n-1}$

(b) t_n – term, a – first term, r – common ratio, n – term index

24. (a) 2, 14, 98, 686, ..., $a = 2$

$$\text{i. } r = \frac{14}{2} = 7$$

$$\text{ii. } t_n = 2(7)^{n-1}$$

$$\text{iii. } t_{12} = 2(7)^{11} = 3\,954\,653\,486$$

(b) -3, -12, -48, -192, ... $a = -3$

$$\text{i. } r = \frac{-12}{-3} = 4$$

$$\text{ii. } t_n = -3(4)^{n-1}$$

$$\text{iii. } t_{12} = -3(4)^{11} = -12\,582\,912$$

(c) 160, 80, 40, 20, ..., $a = 160$

$$\text{i. } r = \frac{80}{160} = \frac{1}{2}$$

$$\text{ii. } t_n = 160\left(\frac{1}{2}\right)^{n-1}$$

$$\text{iii. } t_{12} = 160\left(\frac{1}{2}\right)^{11} = 0.078\,125$$

(d) 7, -21, 63, -189, ... $a = 7$

$$\text{i. } r = \frac{-21}{7} = -3$$

$$\text{ii. } t_n = 7(-3)^{n-1}$$

$$\text{iii. } t_{12} = 7(-3)^{11} = -1\,240\,029$$

(e) 0.4, 0.08, 0.016, 0.0032, ..., $a = 0.4$

$$\text{i. } r = \frac{0.08}{0.4} = 0.2$$

$$\text{ii. } t_n = 0.4(0.2)^{n-1}$$

$$\text{iii. } t_{12} = 0.4(0.2)^{11} = 8.19 \times 10^{-9}$$

(f) 6561, 2187, 729, 243, ..., $a = 6561$

$$\text{i. } r = \frac{2187}{6561} = \frac{1}{3}$$

$$\text{ii. } t_n = 6561 \left(\frac{1}{3} \right)^{n-1} = 3^8 3^{1-n} = 3^{9-n}$$

$$\text{iii. } t_{12} = 3^{9-12} = 3^{-3} = \frac{1}{27}$$

$$\begin{aligned} \text{25. (a) } 1 - 0.15 &= 0.85, \$149\,500(0.85) = \$127\,075, \\ \$127\,075(0.85) &= \$108\,013.75, \\ \$108\,013.75(0.85) &= \$91\,811.69, \\ \$91\,811.69(0.85) &= \$78\,039.93 \\ \$78\,039.93(0.85) &= \$66\,333.94 \end{aligned}$$

$$\text{(b) } a = \$149\,500, r = 0.85, t_n = \$149\,500(0.85)^{n-1}$$

$$\text{(c) } t_8 = \$149\,500(0.85)^7 = \$47\,926.27$$

∴ the truck has a value of \$47 926.27 at the end of the seventh year.

26. In simple interest, only the principle earns interest, whereas in compound interest, the interest earns interest as well.

$$\text{27. (a) } A = P(1 + i)^n$$

(b) A – the amount of the investment, P – the present value of the investment, i – the interest rate per compounding period, n – the number of compounding periods

$$\text{28. Solve for } P \text{ in the formula } A = P(1 + i)^n \text{ to obtain the formula } P = \frac{A}{(1 + i)^n}$$

$$\text{29. (a) i. } A = \$5000(1.06)^7 = \$7518.15$$

$$\text{ii. } I = A - P = \$7518.15 - \$5000 = \$2518.15$$

$$\text{(b) } i = \frac{0.048}{12} = 0.004, n = 4 \times 12 = 48$$

$$\text{i. } A = \$10\,500(1.004)^{48} = \$12\,717.67$$

$$\text{ii. } I = A - P = \$12\,717.67 - \$10\,500 = \$2217.67$$

$$\text{30. (a) i. } i = \frac{0.08}{4} = 0.02, n = 4 \times 6 = 24,$$

$$P = \frac{\$21\,500}{(1.02)^{24}} = \$13\,367.01$$

$$\text{ii. } I = A - P = \$21\,500 - \$13\,367.01 = \$8132.99$$

$$\text{(b) i. } i = \frac{0.05}{2} = 0.025, n = 5 \times 2 = 10,$$

$$P = \frac{\$100\,000}{(1.025)^{10}} = \$78\,119.84$$

$$\text{ii. } I = A - P = \$100\,000 - \$78\,119.84 = \$21\,880.16$$

$$\text{31. } i = \frac{0.06}{2} = 0.03, n = 5 \times 2 = 10; \therefore i = \frac{0.06}{12} = 0.005, n = 8$$

$$A = \$12\,000(1.03)^{10}(1.005)^8 = \$16\,783.48$$

∴ after eight months, the money is worth \$16 783.48.

$$\text{32. } (1 + i) = 1 + \left(\frac{8}{100} \right) \left(\frac{1}{12} \right) = \frac{151}{150}, \therefore n = 3 \times 12 = 36$$

$$P = \frac{\$20\,000}{\left(\frac{151}{150} \right)^{36}} = \$15\,745.09$$

∴ Mandy and her fiancé must invest \$15 745.09 today in order to reach their goal.

$$\text{33. (a) } x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$\text{(b) } x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x} \right)^m$$

$$\text{(c) } x^{\frac{-m}{n}} = \sqrt[n]{x^{-m}} = \frac{1}{\sqrt[n]{x^m}} = \frac{1}{\left(\sqrt[n]{x} \right)^m}$$

$$\text{34. (a) } 9^{\frac{1}{2}} = \sqrt{9} = 3$$

$$\text{(b) } (-27)^{\frac{1}{3}} = \sqrt[3]{(-27)} = -3$$

$$\text{(c) } -16^{\frac{1}{4}} = -\sqrt[4]{16} = -2$$

$$\text{(d) } 8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

$$\text{(e) } 32^{\frac{1}{5}} = \sqrt[5]{32} = 2$$

$$\text{(f) } \left(\frac{1}{64} \right)^{\frac{2}{3}} = \left(\sqrt[3]{\frac{1}{64}} \right)^2 = \left(\frac{1}{4} \right)^2 = \frac{1}{16}$$

$$\text{(g) } (27)^{\frac{-1}{3}} = 3^{-1} = \frac{1}{3}$$

$$\text{(h) } 8^{\frac{-4}{3}} = 2^{-4} = \frac{1}{16}$$

$$\text{(i) } \left(\frac{4}{9} \right)^{\frac{3}{2}} = \left(\frac{2}{3} \right)^3 = \frac{8}{27}$$

$$\text{(j) } (32)^{\frac{-4}{5}} = 2^{-4} = \frac{1}{16}$$

$$\text{35. (a) } 26^{\frac{1}{2}} \doteq 5.10$$

$$\text{(b) } (-20)^{\frac{1}{3}} \doteq -2.71$$

$$\text{(c) } -88^{\frac{1}{4}} \doteq -3.06$$

$$\text{(d) } 35^{\frac{1}{3}} \doteq 3.27$$

$$\text{(e) } 200^{\frac{1}{5}} \doteq 2.89$$

$$\text{(f) } \left(\frac{3}{4} \right)^{\frac{1}{3}} \doteq 0.91$$

$$\text{(g) } (945)^{\frac{-2}{3}} \doteq 0.01$$

$$\text{(h) } (38)^{\frac{-4}{3}} \doteq 0.01$$

$$\text{(i) } (1.35)^{\frac{3}{2}} \doteq 1.57$$

$$\text{(j) } (26.55)^{\frac{-2}{5}} \doteq 0.27$$

$$\text{36. (a) } A = \$5000(1.05)^{\frac{10}{12}} = \$5207.48$$

$$\text{(b) } i = \frac{0.06}{2} = 0.03, A = \$5500(1.03)^{\frac{33}{6}} = \$6470.94$$

$$\text{37. (a) } i = \frac{0.06}{4} = 0.015, P = \frac{\$1500}{(1.015)^{\frac{20}{13}}} = \$1466.03$$

$$\text{(b) } i = \frac{0.07}{2} = 0.035, P = \frac{\$100000}{(1.035)^{\frac{44}{6}}} = \$77\,702.94$$

38. (a) $t_n = \$1200(1.05)^n$, n years after purchase

(b) $t_{10} = \$1200(1.05)^{10} = \$1954.67$

(c) $t_{\frac{28}{12}} = \$1200(1.05)^{\frac{28}{12}} = \$1344.69$

39. product: $a^m \times a^n = a^{m+n}$, e.g. $3^2 \times 3^4 = 3^6$

quotient: $a^m \div a^n = a^{m-n}$, e.g. $5^4 \div 5^3 = 5^1$

power of a power: $(a^m)^n = a^{mn}$, e.g. $(4^4)^4 = 4^{16}$

power of a product: $(xy)^m = x^m y^m$, e.g. $(3x)^2 = 9x^2$

power of a quotient: $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$, e.g. $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$

zero as an exponent: $a^0 = 1$, e.g. $(4xy)^0 = 1$

negative exponents: $a^{-m} = \frac{1}{a^m}$, e.g. $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

rational exponents: $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$, e.g. $2^{\frac{3}{4}} = (\sqrt[4]{2})^3$

40. (a) $25^{\frac{1}{2}} + 16^{\frac{3}{4}} = \sqrt{25} + (\sqrt[4]{16})^3 = 5 + 2^3 = 13$

(b) $8^{\frac{2}{3}} - 81^{\frac{3}{4}} + 4^2 = (\sqrt[3]{8})^2 - (\sqrt[4]{81})^3 + 16 = 2^2 - 3^3 + 16 = -7$

(c) $81^{\frac{-3}{4}} + 16^{\frac{-3}{4}} - 16^{\frac{-1}{2}} = 3^{-3} + 2^{-3} - 4^{-1} = \frac{1}{27} + \frac{1}{8} - \frac{1}{4} = \frac{-19}{216}$

41. (a) $\frac{2x^2y^{-2}}{x^3} = \frac{2}{xy^2} = \frac{2}{(-3)(1)^2} = \frac{2}{-3}$

(b) $(3x^2y^{-3})^{-3} = \frac{1}{27}x^{-6}y^9 = \frac{1}{27}(-3)^{-6}1^9 = \frac{1}{19\ 683}$

(c) $\frac{3x^{-3}y^2}{x^3y^{-1}} = \frac{3y^3}{x^6} = \frac{3(1)^3}{(-3)^6} = \frac{1}{243}$

(d) $\frac{x^{-2} - y}{y^{-1} + x^2} = \frac{\frac{1}{9} - 1}{1 + 9} = \frac{\frac{-8}{9}}{10} = \frac{-8}{90} = \frac{-4}{45}$

42. (a) $(2a^2b^2)^2(3a^4b)^{-2} = \frac{4a^4b^4}{9a^8b^2} = \frac{4b^2}{9a^4}$

(b) $(-4x^2y^{-3}z^4)^2 = \frac{16x^4z^8}{y^6}$

(c) $(5x^3y^{-2})^3 \div (5x^{-2}y^3)^{-1} = \frac{125x^9}{y^6} \times 5x^{-2}y^3 = \frac{625x^7}{y^3}$

43. (a) $a^{10+2p}(a^{-p-8}) = a^{p+2}$

(b) $(2x^2)^{3-2m}\left(\frac{1}{x}\right)^{2m} = 2^{3-2m}x^{6-4m-2m} = 2^{(3-2m)}x^{(6-6m)}$

(c) $[(c)^{2n-3m}](c^3)^m \div (c^2)^n = c^{2n-3m+3m-2n} = c^0 = 1$

(d) $(x^{4n-m})\left(\frac{1}{x^3}\right)^{m+n} = x^{4n-m-3m-3n} = x^{n-4m}$

44. Examples: $2^x = 4^{x-3}$, $21 = 3^x$

45. When both sides of an exponential equation are expressible as a power of a common base, then the equation can be solved algebraically.

46. (1) Consider an exponential equation. (2) Draw the graph of $y = LS$ and $y = RS$. (3) Carefully determine the intersection point.

47. (a)

$$3^{2x-1} = 27$$

$$3^{2x-1} = 3^3$$

$$2x - 1 = 3$$

$$x = 2$$

(b)

$$(2^{2x})(2^{x-1}) = 32$$

$$2^{3x-1} = 2^5$$

$$3x - 1 = 5$$

$$x = 2$$

(c)

$$5^{3x+2} = \frac{1}{5}$$

$$5^{3x+2} = 5^{-1}$$

$$3x + 2 = -1$$

$$x = -1$$

(d)

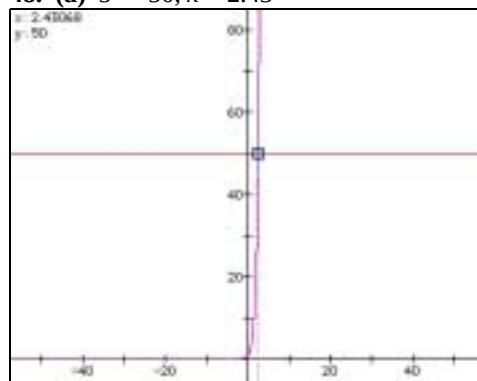
$$6^{2x-3} = \sqrt{6}$$

$$6^{2x-3} = 6^{\frac{1}{2}}$$

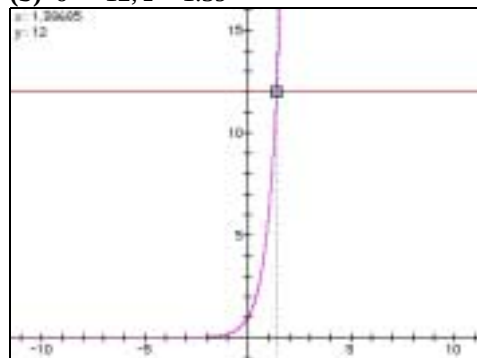
$$2x - 3 = \frac{1}{2}$$

$$x = \frac{7}{4}$$

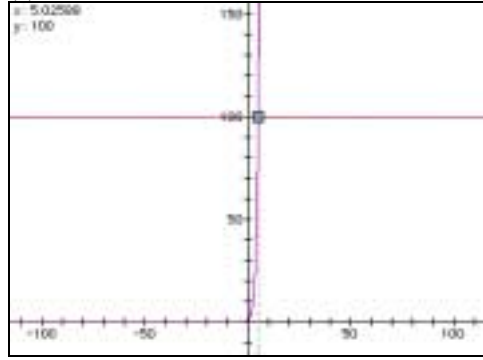
48. (a) $5^x = 50$, $x = 2.43$



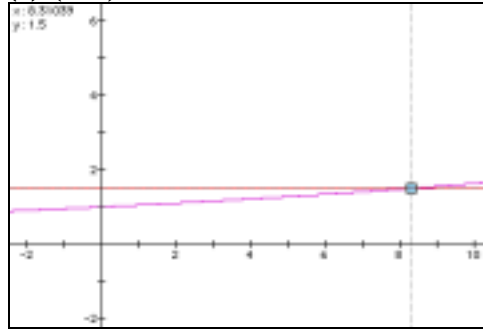
(b) $6^z = 12$, $z = 1.39$



(c) $2.5^c = 100, c = 5.03$



(d) $(1.05)^x = 1.5, x = 8.31$



49. (a)

$$(2)^{x^2} = 2^{-12-7x}$$

$$x^2 = -12 - 7x$$

$$x^2 + 7x + 12 = 0$$

$$(x+3)(x+4) = 0$$

$$x = -3, x = -4$$

(b)

$$(3^{x+2})^x = \frac{1}{3}$$

$$3^{x^2+2x} = 3^{-1}$$

$$x^2 + 2x + 1 = 0$$

$$(x+1)(x+1) = 0$$

$$x = -1$$

(c)

$$9^{x^2+1} = 3^{3x-1}3^{2x}$$

$$3^{2x^2+2} = 3^{5x-1}$$

$$2x^2 - 5x + 3 = 0$$

$$2x^2 - 2x - 3x + 3 = 0$$

$$2x(x-1) - 3(x-1) = 0$$

$$(2x-3)(x-1) = 0$$

$$x = \frac{3}{2}, x = 1$$

(d)

$$4^{x^2} = (16^{1-x})\left(\frac{1}{4}\right)^x$$

$$4^{x^2} = 4^{2-2x}4^{-x}$$

$$x^2 + 3x - 2 = 0$$

$$\left(x + \frac{3}{2}\right)^2 = 2 + \frac{9}{4}$$

$$x = \frac{\pm\sqrt{17}-3}{2}$$

50. (a) $3, 12, 48, \dots, 49152, a = 3, r = \frac{12}{3} = 4$

$$t_n = ar^{n-1}$$

$$49152 = (3)(4)^{n-1}$$

$$16384 = 4^{n-1}$$

$$\ln 16384 = (n-1)\ln 4$$

$$n = \frac{\ln 16384}{\ln 4} + 1$$

$$n = 8$$

(b) $-7, -14, -28, \dots, -3584, a = -7, r = \frac{-14}{-7} = 2$

$$t_n = ar^{n-1}$$

$$-3584 = (-7)(2)^{n-1}$$

$$512 = 2^{n-1}$$

$$\ln 512 = (n-1)\ln 2$$

$$n = \frac{\ln 512}{\ln 2} + 1$$

$$n = 10$$

51. $i = \frac{0.08}{4} = 0.02$

$$3(\$1000) = \$1000(1.02)^{4n}, n \text{ in years}$$

$$3 = (1.02)^{4n}$$

$$\ln 3 = (4n)\ln(1.02)$$

$$4n = \frac{\ln 3}{\ln(1.02)}$$

$$4n \doteq 55.48$$

$$n \doteq 13.87 \text{ years (13 years, 10 months, 13 days)}$$

$\therefore$ a \$1000 investment will triple in value in approximately 13.87 years.