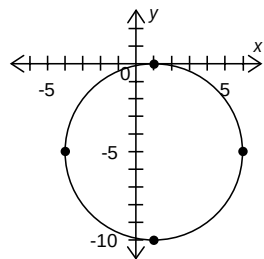
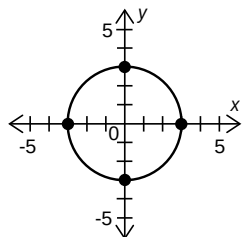


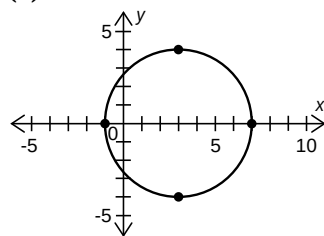
(d) The graph will move 1 unit right and 5 units down.



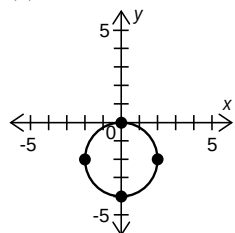
3. (a)



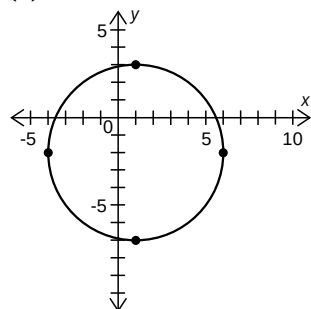
(b)



(c)



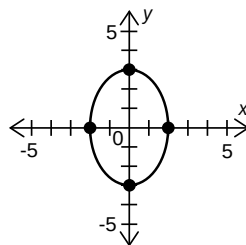
(d)



Practise, Apply, Solve 7.4U page 591

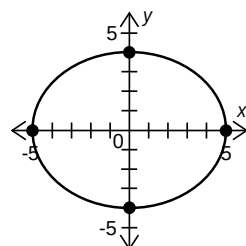
1. (a) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

The length of the major axis is 6. The length of the minor axis is 4.



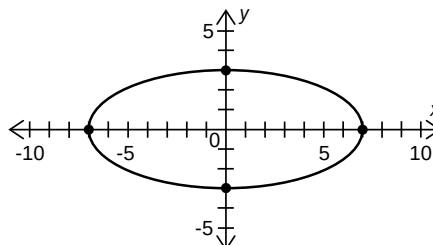
(b) $\frac{x^2}{25} + \frac{y^2}{16} = 1$

The length of the major axis is 10. The length of the minor axis is 8.



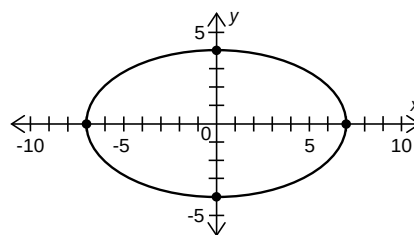
(c) $\frac{x^2}{49} + \frac{y^2}{9} = 1$

The length of the major axis is 14. The length of the minor axis is 6.



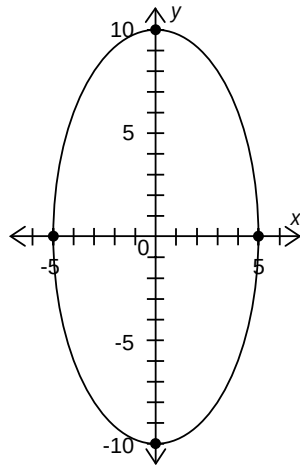
(d) $\frac{x^2}{49} + \frac{y^2}{16} = 1$

The length of the major axis is 14. The length of the minor axis is 8.



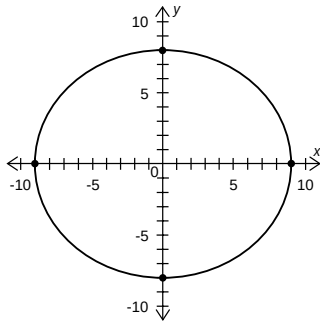
$$(e) \frac{x^2}{25} + \frac{y^2}{100} = 1$$

The length of the major axis is 20. The length of the minor axis is 10.



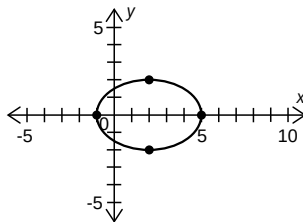
$$(f) \frac{x^2}{81} + \frac{y^2}{64} = 1$$

The length of the major axis is 18. The length of the minor axis is 16.



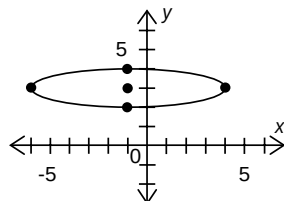
$$2. (a) \frac{(x-2)^2}{9} + \frac{y^2}{4} = 1$$

The centre is (2, 0). The length of the major axis is 6 and is horizontal. The length of the minor axis is 4.



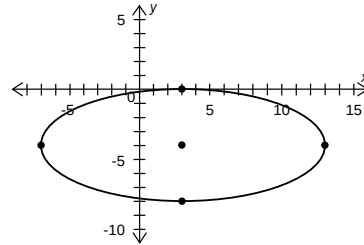
$$(b) \frac{(x+1)^2}{25} + (y-3)^2 = 1$$

The centre is (-1, 3). The length of the major axis is 10 and is horizontal. The length of the minor axis is 2.



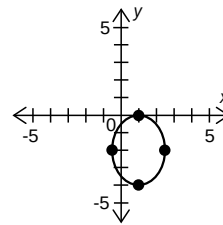
$$(c) \frac{(x-3)^2}{100} + \frac{(y+4)^2}{16} = 1$$

The centre is (3, -4). The length of the major axis is 20 and is horizontal. The length of the minor axis is 8.



$$(d) \frac{(x-1)^2}{2} + \frac{(y+2)^2}{4} = 1$$

The centre is (1, -2). The length of the major axis is 4 and is vertical. The length of the minor axis is $2\sqrt{2}$.

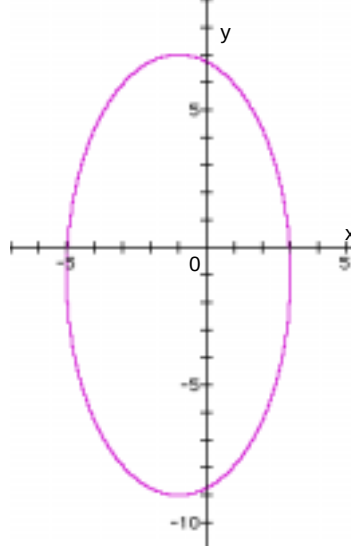


(e)

$$(x+1)^2 + \frac{(y+1)^2}{4} = 16$$

$$\frac{(x+1)^2}{16} + \frac{(y+1)^2}{64} = 1$$

The centre is (-1, -1). The length of major axis is 16 and is vertical. The length of minor axis is 8.

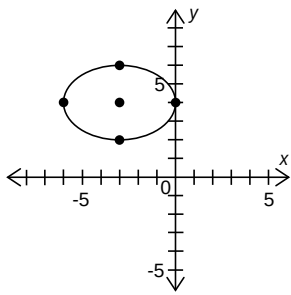


(f)

$$4(x+3)^2 + 9(y-4)^2 = 36$$

$$\frac{(x+3)^2}{9} + \frac{(y-4)^2}{4} = 1$$

The centre is (-3, 4). The length of the major axis is 6 and is horizontal. The length of the minor axis is 4.



3. (a)

$$9x^2 + 4y^2 = 144 \quad \text{Divide each term by 144.}$$

$$\frac{x^2}{16} + \frac{y^2}{36} = 1$$

The length of the major axis is 12. The length of the minor axis is 8.

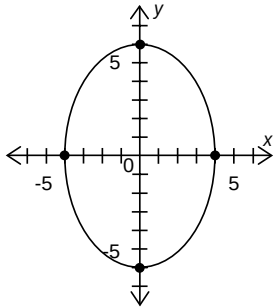
$$a = 6, b = 4$$

$$a^2 = b^2 + c^2$$

$$36 = 16 + c^2$$

$$2\sqrt{5} = c$$

$\therefore$ the foci are $(0, 2\sqrt{5})$ and $(0, -2\sqrt{5})$.



(b)

$$2x^2 + y^2 = 36 \quad \text{Divide each term by 36.}$$

$$\frac{x^2}{18} + \frac{y^2}{36} = 1$$

The length of the major axis is 12. The length of the minor axis is $2\sqrt{18} = 6\sqrt{2}$.

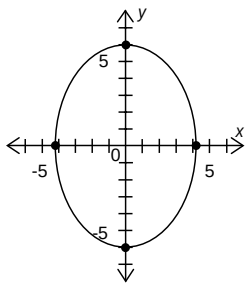
$$a = 6, b = \sqrt{18}$$

$$a^2 = b^2 + c^2$$

$$36 = 18 + c^2$$

$$3\sqrt{2} = c$$

$\therefore$ the foci are $(0, 3\sqrt{2})$ and $(0, -3\sqrt{2})$.



(c)

$$6x^2 + 4y^2 = 48$$

Divide each term by 48.

$$\frac{x^2}{8} + \frac{y^2}{12} = 1$$

The length of the major axis is $4\sqrt{3}$. The length of the minor axis is $4\sqrt{2}$.

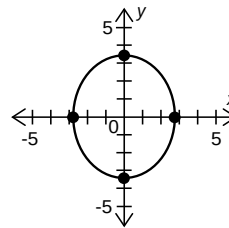
$$a = 2\sqrt{3}, b = 2\sqrt{2},$$

$$a^2 = b^2 + c^2$$

$$12 = 8 + c^2$$

$$2 = c$$

$\therefore$ the foci are $(0, 2)$ and $(0, -2)$.



(d)

$$x^2 + 8y^2 = 72$$

Divide each term by 72.

$$\frac{x^2}{72} + \frac{y^2}{9} = 1$$

The length of the major axis is $2\sqrt{72}$ or $12\sqrt{2}$. The length of the minor axis is 6.

$$a = \sqrt{72}, b = 3$$

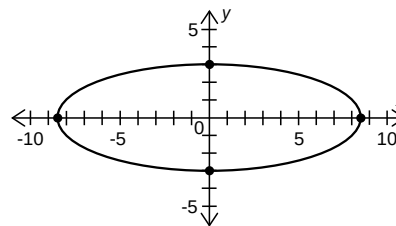
$$a^2 = b^2 + c^2$$

$$72 = 9 + c^2$$

$$\sqrt{63} = c$$

$$c = 3\sqrt{7}$$

$\therefore$ the foci are $(3\sqrt{7}, 0)$ and $(-3\sqrt{7}, 0)$.



(e)

$$4x^2 + 16y^2 = 64$$

Divide each term by 64.

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

The length of the major axis is 8. The length of the minor axis is 4.

$$a = 4, b = 2$$

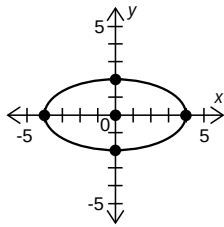
$$a^2 = b^2 + c^2$$

$$16 = 4 + c^2$$

$$\sqrt{12} = c$$

$$c = 2\sqrt{3}$$

$\therefore$ the foci are $(2\sqrt{3}, 0)$ and $(-2\sqrt{3}, 0)$.



(f)

$$25x^2 + 4y^2 = 100$$

Divide each term by 100.

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

The length of the major axis is 10. The length of the minor axis is 4.

$$a = 5, b = 2$$

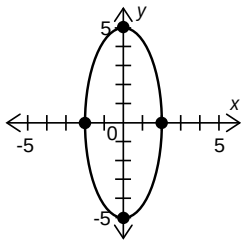
$$a^2 = b^2 + c^2$$

$$25 = 4 + c^2$$

$$21 = c^2$$

$$c = \sqrt{21}$$

$\therefore$ the foci are $(0, \sqrt{21})$ and $(0, -\sqrt{21})$.



4. (a) $\frac{x^2}{9} + \frac{y^2}{20.25} = 1$

(b) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

(c) $a^2 = b^2 + c^2$ $a = 10, c = 8$

$$10^2 = b^2 + 8^2$$

$$100 = b^2 + 64$$

$$b^2 = 36$$

$$b = 6$$

$\therefore$ the equation is $\frac{x^2}{100} + \frac{y^2}{36} = 1$.

(d)

$$a^2 = b^2 + c^2 \quad a = 4, c = 2$$

$$16 = b^2 + 4$$

$$\sqrt{12} = b$$

$$2\sqrt{3} = b$$

$\therefore$ the equation is $\frac{x^2}{16} + \frac{y^2}{12} = 1$.

(e)

$$a^2 = b^2 + c^2 \quad a = 8, c = 6$$

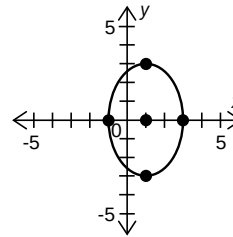
$$64 = b^2 + 36$$

$$28 = b^2$$

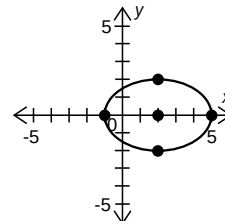
$$\sqrt{28} = b$$

$\therefore$ the equation is $\frac{x^2}{28} + \frac{y^2}{64} = 1$.

5. (a) $\frac{(x-1)^2}{4} + \frac{y^2}{9} = 1$

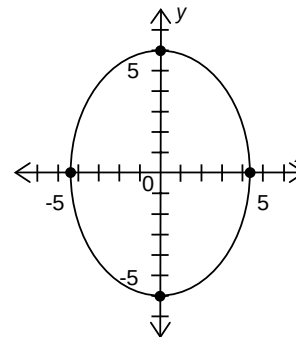


(b) $\frac{(x-2)^2}{9} + \frac{y^2}{4} = 1$

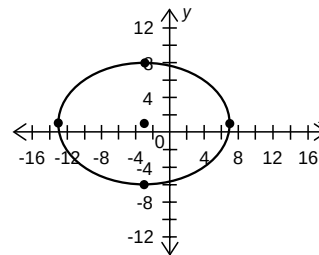


(c) $2x^2 + y^2 = 36$

$$\frac{x^2}{18} + \frac{y^2}{36} = 1$$



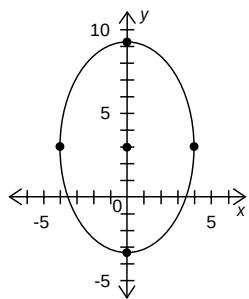
(d) $\frac{(x+3)^2}{100} + \frac{(y-1)^2}{49} = 1$



(e)

$$5x^2 + 2(y-3)^2 = 80$$

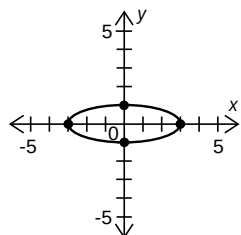
$$\frac{x^2}{16} + \frac{(y-3)^2}{40} = 1$$



(f)

$$(x-1)^2 + 9y^2 = 9$$

$$\frac{(x-1)^2}{9} + y^2 = 1$$



6. (a) i. Vertices are $(0, 4)$ and $(0, -4)$.

ii. Length of major axis is 8. Length of minor axis is 6.

iii.

$$a = 4, b = 3$$

$$a^2 = b^2 + c^2$$

$$16 = 9 + c^2$$

$$\sqrt{7} = c$$

$\therefore$ foci are $(0, \sqrt{7})$ and $(0, -\sqrt{7})$.

iv. Equation of ellipse is: $\frac{x^2}{9} + \frac{y^2}{16} = 1$.

v. Domain = $\{x \mid -3 \leq x \leq 3, x \in \mathbf{R}\}$

Range = $\{y \mid -4 \leq y \leq 4, y \in \mathbf{R}\}$

(b) i. Vertices are $(6, 0)$ and $(-6, 0)$.

ii. Length of major axis is 12. Length of minor axis is 10.

iii.

$$a = 6, b = 5$$

$$a^2 = b^2 + c^2$$

$$36 = 25 + c^2$$

$$c = \sqrt{11}$$

$\therefore$ foci are $(\sqrt{11}, 0)$ and $(-\sqrt{11}, 0)$.

iv. Equation of ellipse is: $\frac{x^2}{36} + \frac{y^2}{25} = 1$

v. Domain = $\{x \mid -6 \leq x \leq 6, x \in \mathbf{R}\}$

Range = $\{y \mid -5 \leq y \leq 5, y \in \mathbf{R}\}$

(c) i. Vertices are $(0, 8)$ and $(0, -8)$.

ii. Length of major axis is 16. Length of minor axis is 12.

iii.

$$a = 8, b = 6$$

$$64 = 36 + c^2$$

$$28 = c^2$$

$$c = 2\sqrt{7}$$

$\therefore$ foci are $(0, 2\sqrt{7})$ and $(0, -2\sqrt{7})$.

iv. Equation of ellipse is: $\frac{x^2}{36} + \frac{y^2}{64} = 1$

v. Domain = $\{x \mid -6 \leq x \leq 6, x \in \mathbf{R}\}$

Range = $\{y \mid -8 \leq y \leq 8, y \in \mathbf{R}\}$

(d) i. Vertices are $(4, 0)$ and $(-4, 0)$.

ii. Length of major axis is 8. Length of minor axis is 4.

iii.

$$a = 4, b = 2$$

$$a^2 = b^2 + c^2$$

$$16 = 4 + c^2$$

$$c = \sqrt{12}$$

$$c = 2\sqrt{3}$$

iv. Equation of ellipse is: $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

v. Domain = $\{x \mid -4 \leq x \leq 4, x \in \mathbf{R}\}$

Range = $\{y \mid -2 \leq y \leq 2, y \in \mathbf{R}\}$

7. The ellipse will be very wide and not very tall.

$$a = 50, c = 49$$

$$a^2 = b^2 + c^2$$

$$50^2 = b^2 + 2401$$

$$2500 = b^2 + 2401$$

$$2500 - 2401 = b^2$$

$$9.9 = b$$

The length of the major axis is 100 and the length of the minor axis is approximately 20.

8. (a) $4(x-2)^2 + (y+1)^2 = 16$

$$\frac{(x-2)^2}{4} + \frac{(y+1)^2}{16} = 1$$

Centre is $(2, -1)$. Length of major axis is 8; length of minor axis is 4.

(b) $2(x+1)^2 + 16(y+2)^2 = 32$

$$\frac{(x+1)^2}{16} + \frac{(y+2)^2}{2} = 1$$

Centre is $(-1, -2)$. Length of major axis is 8; length of minor axis is $2\sqrt{2}$.

(c) $3(x+1)^2 + 16(y-2)^2 = 48$

$$\frac{(x+1)^2}{16} + \frac{(y-2)^2}{3} = 1$$

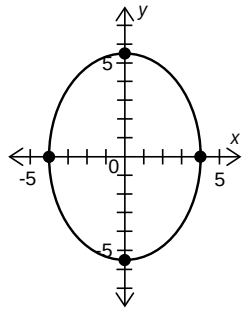
Centre is $(-1, 2)$. Length of major axis is 8; length of minor axis is $2\sqrt{3}$.

(d) $5x^2 + 2(y-3)^2 = 80$

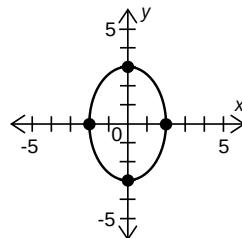
$$\frac{x^2}{16} + \frac{(y-3)^2}{40} = 1$$

Centre is $(0, 3)$. Length of major axis is $2\sqrt{40}$ or $4\sqrt{10}$; length of minor axis is 8.

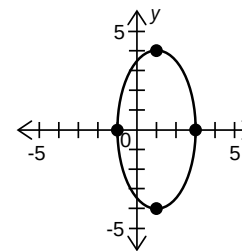
9. (a) $y = \pm \sqrt{32 - x^2}$



(b) $y = \pm 3\sqrt{1 - \frac{x^2}{5}}$



(c) $y = \pm \sqrt{-4x^2 + 8x + 12}$



10. $a^2 = b^2 + c^2$
 $a^2 = 12^2 + 5^2$
 $a^2 = 144 + 25$
 $a^2 = 169$
 $a = 13$

∴ the equation is $\frac{x^2}{169} + \frac{y^2}{144} = 1$.

11. Let $P(x, y)$ be a point on the ellipse.
 $PF_1 + PF_2 = 10$

$$\begin{aligned} \sqrt{x^2 + (y-4)^2} + \sqrt{x^2 + (y+4)^2} &= 10 \\ x^2 + (y-4)^2 &= \left(10 - \sqrt{x^2 + (y+4)^2}\right)^2 \\ x^2 + y^2 - 8y + 16 &= 100 - 20\sqrt{x^2 + (y+4)^2} + x^2 + y^2 + 8y + 16 \\ -16y - 100 &= -20\sqrt{x^2 + (y+4)^2} \\ 4y + 25 &= 5\sqrt{x^2 + (y+4)^2} \\ (4y + 25)^2 &= 25[x^2 + (y+4)^2] \\ 16y^2 + 200y + 625 &= 25x^2 + 25y^2 + 200y + 400 \\ 225 &= 25x^2 + 9y^2 \\ 1 &= \frac{x^2}{9} + \frac{y^2}{25} \end{aligned}$$

12. (a) $a = 1.5 \times 10^8$ $c = 2.5 \times 10^6$
 $a^2 = b^2 + c^2$
 $(1.5 \times 10^8)^2 = b^2 + (2.5 \times 10^6)^2$
 $2.25 \times 10^{16} = b^2 + 6.25 \times 10^{12}$
 $22\,500 \times 10^{12} - 6.25 \times 10^{12} = b^2$
 $22\,493.75 \times 10^{12} = b^2$
 $\sqrt{22\,493.75 \times 10^{12}} = b$
 $149.98 \times 10^6 = b$
 $1.4998 \times 10^8 = b$

∴ the length of the minor axis is 2.9996×10^8 .

(b) The equation of the Earth's orbit is:

$$\frac{x^2}{2.2500 \times 10^{16}} + \frac{y^2}{2.2493 \times 10^{16}} = 1$$

13. The equation will be of the form:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Substitute the 2 points into the equation:

Using (2, -2):

$$\frac{2^2}{a^2} + \frac{(-2)^2}{b^2} = 1$$

① $\frac{4}{a^2} + \frac{4}{b^2} = 1$

Using $\left(-\frac{2}{3}, \frac{10}{3}\right)$ with the equation:

$$\frac{\left(-\frac{2}{3}\right)^2}{a^2} + \frac{\left(\frac{10}{3}\right)^2}{b^2} = 1$$

$$\frac{\left(\frac{4}{9}\right)}{a^2} + \frac{\left(\frac{100}{9}\right)}{b^2} = 1$$

② $\frac{4}{9a^2} + \frac{100}{9b^2} = 1$

Solve equation ① for a^2 :

$$\frac{4}{a^2} = 1 - \frac{4}{b^2}$$

$$\frac{4}{a^2} = \frac{b^2 - 4}{b^2}$$

$$4 = a^2 \left(\frac{b^2 - 4}{b^2} \right)$$

$$\frac{4b^2}{b^2 - 4} = a^2$$

Solve equation ② for a^2 :

$$\frac{4}{9a^2} = 1 - \frac{100}{9b^2}$$

$$\frac{4}{9a^2} = \frac{9b^2 - 100}{9b^2}$$

$$4 = 9a^2 \left(\frac{9b^2 - 100}{9b^2} \right)$$

$$\frac{36b^2}{9b^2 - 100} = 9a^2$$

$$\frac{4b^2}{9b^2 - 100} = a^2$$

Equate both equations and solve for b .

$$\frac{4b^2}{b^2 - 4} = \frac{4b^2}{9b^2 - 100}$$

$$4b^2(b^2 - 4) = 4b^2(9b^2 - 100)$$

$$b^2 - 4 = 9b^2 - 100$$

$$0 = 8b^2 - 96$$

$$0 = 8(b^2 - 12)$$

$$b = \pm\sqrt{12}$$

Substitute b into one of the equations.

$$a^2 = \frac{4b^2}{b^2 - 4}$$

$$a^2 = \frac{4(\sqrt{12})^2}{(\sqrt{12})^2 - 4}$$

$$a^2 = \frac{48}{12 - 4}$$

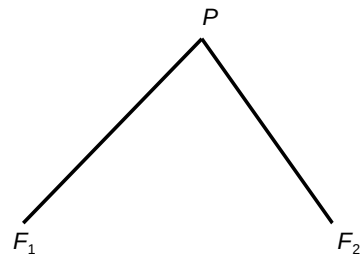
$$a^2 = \frac{48}{8}$$

$$a^2 = 6$$

$$a = \pm\sqrt{6}$$

Now, if $a = \pm\sqrt{6}$ and $b = \pm\sqrt{12}$. Then the equation of the ellipse is: $\frac{x^2}{6} + \frac{y^2}{12} = 1$.

14.



$$PF_1 + PF_2 = 8$$

The dog has an elliptical area in which to run.

$$a = 4, c = 2$$

$$a^2 = b^2 + c^2$$

$$16 = b^2 + 4$$

$$16 - 4 = b^2$$

$$\pm\sqrt{12} = b$$

The dimensions of the ellipse are a width of 8 m and a height of $4\sqrt{3}$ m.

15. (a)

$$4x^2 + y^2 - 2y - 3 = 0$$

$$4x^2 + y^2 - 2y = 3$$

$$4x^2 + y^2 - 2y + 1 = 3 + 1$$

$$4x^2 + (y - 1)^2 = 4$$

$$x^2 + \frac{(y - 1)^2}{4} = 1$$

(b)

$$4x^2 + 9y^2 - 8x + 18y = 23$$

$$4x^2 - 8x + 9y^2 + 18y = 23$$

$$4(x^2 - 2x) + 9(y^2 + 2y) = 23$$

$$4(x^2 - 2x + 1) + 9(y^2 + 2y + 1) = 23 + 4 + 9$$

$$4(x - 1)^2 + 9(y + 1)^2 = 36$$

$$\frac{(x - 1)^2}{9} + \frac{(y + 1)^2}{4} = 1$$

(c)

$$4x^2 + y^2 - 16x + 2y + 1 = 0$$

$$4x^2 - 16x + y^2 + 2y = -1$$

$$4(x^2 - 4x) + y^2 + 2y = -1$$

$$4(x^2 - 4x + 4) + y^2 + 2y + 1 = -1 + 16 + 1$$

$$4(x - 2)^2 + (y + 1)^2 = 16$$

$$\frac{(x - 2)^2}{4} + \frac{(y + 1)^2}{16} = 1$$

(d)

$$16x^2 + 9y^2 + 90y + 81 = 0$$

$$16x^2 + 9y^2 + 90y = -81$$

$$16x^2 + 9(y^2 + 10y) = -81$$

$$16x^2 + 9(y^2 + 10y + 25) = -81 + 225$$

$$16x^2 + 9(y + 5)^2 = 144$$

$$\frac{x^2}{9} + \frac{(y + 5)^2}{16} = 1$$

$$16. (a) \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Length of major axis (horizontal) is 10. Length of minor axis (vertical) is 6.

$$\text{Domain} = \{x \mid -5 \leq x \leq 5\} \quad \text{Range} = \{y \mid -3 \leq y \leq 3\}$$

$$(b) \frac{x^2}{16} + \frac{y^2}{36} = 1$$

Length of major axis (vertical) is 12. Length of minor axis (horizontal) is 8.

$$\text{Domain} = \{x \mid -4 \leq x \leq 4\} \quad \text{Range} = \{y \mid -6 \leq y \leq 6\}$$

(c)

$$4x^2 + 25y^2 = 100$$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

Length of major axis (horizontal) is 10. Length of minor axis (vertical) is 4.

$$\text{Domain} = \{x \mid -5 \leq x \leq 5\} \quad \text{Range} = \{y \mid -2 \leq y \leq 2\}$$

(d)

$$16x^2 + 9y^2 = 144$$

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

Length of major axis (vertical) is 8. Length of minor axis (horizontal) is 6.

$$\text{Domain} = \{x \mid -3 \leq x \leq 3\} \quad \text{Range} = \{y \mid -4 \leq y \leq 4\}$$

(e)

$$x^2 + 4y^2 + 4x + 8y = 8$$

$$x^2 + 4x + 4y^2 + 8y = 8$$

$$x^2 + 4x + 4(y^2 + 2y) = 8$$

$$x^2 + 4x + 4 + 4(y^2 + 2y + 1) = 8 + 4 + 4$$

$$(x + 2)^2 + 4(y + 1)^2 = 16$$

$$\frac{(x + 2)^2}{16} + \frac{(y + 1)^2}{4} = 1$$

Length of major axis (horizontal) is 8. Length of minor axis (vertical) is 4.

$$\text{Domain} = \{x \mid -4 \leq x \leq 4\} \quad \text{Range} = \{y \mid -2 \leq y \leq 2\}$$

(f)

$$x^2 + 9y^2 + 2x - 18y = 26$$

$$x^2 + 2x + 9y^2 - 18y = 26$$

$$x^2 + 2x + 1 + 9(y^2 - 2y) = 26 + 1$$

$$x^2 + 2x + 1 + 9(y^2 - 2y + 1) = 26 + 1 + 9$$

$$(x + 1)^2 + 9(y - 1)^2 = 36$$

$$\frac{(x + 1)^2}{36} + \frac{(y - 1)^2}{4} = 1$$

Length of major axis (horizontal) is 12. Length of minor axis (vertical) is 4.

$$\text{Domain} = \{x \mid -6 \leq x \leq 6\} \quad \text{Range} = \{y \mid -2 \leq y \leq 2\}$$

17.

$$\frac{x^2}{(3b)^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{9b^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2 + 9y^2}{9b^2} = 1$$

$$x^2 + 9y^2 = 9b^2$$

Substitute (3, 3) into the equation.

$$3^2 + 9(3)^2 = 9b^2$$

$$9 + 81 = 9b^2$$

$$90 = 9b^2$$

$$10 = b^2$$

$$b = \pm\sqrt{10}$$

$$\therefore a = 3\sqrt{10}$$

$$\therefore \text{the equation is } \frac{x^2}{90} + \frac{y^2}{10} = 1.$$

18. A circle could be an ellipse with the foci both at the centre. The length of the major and minor axes is the length of the diameter of the circle. The circle could also satisfy the locus definition of the ellipse.

19. Find the equation of the semi-ellipse, $b = 2$.

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{a^2} = 1$$

$$\frac{(1.6)^2}{4} + \frac{(3.8)^2}{a^2} = 1$$

$$\frac{2.56}{4} + \frac{14.44}{a^2} = 1$$

$$2.56a^2 + 57.76 = 4a^2$$

$$57.76 = 1.44a^2$$

$$40\frac{1}{9} = a^2$$

$$\frac{361}{9} = a^2$$

$$\pm\frac{19}{3} = a$$

$$\text{The equation is } \frac{x^2}{4} + \frac{9y^2}{361} = 1.$$

Now, check to see if (1.75, 3) will be inside or outside the ellipse.

Where $y = 3$,

$$\frac{x^2}{4} + \frac{9(3)^2}{361} = 1$$

$$\frac{x^2}{4} = 1 - \frac{81}{361}$$

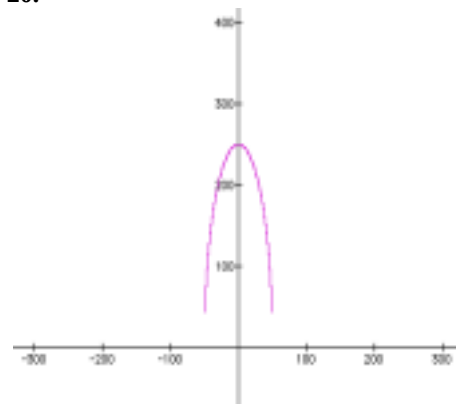
$$\frac{x^2}{4} = \frac{280}{361}$$

$$x^2 = \frac{1120}{361}$$

$$x \approx 1.76$$

$\therefore$ the driver's truck will just fit under the arch.

20.



One side of the sofa is 120×80 , so we can test out the ordered pairs (40, 120) and (60, 80).

The equation for this ellipse is:

$$\frac{x^2}{2500} + \frac{y^2}{62\,500} = 1$$

When $x = 40$,

$$\frac{40^2}{2500} + \frac{y^2}{62\,500} = 1$$

$$\frac{y^2}{62\,500} = 1 - \frac{1600}{2500}$$

$$\frac{y^2}{62\,500} = 1 - \frac{16}{25}$$

$$\frac{y^2}{62\,500} = \frac{9}{25}$$

$$y^2 = \left(\frac{9}{25}\right)(62\,500)$$

$$y^2 = 22\,500$$

$$y = 150 \quad (\text{take positive root})$$

$\therefore$ the sofa will fit through as shown.

Practise, Apply, Solve 7.6U, page 602

1. (a) $p = -4$ Focus $(0, -4)$ Directrix $y = 4$

(b) $p = \frac{15}{4}$ Focus $\left(\frac{15}{4}, 0\right)$ Directrix $x = -\frac{15}{4}$

(c) $p = \frac{1}{4}$ Focus $\left(0, \frac{1}{4}\right)$ Directrix $y = -\frac{1}{4}$

(d) $p = \frac{1}{8}$ Focus $\left(0, \frac{1}{8}\right)$ Directrix $y = -\frac{1}{8}$

(e) $p = 2$ Focus $(2, 0)$ Directrix $x = -2$

(f) $p = \frac{1}{4}$ Focus $\left(0, \frac{1}{4}\right)$ Directrix $y = -\frac{1}{4}$

2. (a) Opens upward Vertex $(0, 4)$

(b) Opens to the right Vertex $(2, 0)$

(c) Opens to the right Vertex $(0, -12)$

(d) Opens upward Vertex $(3, 0)$

(e) Opens downward Vertex $(3, 2)$

(f) Opens to the right Vertex $(-2, 1)$

3. (a) $y^2 - 4y = 4x - 8$

$$y^2 - 4y + 4 = 4x - 4$$

$$(y - 2)^2 = 4(x - 1)$$

Opens to the right Vertex $(1, 2)$

(b) $x^2 = 4y + 8$

$$x^2 = 4(y + 2)$$

Opens upward Vertex $(0, -2)$

(c) $x^2 - 2x - 4y + 5 = 0$

$$x^2 - 2x + 1 = 4y - 5 + 1$$

$$(x - 1)^2 = 4(y - 1)$$

Opens upward Vertex $(1, 1)$

(d) $y^2 - 2x + 6y + 11 = 0$

$$y^2 + 6y + 9 = 2x - 11 + 9$$

$$(y + 3)^2 = 2(x - 1)$$

Opens to the right Vertex $(1, -3)$

(e) $y^2 - 2y + 1 = 8x + 15 + 1$

$$(y - 1)^2 = 8(x + 2)$$

Opens to the right Vertex $(-2, 1)$

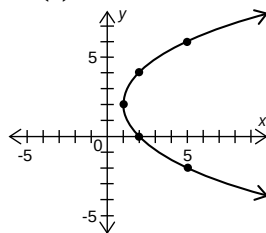
$$(f) \quad x^2 - 4x - 2y - 16 = 0$$

$$(x - 2)^2 = 2(y + 10)$$

Opens upward

Vertex $(2, -10)$

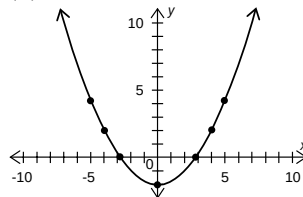
4. (a)



$$D = \{x \mid x \geq 1, x \in \mathbf{R}\}$$

$$R = \{y \mid y \in \mathbf{R}\}$$

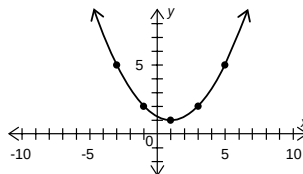
(b)



$$D = \{x \mid x \in \mathbf{R}\}$$

$$R = \{y \mid y \geq -2, y \in \mathbf{R}\}$$

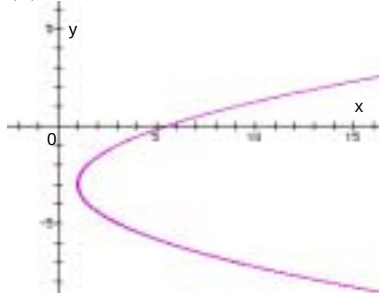
(c)



$$D = \{x \mid x \in \mathbf{R}\}$$

$$R = \{y \mid y \geq 1, y \in \mathbf{R}\}$$

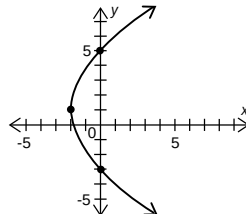
(d)



$$D = \{x \mid x \geq 1, x \in \mathbf{R}\}$$

$$R = \{y \mid y \in \mathbf{R}\}$$

(e)



$$D = \{x \mid x \geq -1, x \in \mathbf{R}\}$$

$$R = \{y \mid y \in \mathbf{R}\}$$