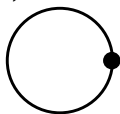


Chapter 7

Practise, Apply, Solve 7.1U, page 574

1. (a)



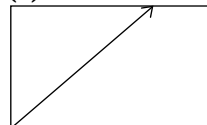
(b)



(c)

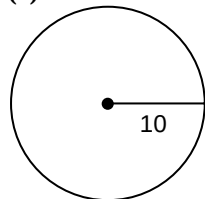


(d)

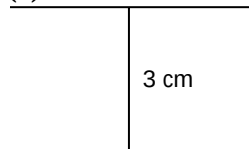


2. The locus will be:

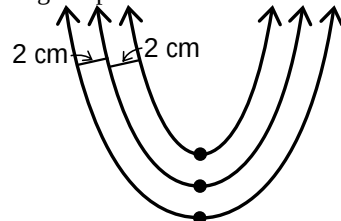
(a) A circle with radius 10 cm.



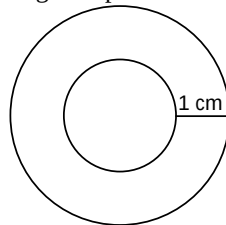
(b) A line



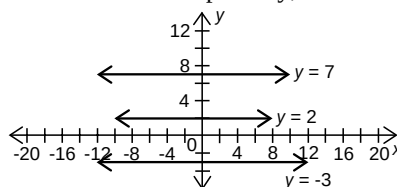
(c) Two other parabolas – one inside and one outside the original parabola.



(d) Another sphere with a radius that is 1 cm greater than the original sphere.

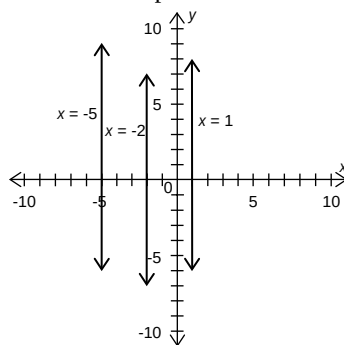


3. (a) The locus of points that are 5 units away from the line $y = 2$ would be 2 horizontal lines, one below the line and one above the line. Graphically, the locus is:



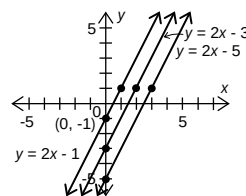
Algebraically: the two lines would be $y = 7$ and $y = -3$.

(b) The locus of points that are 3 units away from $x = -2$ would be 2 vertical lines, one to the right and one to the left of $x = -2$. Graph:



Algebraic expression: The equations will be $x = 1$ and $x = -5$.

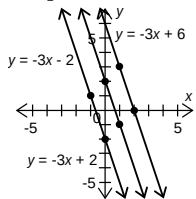
(c) Description: The locus of points 2 units from the line $y = 2x - 3$ would be 2 lines that are parallel to the original line. One would be above the original line and the other would be below the original line. Graph:



Algebraic expression: The slope of the new lines is the same as the original but the y -intercept of the new graphs will be different (± 2). New equations: $y = 2x - 5$ and $y = 2x - 1$.

(d) Description: The locus of points 4 units away from the line $y = -3x + 2$ would be 2 lines parallel to the original.

Graph:



Algebraic expression: The slope of the new lines is the same as the original but the y-intercept of the new graphs will be different (± 4). New equations: $y = -3x + 6$ and $y = -3x - 2$.

4. (a) The locus of points that are 5 units away from the origin is a circle with a radius of 5. The equation for this circle would be $x^2 + y^2 = 25$.

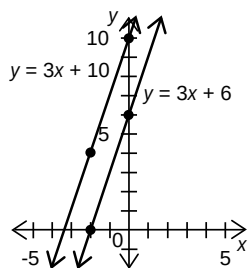
(b) Using *Geometer's Sketchpad* would show that the locus of points is a circle with radius of $5\sqrt{2}$. The equation for this circle would be $x^2 + y^2 = 100$.

5. (a) The locus of points 6 units from $(3, -1)$ will be a circle with a centre of $(3, -1)$ and a radius of 6.

(b) The locus of points 3 units from $(0, 3)$ will be a circle with a centre of $(0, 3)$ and a radius of 3.

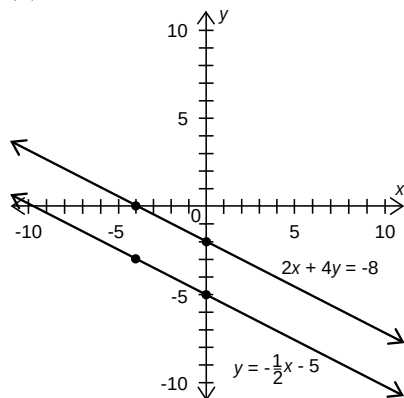
(c) The locus of points 4 units from $(2, 3)$ will be a circle with centre $(2, 3)$ and a radius of 4.

6. (a) i.



ii. Equation of locus 4 units above $y = 3x + 6$ is $y = 3x + 10$.

(b) i.



ii. To graph $2x + 4y = -8$, you could find the x- and y-intercepts. x-intercept:

$$2x = -8$$

$$x = -4$$

$\therefore$ x-intercept is -4 .

y-intercept:

$$4y = -8$$

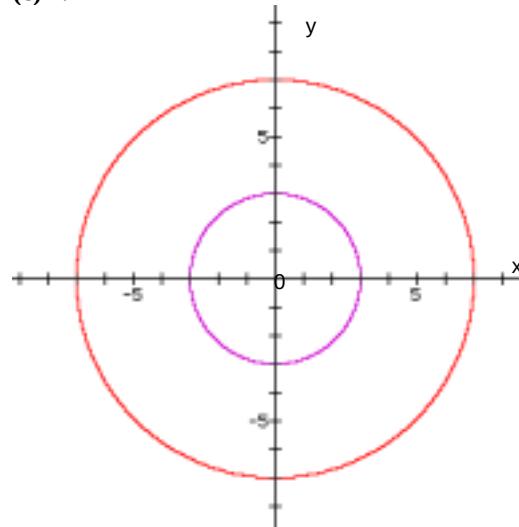
$$y = -2$$

$\therefore$ y-intercept is -2 .

The graph of the locus of points 3 units below the given line has a y-intercept of -5 and has the same slope as the original.

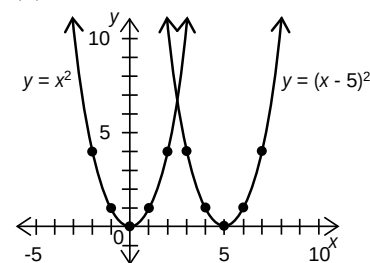
Therefore, the equation of the new line is $y = -\frac{1}{2}x - 5$.

(c) i.



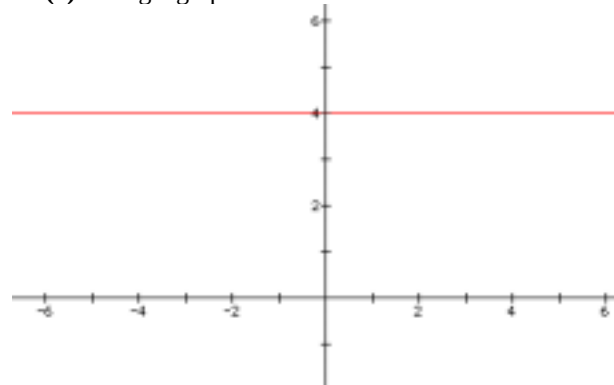
ii. The locus of points 4 units from the circumference of a circle centred at the origin with a radius of 3 would be a circle with radius of 7 and centred at the origin. The equation will be: $x^2 + y^2 = 49$.

(d) i.



ii. The locus of points 5 units to the right of $y = x^2$ will be another parabola with the same shape as $y = x^2$ and with a vertex of $(5, 0)$. The equation will be: $y = (x - 5)^2$.

7. (a) Using a graph:



We can see that the locus of points equidistant from $(0, 2)$ and $(0, 6)$ is a line with the equation $y = 4$.

Algebraic solution: To find points equidistant from $A(0, 2)$ and $B(0, 6)$, let $P(x, y)$ be a locus point. Equate AP and PB .

$$AP = PB$$

$$\sqrt{x^2 + (y-2)^2} = \sqrt{x^2 + (y-6)^2}$$

$$x^2 + (y-2)^2 = x^2 + (y-6)^2$$

$$x^2 + y^2 - 4y + 4 = x^2 + y^2 - 12y + 36$$

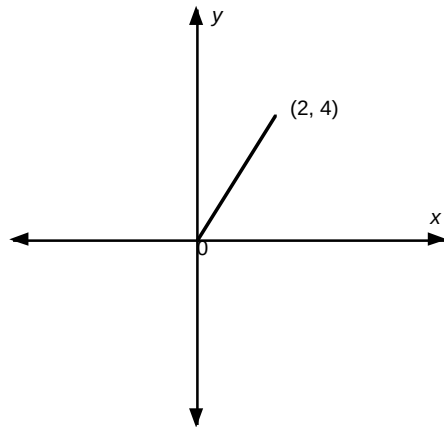
$$8y = 32$$

Collect like terms.

$$y = 4$$

Therefore, $y = 4$ is the equation of the locus of points.

(b) Using a graph:



The locus of points that are equidistant from $(0, 0)$ and $(2, 4)$ would be the perpendicular bisector of the line segment connecting $(0, 0)$ and $(2, 4)$. One way to approach this is to find the midpoint of the line segment as well as its slope. We can then use this information to find the equation of the perpendicular bisector. The midpoint of the segment is $(1, 2)$. The slope of the segment is 2. Therefore, the perpendicular bisector is the line that passes through the point $(1, 2)$ and has a slope of $-\frac{1}{2}$ (the negative reciprocal of 2).

$$\therefore y = mx + b \quad \text{Substitute } m = -\frac{1}{2}.$$

$$y = -\frac{1}{2}x + b \quad \text{Now, substitute } (1, 2).$$

$$2 = -\frac{1}{2} + b$$

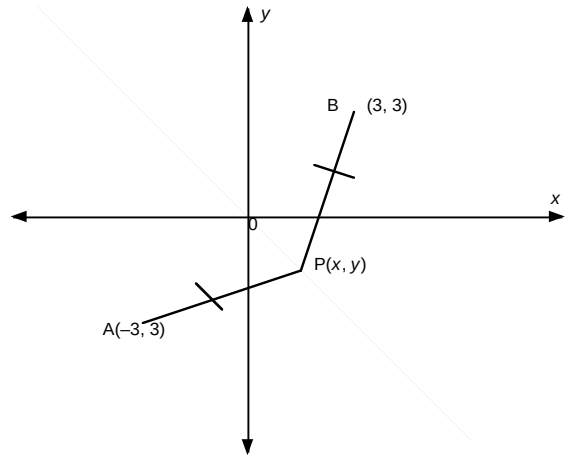
$$2 + \frac{1}{2} = b$$

$$\frac{5}{2} = b$$

$$\therefore \text{the equation of the locus is } y = -\frac{1}{2}x + \frac{5}{2}.$$

(c) The points that are equidistant from the points $(3, 3)$ and $(-3, -3)$ would form a line.

Graph:



The points on the line are equidistant from the given points.

Let $P(x, y)$ be a locus point.

$$AP = AB$$

Use distance formula.

$$\sqrt{(x+3)^2 + (y+3)^2} = \sqrt{(x-3)^2 + (y-3)^2}$$

Square both sides.

$$(x+3)^2 + (y+3)^2 = (x-3)^2 + (y-3)^2$$

Expand.

$$x^2 + 6x + 9 + y^2 + 6y + 9 = x^2 - 6x + 9 + y^2 - 6y + 9$$

Collect like terms.

$$x^2 + y^2 + 6x + 6y + 18 = x^2 + y^2 - 6x - 6y + 18$$

$$12x + 12y = 0$$

Divide by 12.

$$x + y = 0$$

$\therefore$ the equation is $x + y = 0$.

Another solution would be to recognize that the locus of points is the perpendicular bisector of the segment between the given points. Since $y = x$ is the line through A and B, it could be seen from the diagram that the perpendicular bisector of AB would be the line $y = -x$.

(d) To find the locus of points equidistant from 2 points, you could use the method used in 7**(b)** or the method used in 7**(c)**.

Let $P(x, y)$ be a point in the locus

$$\sqrt{(x-1)^2 + (y-5)^2} = \sqrt{(x+2)^2 + (y-6)^2}$$

$$(x-1)^2 + (y-5)^2 = (x+2)^2 + (y-6)^2$$

$$x^2 - 2x + 1 + y^2 - 10y + 25 = x^2 + 4x + 4 + y^2 - 12y + 36$$

$$-2x - 10y + 26 = 4x - 12y + 40$$

$$-6x + 2y - 14 = 0$$

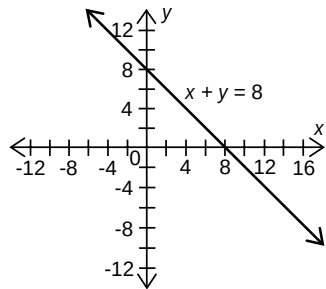
$$-3x + y - 7 = 0$$

$\therefore -3x + y - 7 = 0$ is the equation of the locus of points.

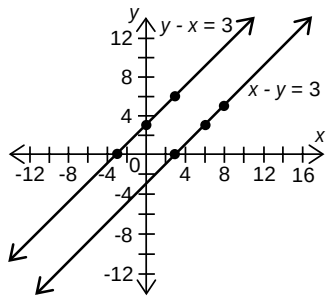
(e) The locus of points that are parallel to $y = 2x + 6$ and have a y-intercept of 3 is the line $y = 2x + 3$.

(f) The locus of points that are perpendicular to $y = 3x + 7$ and passes through the origin is the line $y = -\frac{1}{3}x$.

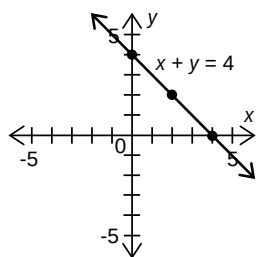
8. (a) $P(x, y)$ is a point such that $x + y = 8$.



- (b) $P(x, y)$ is a point such that $x - y = 3$ or $y - x = 3$.

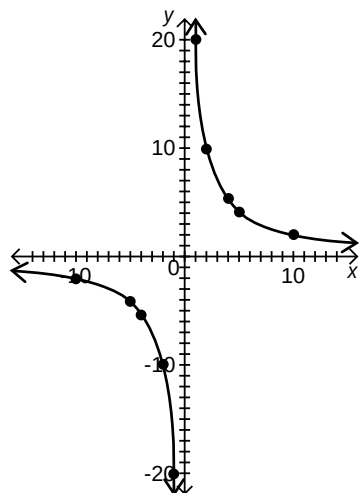


- (c) $P(x, y)$ is a point such that the average of x and y is 2. In other words, $\frac{x+y}{2} = 2$ or simplifying $x + y = 4$.



- (d) $P(x, y)$ is a point such that $xy = 20$. A table of values will help to determine the graph of this relation.

x	y
5	4
2	10
4	5
-4	-5
-5	-4
1	20
-1	-20



9. Multiple solutions are possible.

(a) $y = x$ is the locus of points that are equidistant from the positive x and y axes or from the negative x - and y - axes.

(b) $x^2 + y^2 = 5$ represents the locus of points that are $\sqrt{5}$ units from the origin.

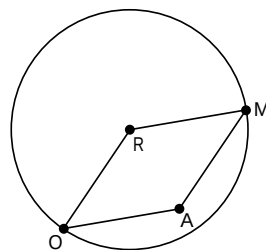
(c) $x + y = 5$ is the locus of points $P(x, y)$ whose coordinates have a sum of 5.

(d) $x = -2$ could be the locus of points 2 units to the left of the y -axis.

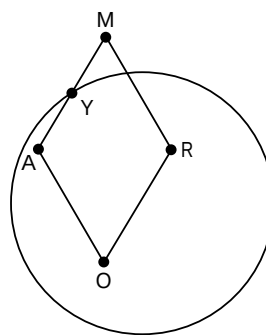
(e) $y = 5$ could be defined as the locus of points 2 units above the line $y = 3$.

(f) $y = \sqrt{16 - x^2}$ could be defined as the locus of points above the x -axis that are 4 units away from the origin.

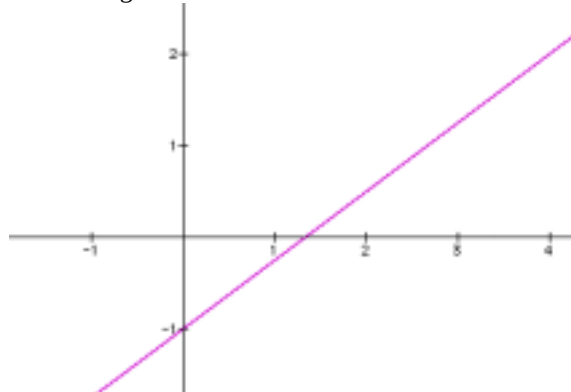
10. The path of M will be a circle centred at R .



The path of Y is also a circle with the midpoint of RO as the centre.



11. A diagram illustrates the situation.



The locus will be a line through $(4, 2)$ with a slope of $\frac{3}{4}$.

$$\therefore y = mx + b \quad \text{Substitute } m = \frac{3}{4}.$$

$$y = \frac{3}{4}x + b \quad \text{Substitute } (4, 2).$$

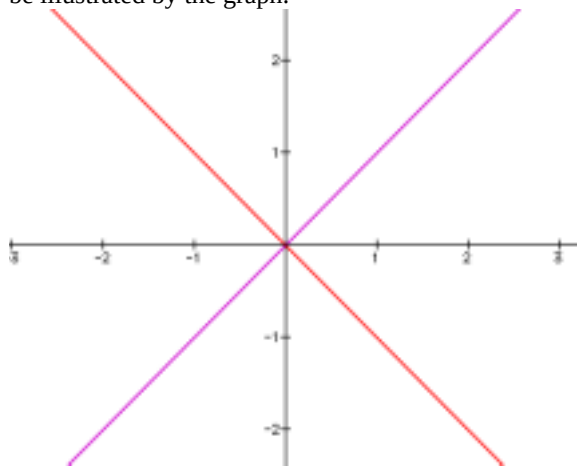
$$2 = \frac{3}{4}(4) + b \quad \text{Solve for } b.$$

$$2 = 3 + b$$

$$-1 = b$$

$$\therefore y = \frac{3}{4}x - 1 \text{ is the equation of the locus.}$$

12. The points that are equidistant from the x - and y -axes can be illustrated by the graph.



The locus is 2 lines as shown. The equation of one line is $y = x$ and the equation of the other line is $y = -x$.

13. Drawing a graph of the situation helps to solve the problem. To graph the original equation $y = x^2 + 2x + 4$, it helps to put this in a form that will yield the vertex.

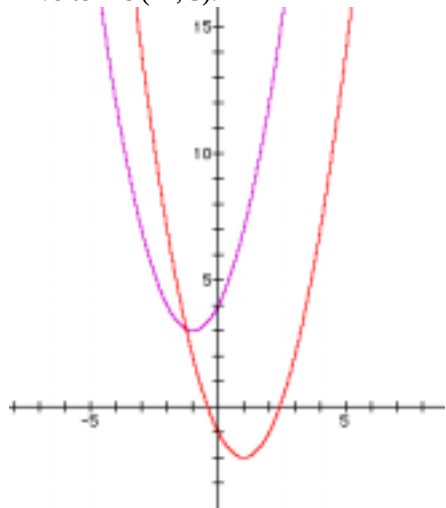
$$y = x^2 + 2x + 4$$

$$y = x^2 + 2x + 1 - 1 + 4$$

$$y = x^2 + 2x + 1 + 3$$

$$y = (x + 1)^2 + 3$$

$$\therefore \text{vertex is } (-1, 3).$$

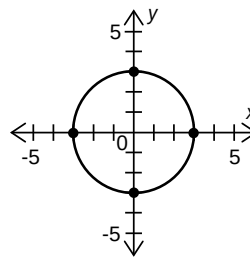


The locus of points lying 2 units to the right and 5 units below the curve is illustrated by the red line. As shown on the

graph, the vertex is $(1, -2)$ and the graph has the same shape as the given curve. Therefore, the equation of the locus is $y = (x - 1)^2 - 2$.

14. $P(x, y)$ is such that the sum of the squares of the co-ordinates is 9.

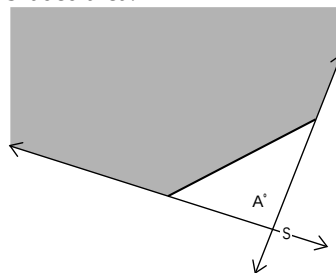
(a)



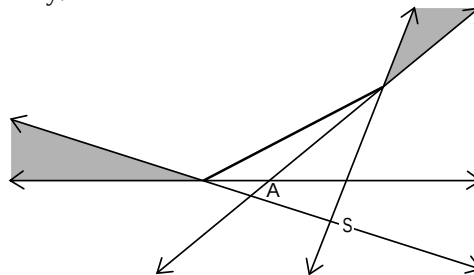
(b) Algebraically, the relationship would be $x^2 + y^2 = 9$.

15. Answers will vary.

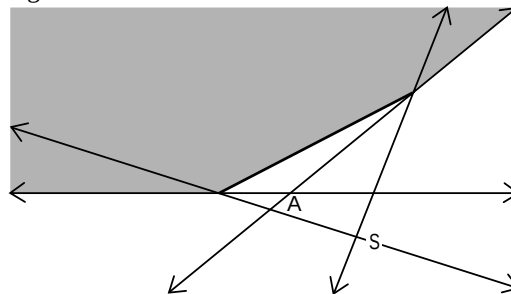
16. (a) Neither Amy nor Sara can be seen from within the shaded area.



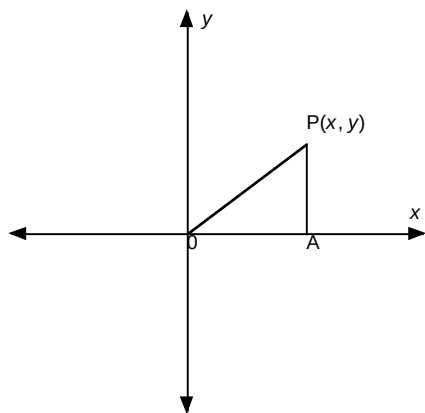
(b) From within the shaded regions, Sara can be seen but not Amy.



(c) They can both be seen from the area outside the shaded regions.



17. (a) The locus will be the y -axis.



Let $P(x, y)$ be a point such that the distance from the origin is the same as the distance from the x -axis.

$$\therefore OP = OA \quad \text{Using the distance formula.}$$

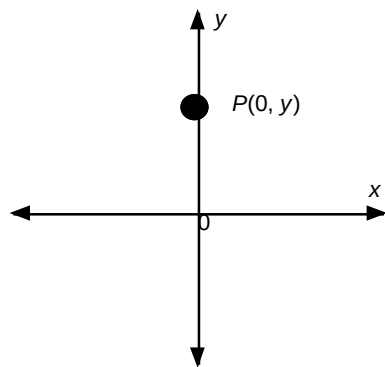
$$\sqrt{x^2 + y^2} = y \quad \text{Squaring both sides.}$$

$$x^2 + y^2 = y^2 \quad \text{Simplifying.}$$

$$x^2 = 0$$

$\therefore x = 0$ is the equation of the locus.

There is no restriction on y . Notice that any point on the y -axis is the same distance from the origin as from the x -axis.



(b) The locus of points equidistant from $(-2, 1)$ and $(6, 3)$ is a line that is the perpendicular bisector of the segment joining $(-2, 1)$ and $(6, 3)$.

To find the equation, several methods could be used. In this solution, the midpoint of the segment will be found, and the slope of the segment will be found. Then the equation of the line will be derived by finding the equation of a line that passes through the midpoint and has a slope that is the negative reciprocal of the slope of the segment.

Midpoint of Line Segment:

$$\text{midpoint} = \left(\frac{-2+6}{2}, \frac{1+3}{2} \right) = (2, 2)$$

$$\text{Slope of Segment} = \frac{3-1}{6-(-2)} = \frac{2}{8} = \frac{1}{4}$$

Find the equation of the line through $(2, 2)$ with a slope of -4 .

$$y = -4x + b$$

$$2 = -4(2) + b$$

$$2 = -8 + b$$

$$10 = b$$

$\therefore$ the equation is $y = -4x + 10$.

(c) The locus of points equidistant from the lines $y = 4$ and $y = -2$ will be a line. The distance between the lines is 6 units, therefore the locus of points will be 3 units away from both lines. The equation of this line is $y = 1$.

(d) The locus of points equidistant from $A(3, 4)$ and $B(7, 8)$ is a line. Several methods could be used to find the equation. The distance formula will be used in this solution. Let $P(x, y)$ be a locus point.

$$AP = PB$$

Use distance formula.

$$\sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x-7)^2 + (y-8)^2}$$

Square both sides.

$$(x-3)^2 + (y-4)^2 = (x-7)^2 + (y-8)^2$$

Expand.

$$x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 - 14x + 49 + y^2 - 16y + 64$$

Simplify.

$$x^2 + y^2 - 6x - 8y + 25 = x^2 + y^2 - 14x - 16y + 113$$

Simplify further.

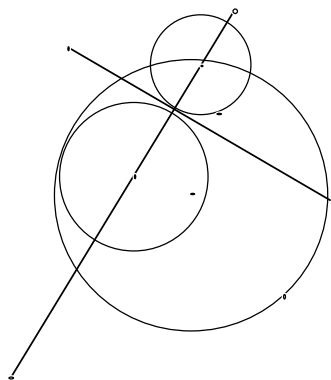
$$-6x - 8y + 25 = -14x - 16y + 113$$

$$8x + 8y - 88 = 0$$

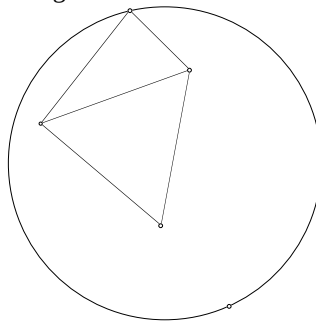
Divide by 8.

$\therefore x + y - 11 = 0$ is the equation of the locus.

18. The path of the point will be a larger circle that has a diameter that is equal to the sum of the diameters of the other 2 circles.

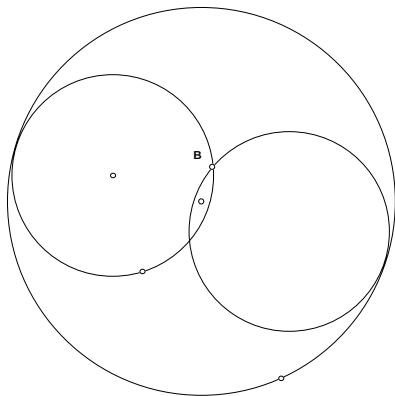


19. The object that will be traced will be a circle. The centre of the circle will be at the orthocentre of the equilateral triangle.



20. Using *The Geometer's Sketchpad*, construct a circle. Choose a point on the circle and connect that to the centre to form a radius. Construct the smaller circle inside the first by

finding the midpoint of the radius of the larger circle. Use this midpoint as the centre of the new smaller circle. Place a point on the smaller circle and choose “Trace point.” Move the circle around the larger by moving the common point of the 2 circles. B will trace a smaller circle still inside the first large circle. The locus of B is another circle with a radius of r , also inside the first circle.

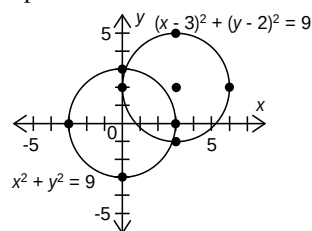


Practise, Apply, Solve 7.2U, page 581

1. (a) $x^2 + (y - 2)^2 = 16$
- (b) $(x - 3)^2 + y^2 = 1$
- (c) $(x - 1)^2 + (y - 5)^2 = 81$
- (d) $(x + 2)^2 + (y - 4)^2 = 64$
- (e) $(x + 2)^2 + (y + 5)^2 = 6$
- (f) $(x - 4)^2 + (y + 6)^2 = 20$

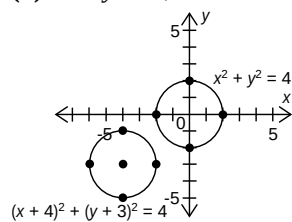
2. (a) Centre is $(0, 0)$ Radius is $\sqrt{18}$ or $3\sqrt{2}$
- (b) Centre is $(2, 0)$ Radius is 3
- (c) Centre is $(6, -8)$ Radius is 9
- (d) Centre is $(-4, 3)$ Radius is $\sqrt{32}$ or $4\sqrt{2}$
- (e) Centre is $(5, 4)$ Radius is $\sqrt{27}$ or $3\sqrt{3}$
- (f) Centre is $(1, -6)$ Radius is $\sqrt{40}$ or $2\sqrt{10}$

3. (a) $x^2 + y^2 = 9$, translated 3 units to the right and 2 units up.



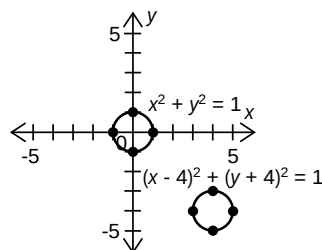
The new equation is $(x - 3)^2 + (y - 2)^2 = 9$.

- (b) $x^2 + y^2 = 4$, translated 4 units to the left, 3 units down.



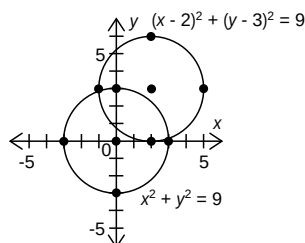
The new equation is $(x + 4)^2 + (y + 3)^2 = 4$.

- (c) $x^2 + y^2 = 1$, translated 4 units to the right and 4 units down.



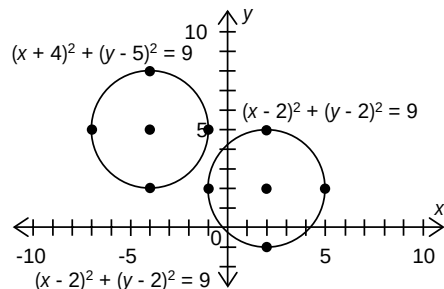
The new equation is $(x - 4)^2 + (y + 4)^2 = 1$.

- (d) $(x - 2)^2 + (y - 3)^2 = 9$, translated 2 units to the left and 3 units down.



The new equation is $x^2 + y^2 = 9$.

- (e) $(x + 4)^2 + (y - 5)^2 = 9$, translated 6 units to the right and 3 units down.



The new equation is $(x - 2)^2 + (y - 2)^2 = 9$.

4. (a) $x^2 + y^2 = 25$ standard form
 $x^2 + y^2 - 25 = 0$ general form
- (b) $(x - 1)^2 + (y - 4)^2 = 16$ standard form
 $x^2 - 2x + 1 + y^2 - 8y + 16 = 16$
 $x^2 + y^2 - 2x - 8y + 1 = 0$ general form
- (c) $(x + 4)^2 + (y - 5)^2 = 1$ standard form
 $x^2 + 8x + 16 + y^2 - 10y + 25 = 1$
 $x^2 + y^2 + 8x - 10y + 40 = 0$ general form
- (d) $(x + 2)^2 + (y - 3)^2 = 9$ standard form
 $x^2 + 4x + 4 + y^2 - 6y + 9 = 9$
 $x^2 + y^2 + 4x - 6y + 4 = 0$ general form

5. (a) $x^2 + 8x + y^2 + 6y = 0$
 $x^2 + 8x + 16 + y^2 + 6y + 9 = 16 + 9$
 $(x + 4)^2 + (y + 3)^2 = 25$ standard form
- (b) $x^2 + y^2 - 4x + 2y - 11 = 0$
 $x^2 - 4x + 4 + y^2 + 2y + 1 = 11 + 1 + 4$
 $(x - 2)^2 + (y + 1)^2 = 16$ standard form
- (c) $x^2 + y^2 - 2x + 10y = -22$
 $x^2 - 2x + 1 + y^2 + 10y + 25 = -22 + 1 + 25$
 $(x - 1)^2 + (y + 5)^2 = 4$ standard form
- (d) $x^2 + y^2 - 12x + 6y = 4$
 $x^2 - 12x + 36 + y^2 + 6y + 9 = 4 + 36 + 9$
 $(x - 6)^2 + (y + 3)^2 = 49$ standard form