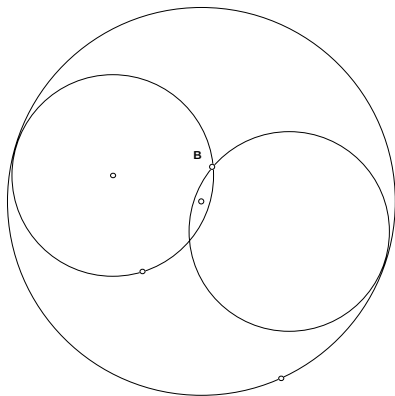


finding the midpoint of the radius of the larger circle. Use this midpoint as the centre of the new smaller circle. Place a point on the smaller circle and choose “Trace point.” Move the circle around the larger by moving the common point of the 2 circles. B will trace a smaller circle still inside the first large circle. The locus of B is another circle with a radius of r , also inside the first circle.

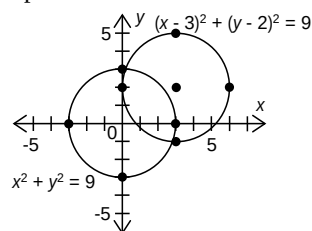


Practise, Apply, Solve 7.2U, page 581

1. (a) $x^2 + (y - 2)^2 = 16$
- (b) $(x - 3)^2 + y^2 = 1$
- (c) $(x - 1)^2 + (y - 5)^2 = 81$
- (d) $(x + 2)^2 + (y - 4)^2 = 64$
- (e) $(x + 2)^2 + (y + 5)^2 = 6$
- (f) $(x - 4)^2 + (y + 6)^2 = 20$

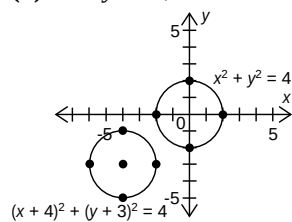
2. (a) Centre is $(0, 0)$ Radius is $\sqrt{18}$ or $3\sqrt{2}$
- (b) Centre is $(2, 0)$ Radius is 3
- (c) Centre is $(6, -8)$ Radius is 9
- (d) Centre is $(-4, 3)$ Radius is $\sqrt{32}$ or $4\sqrt{2}$
- (e) Centre is $(5, 4)$ Radius is $\sqrt{27}$ or $3\sqrt{3}$
- (f) Centre is $(1, -6)$ Radius is $\sqrt{40}$ or $2\sqrt{10}$

3. (a) $x^2 + y^2 = 9$, translated 3 units to the right and 2 units up.



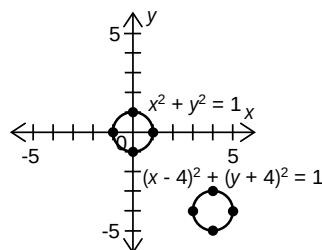
The new equation is $(x - 3)^2 + (y - 2)^2 = 9$.

- (b) $x^2 + y^2 = 4$, translated 4 units to the left, 3 units down.



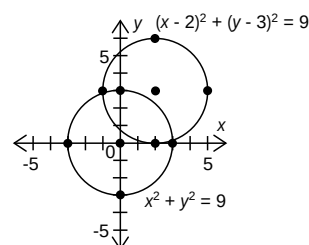
The new equation is $(x + 4)^2 + (y + 3)^2 = 4$.

- (c) $x^2 + y^2 = 1$, translated 4 units to the right and 4 units down.



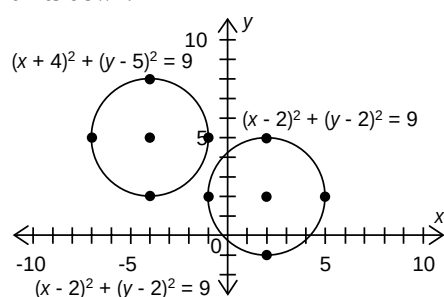
The new equation is $(x - 4)^2 + (y + 4)^2 = 1$.

- (d) $(x - 2)^2 + (y - 3)^2 = 9$, translated 2 units to the left and 3 units down.



The new equation is $x^2 + y^2 = 9$.

- (e) $(x + 4)^2 + (y - 5)^2 = 9$, translated 6 units to the right and 3 units down.



The new equation is $(x - 2)^2 + (y - 2)^2 = 9$.

4. (a) $x^2 + y^2 = 25$ standard form
 $x^2 + y^2 - 25 = 0$ general form
- (b) $(x - 1)^2 + (y - 4)^2 = 16$ standard form
 $x^2 - 2x + 1 + y^2 - 8y + 16 = 16$
 $x^2 + y^2 - 2x - 8y + 1 = 0$ general form
- (c) $(x + 4)^2 + (y - 5)^2 = 1$ standard form
 $x^2 + 8x + 16 + y^2 - 10y + 25 = 1$
 $x^2 + y^2 + 8x - 10y + 40 = 0$ general form
- (d) $(x + 2)^2 + (y - 3)^2 = 9$ standard form
 $x^2 + 4x + 4 + y^2 - 6y + 9 = 9$
 $x^2 + y^2 + 4x - 6y + 4 = 0$ general form

5. (a) $x^2 + 8x + y^2 + 6y = 0$
 $x^2 + 8x + 16 + y^2 + 6y + 9 = 16 + 9$
 $(x + 4)^2 + (y + 3)^2 = 25$ standard form
- (b) $x^2 + y^2 - 4x + 2y - 11 = 0$
 $x^2 - 4x + 4 + y^2 + 2y + 1 = 11 + 1 + 4$
 $(x - 2)^2 + (y + 1)^2 = 16$ standard form
- (c) $x^2 + y^2 - 2x + 10y = -22$
 $x^2 - 2x + 1 + y^2 + 10y + 25 = -22 + 1 + 25$
 $(x - 1)^2 + (y + 5)^2 = 4$ standard form
- (d) $x^2 + y^2 - 12x + 6y = 4$
 $x^2 - 12x + 36 + y^2 + 6y + 9 = 4 + 36 + 9$
 $(x - 6)^2 + (y + 3)^2 = 49$ standard form

6. (a) $x^2 + y^2 - 4y = 0$

$$x^2 + y^2 - 4y + 4 = 4$$

$$x^2 + (y - 2)^2 = 4$$

Centre is (0, 2) and radius is 2.

(b) $4x^2 + 4y^2 = 16$

$$x^2 + y^2 = 4$$

Centre is (0, 0) and radius is 2.

(c) $2x^2 + 2y^2 - 6x + 4y + 1 = 0$

$$x^2 + y^2 - 3x + 2y + \frac{1}{2} = 0$$

$$x^2 - 3x + y^2 + 2y = -\frac{1}{2}$$

Complete the square.

Factor.

Divide by 4.

Divide by 2.

Rearrange.

Complete the squares.

$$x^2 - 3x + \frac{9}{4} + y^2 + 2y + 1 = -\frac{1}{2} + \frac{9}{4} + 1$$

$$\left(x - \frac{3}{2}\right)^2 + (y + 1)^2 = \frac{11}{4}$$

Centre is $\left(\frac{3}{2}, -1\right)$ Radius is $\frac{\sqrt{11}}{2}$.

(d) $x^2 + y^2 - 6x + 2y + 1 = 0$

$$x^2 - 6x + y^2 + 2y = -1$$

$$x^2 - 6x + 9 + y^2 + 2y + 1 = -1 + 9 + 1$$

$$(x - 3)^2 + (y + 1)^2 = 9$$

Centre is (3, -1) Radius is 3.

(e) $x^2 + y^2 - 8x + 6y = 11$

$$x^2 - 8x + y^2 + 6y = 11$$

$$x^2 - 8x + 16 + y^2 + 6y + 9 = 11 + 16 + 9$$

$$(x - 4)^2 + (y + 3)^2 = 36$$

Centre is (4, -3) Radius is 6.

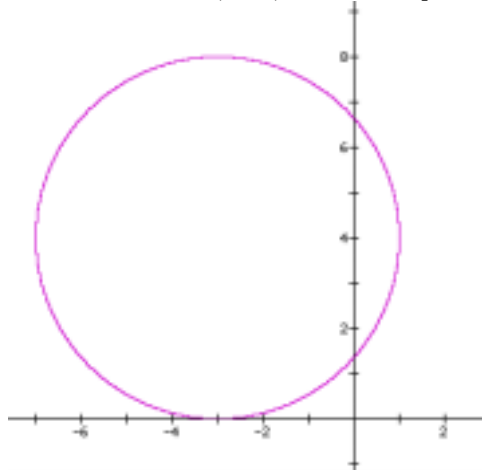
7. (a) The locus of points 2 units away from the origin would be a circle with a centre at (0, 0) and a radius of 2. The equation would be $x^2 + y^2 = 4$.

(b) The locus of points 3 units away from (-1, 2) would be a circle with centre (-1, 2) and radius 3. The equation would be $(x + 1)^2 + (y - 2)^2 = 9$.

(c) The equation for the locus of points $\sqrt{5}$ units away from (0, 4) would be $x^2 + (y - 4)^2 = 5$.

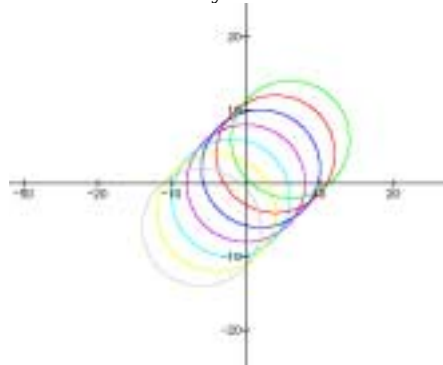
(d) The equation for the locus of points $2\sqrt{6}$ units away from (-2, 7) is $(x + 2)^2 + (y - 7)^2 = 24$.

8. To find the equation of the circle that is tangent to the x-axis and has centre (-3, 4), it would help to draw a diagram.



Since the centre is (-3, 4), the equation of the circle would be $(x + 3)^2 + (y - 4)^2 = r^2$. Since the circle touches the x-axis 4 units from the centre, the radius is 4. Therefore, the equation is $(x + 3)^2 + (y - 4)^2 = 16$.

9. Call the centre of the circles (a, a), since the x and y co-ordinates should be equal. Since the radius is 8, the equations of these circles will all be of the form: $(x - a)^2 + (y - a)^2 = 64$. Graphically, the circles' centres will all be on the line $y = x$. A sketch of this family of circles would look like this:



10. (a) If the diameter has endpoints A(-2, 4) and B(2, 8), then the centre of the circle is the midpoint of AB, which is C(0, 6). The radius of the circle will be the distance from C(0, 6) to either A or B.

$$\begin{aligned} \text{The distance from A to C} &= \sqrt{(0 - (-2))^2 + (6 - 4)^2} \\ &= \sqrt{4 + 4} \end{aligned}$$

$$= \sqrt{8}$$

$$\therefore \text{the equation is } x^2 + (y - 6)^2 = 8.$$

(b) First, find the midpoint of AB to find the centre of the circle. The midpoint C is (2, -2). The radius is the distance from either A or B to C.

$$\begin{aligned} \text{The distance from A to C} &= \sqrt{(2 - (-2))^2 + (-2 - 4)^2} \\ &= \sqrt{1^2 + (-4)^2} \\ &= \sqrt{1 + 16} \\ &= \sqrt{17} \end{aligned}$$

$$\therefore \text{the equation of the circle is } (x - 2)^2 + (y + 2)^2 = 17.$$

(c) If the centre is (0, 0) and the x-intercept is 8, this gives us the point (8, 0). Thus, the radius is 8. The equation is $x^2 + y^2 = 64$.

(d) If the centre is (0, 2) and the y-intercept is -2, this gives us the point (0, -2). Thus the radius is 4, i.e. the distance between the two points:

$$\begin{aligned} d &= \sqrt{(0 - 0)^2 + (2 + 2)^2} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

Thus the equation is $x^2 + (y - 2)^2 = 16$.

(e) If the centre is (-1, 2) the equation is of the form: $(x + 1)^2 + (y - 2)^2 = r^2$. The x-intercept is 3, so to find r^2 , substitute (3, 0) in the equation:

$$(4)^2 + (-2)^2 = r^2$$

$$20 = r^2$$

$$r = \sqrt{20}$$

$$r = 2\sqrt{5}$$

$\therefore$ the equation is $(x + 1)^2 + (y - 2)^2 = 20$.

11. First, put $x^2 + y^2 - 8x + 6y + 30 = 0$ into standard form:

$$x^2 + y^2 - 8x + 6y + 30 = 0$$

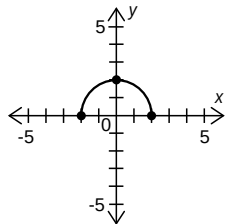
$$x^2 - 8x + 16 + y^2 + 6y + 9 = -30 + 16 + 9$$

$$(x - 4)^2 + (y + 3)^2 = -5$$

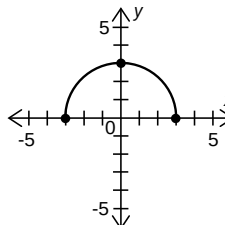
Since r^2 cannot be a negative number,

$x^2 + y^2 - 8x + 6y + 30 = 0$ is not a circle.

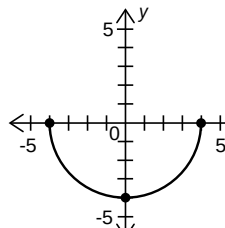
12. (a) Semi-circle with centre at origin and radius of 2, and above x-axis.



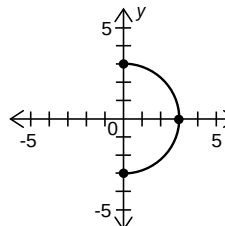
(b) Semi-circle with centre at the origin, radius of 3, and above the x-axis.



(c) Semi-circle with centre at the origin and radius of 4, and below the x-axis.



(d) Semi-circle with centre at origin, radius of 3, and to the right of the y-axis since x must be positive.



(e) Rearranging the equation $y = 2 + \sqrt{9 - x^2}$:

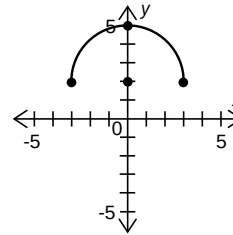
$$y - 2 = \sqrt{9 - x^2}$$

$$(y - 2)^2 = 9 - x^2$$

$$x^2 + (y - 2)^2 = 9$$

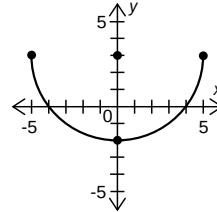
This will be a semi-circle with centre (0, 2) and radius 3.

Since there is a positive sign in front of the radical it is the top half of the circle.



(f) Rearranging the equation will yield $x^2 + (y - 3)^2 = 25$.

This is a semi-circle with centre (0, 3) and radius 5. Because of the negative sign in front of the radical sign it will be the lower part of the circle.



13. (a) $x^2 + y^2 = r^2$

$$3^2 + 4^2 = r^2$$

$$25 = r^2$$

$\therefore$ the equation is $x^2 + y^2 = 25$.

(b) $(x - 2)^2 + (y - 3)^2 = r^2$

$$(4 - 2)^2 + (7 - 3)^2 = r^2$$

$$2^2 + 4^2 = r^2$$

$$20 = r^2$$

$\therefore (x - 2)^2 + (y - 3)^2 = 20$ is the equation.

(c) $x^2 + y^2 = r^2$

$$1^2 + 2^2 = r^2$$

$$5 = r^2$$

$\therefore$ the equation is $x^2 + y^2 = 5$.

(d) $(x + 1)^2 + y^2 = r^2$

$$(3 + 1)^2 + 1^2 = r^2$$

$$4^2 + 1^2 = r^2$$

$$17 = r^2$$

$\therefore$ the equation is $(x + 1)^2 + y^2 = 17$.

(e) $(x - 8)^2 + (y - 2)^2 = r^2$

$$(1 - 8)^2 + (4 - 2)^2 = r^2$$

$$(-7)^2 + 2^2 = r^2$$

$$49 + 4 = r^2$$

$$53 = r^2$$

$\therefore$ the equation is $(x - 8)^2 + (y - 2)^2 = 53$.

14. The equation of the circle is $(x - 1)^2 + (y - 2)^2 = 25$. If the chord is 4 units above the centre, then the y-coordinate of the endpoints of the chord is 6. Now, substitute $y = 6$ into the equation of the circle:

$$(x - 1)^2 + (6 - 2)^2 = 25$$

$$(x - 1)^2 + 16 = 25$$

$$(x - 1)^2 = 9$$

$$x - 1 = \pm 3$$

$$x = 1 \pm 3$$

The 2 endpoints are (4, 6) and (-2, 6). Now, to find the length of the chord, use the distance formula. So,

$$\begin{aligned} d &= \sqrt{(-2-4)^2 + (6-6)^2} \\ &= \sqrt{(-6)^2} \\ &= 6 \end{aligned}$$

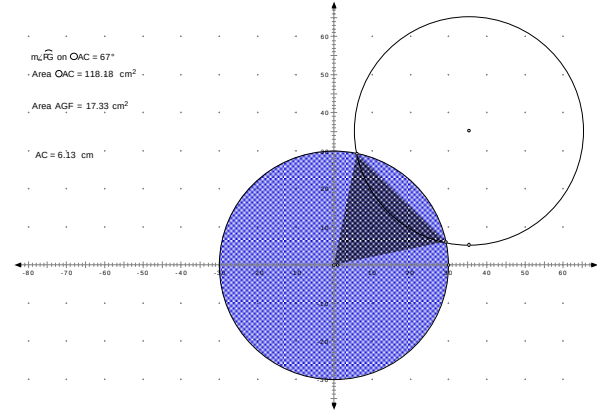
Thus, the length of the chord is 6 units.

15. (a) Answers will vary.

(b) $x^2 - 8x + 16 + y^2 + 6y + 9 = -13 + 16 + 9$
 $(x-4)^2 + (y+3)^2 = 12$

(c) The standard form will give the centre and radius.

16.



The sector is $\frac{67}{360}$ of the circle. Therefore, the area of the

sector is 12.37 cm^2 . The area of the triangle is 9.75 cm^2 . Therefore, the difference of 2.62 represents half the area that we need. Therefore, the area is 5.24. The scale on this diagram is $4.6 \text{ cm} = 30 \text{ km}$, or $1 \text{ km} = 6.52 \text{ km}$. Therefore, the area required is approximately 222.75 km^2 .

To find the centre of the 2nd tower,

$$x^2 + y^2 = 50^2$$

Since x and y are equal, we can replace them with a ,

$$a^2 + a^2 = 50^2$$

$$2a^2 = 2500$$

$$a^2 = 1250$$

$$a = \sqrt{1250}$$

$$a \doteq 35$$

$\therefore$ the centre is approximately (35, 35).

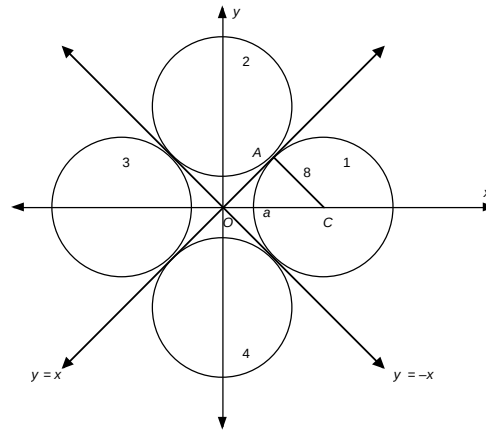
The equation is $(x-35)^2 + (y-35)^2 = 900$.

(a) Yes, you are within the reach of the second tower.

(b) (from diagram) about 222.75 km^2

(c) (from diagram) 20.01 km

17. A diagram will help explain the problem:



In $\triangle AOC$,

$$\sin 45^\circ = \frac{8}{a}$$

$$a = \frac{8}{\sin 45^\circ}$$

$$a = \frac{8}{\frac{1}{\sqrt{2}}}$$

$$\therefore a = 8\sqrt{2}$$

Since the radius of all 4 circles is 8, all of the circles can be expressed in the following form:

$$(x-a)^2 + (y-b)^2 = 64$$

As shown in the diagram, the equations of the circles are:

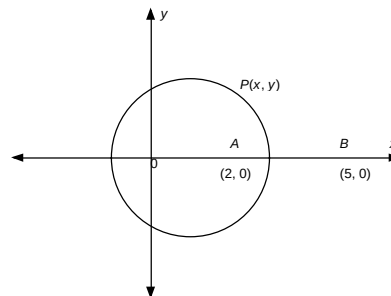
$$1. (x-8\sqrt{2})^2 + y^2 = 64$$

$$2. x^2 + (y-8\sqrt{2})^2 = 64$$

$$3. (x+8\sqrt{2})^2 + y^2 = 64$$

$$4. x^2 + (y+8\sqrt{2})^2 = 64$$

18.



Let $P(x, y)$ be an arbitrary point which moves in the way described in the question. First, use the distance formula to find the distance from P to A and from P to B :

$$\text{From } P \text{ to } A: \sqrt{(x-2)^2 + y^2}$$

$$\text{From } P \text{ to } B: \sqrt{(x-5)^2 + y^2}$$

Since it is given that the point is always twice as far from (5, 0) as it is from (2, 0), the following relationship is true:

$$2\sqrt{(x-2)^2 + y^2} = \sqrt{(x-5)^2 + y^2}$$

$$4[(x-2)^2 + y^2] = (x-5)^2 + y^2$$

$$4(x-2)^2 + 4y^2 = (x-5)^2 + y^2$$

$$4(x^2 - 4x + 4) + 4y^2 = x^2 - 10x + 25 + y^2$$

$$4x^2 - 16x + 16 + 4y^2 = x^2 - 10x + 25 + y^2$$

$$3x^2 - 6x + 3y^2 - 9 = 0$$

$$x^2 - 2x + y^2 - 3 = 0$$

$$x^2 + y^2 - 2x - 3 = 0$$

Completing the square,

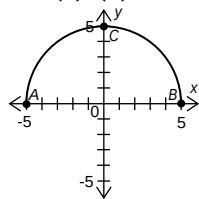
$$x^2 - 2x + 1 - 1 + y^2 - 3 = 0$$

$$(x-1)^2 + y^2 = 4$$

∴ its path is a circle with centre (1, 0), radius 2, and equation

$$(x-1)^2 + y^2 = 4.$$

19. (a) (b)



(c) In $\triangle AOC$,

$$|AO| = |CO| = 5 \text{ (given)}$$

From this, we can conclude that $\triangle AOC$ is an isosceles triangle. Thus, $\angle OCA = 45^\circ$.

Similarly, in $\triangle BOC$,

$$|BO| = |CO| = 5 \text{ (given)},$$

$$\therefore \angle OCB = 45^\circ.$$

Hence, $\angle ACB = 90^\circ$ and $\triangle ABC$ is a right triangle.

20. This problem can be solved graphically or algebraically.

The algebraic solution will be shown here:

$$\therefore (x+4)^2 + (y-1)^2 = 25 \text{ ----- ①}$$

$$\text{and } (x-2)^2 + (y-3)^2 = 25 \text{ ----- ②}$$

$$\therefore (x+4)^2 + (y-1)^2 = (x-2)^2 + (y-3)^2$$

$$x^2 + 8x + 16 + y^2 - 2y + 1 = x^2 - 4x + 4 + y^2 - 6y + 9$$

$$8x - 2y + 17 = -4x - 6y + 13$$

$$12x + 4y + 4 = 0$$

$$3x + y + 1 = 0$$

$$\therefore y = -3x - 1 \text{ ----- ③}$$

Substitute ③ into ①,

$$(x+4)^2 + (-3x-1-1)^2 = 25$$

$$x^2 + 8x + 16 + (-3x-2)^2 = 25$$

$$x^2 + 8x + 16 + 9x^2 + 12x + 4 = 25$$

$$10x^2 + 20x + 20 = 25$$

$$10x^2 + 20x - 5 = 0$$

Solve for x using the quadratic formula,

$$x = \frac{-20 \pm \sqrt{20^2 - 4(10)(-5)}}{2(10)}$$

$$x = \frac{-20 \pm \sqrt{400 + 200}}{20}$$

$$x = \frac{-20 \pm \sqrt{600}}{20}$$

$$x = 0.22 \text{ or } x = -2.22$$

Substitute $x = 0.22$ and $x = -2.22$ into ③,

$$y = -3(0.22) - 1$$

$$y = -1.66$$

$$y = -3(-2.22) - 1$$

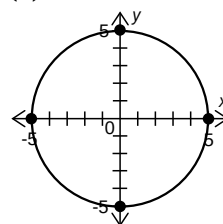
$$y = 5.66$$

∴ the points of intersection are (0.22, -1.66) and (-2.22, 5.66).

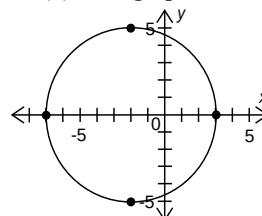
Practice 7.3U, page 584

1. (a) $y = \sqrt{25 - x^2}$ and $y = -\sqrt{25 - x^2}$

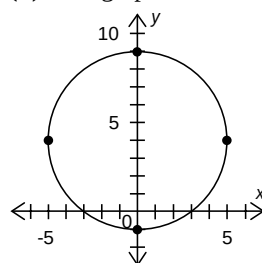
(b)



2. (a) The graph will move 2 spaces to the left.



(b) The graph will move 4 units up.



(c) The graph will be centred at the origin with radius 4 units.

