

$$\frac{x^2}{2500} + \frac{y^2}{62\,500} = 1$$

When $x = 40$,

$$\frac{40^2}{2500} + \frac{y^2}{62\,500} = 1$$

$$\frac{y^2}{62\,500} = 1 - \frac{1600}{2500}$$

$$\frac{y^2}{62\,500} = 1 - \frac{16}{25}$$

$$\frac{y^2}{62\,500} = \frac{9}{25}$$

$$y^2 = \left(\frac{9}{25}\right)(62\,500)$$

$$y^2 = 22\,500$$

$$y = 150 \quad (\text{take positive root})$$

$\therefore$ the sofa will fit through as shown.

Practise, Apply, Solve 7.6U, page 602

1. (a) $p = -4$ Focus $(0, -4)$ Directrix $y = 4$

(b) $p = \frac{15}{4}$ Focus $\left(\frac{15}{4}, 0\right)$ Directrix $x = -\frac{15}{4}$

(c) $p = \frac{1}{4}$ Focus $\left(0, \frac{1}{4}\right)$ Directrix $y = -\frac{1}{4}$

(d) $p = \frac{1}{8}$ Focus $\left(0, \frac{1}{8}\right)$ Directrix $y = -\frac{1}{8}$

(e) $p = 2$ Focus $(2, 0)$ Directrix $x = -2$

(f) $p = \frac{1}{4}$ Focus $\left(0, \frac{1}{4}\right)$ Directrix $y = -\frac{1}{4}$

2. (a) Opens upward Vertex $(0, 4)$

(b) Opens to the right Vertex $(2, 0)$

(c) Opens to the right Vertex $(0, -12)$

(d) Opens upward Vertex $(3, 0)$

(e) Opens downward Vertex $(3, 2)$

(f) Opens to the right Vertex $(-2, 1)$

3. (a) $y^2 - 4y = 4x - 8$

$$y^2 - 4y + 4 = 4x - 4$$

$$(y - 2)^2 = 4(x - 1)$$

Opens to the right

Vertex $(1, 2)$

(b) $x^2 = 4y + 8$

$$x^2 = 4(y + 2)$$

Opens upward

Vertex $(0, -2)$

(c) $x^2 - 2x - 4y + 5 = 0$

$$x^2 - 2x + 1 = 4y - 5 + 1$$

$$(x - 1)^2 = 4(y - 1)$$

Opens upward

Vertex $(1, 1)$

(d) $y^2 - 2x + 6y + 11 = 0$

$$y^2 + 6y + 9 = 2x - 11 + 9$$

$$(y + 3)^2 = 2(x - 1)$$

Opens to the right

Vertex $(1, -3)$

(e) $y^2 - 2y + 1 = 8x + 15 + 1$

$$(y - 1)^2 = 8(x + 2)$$

Opens to the right

Vertex $(-2, 1)$

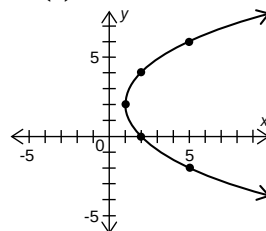
(f) $x^2 - 4x - 2y - 16 = 0$

$$(x - 2)^2 = 2(y + 10)$$

Opens upward

Vertex $(2, -10)$

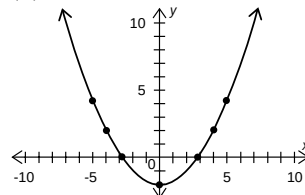
4. (a)



$$D = \{x \mid x \geq 1, x \in \mathbf{R}\}$$

$$R = \{y \mid y \in \mathbf{R}\}$$

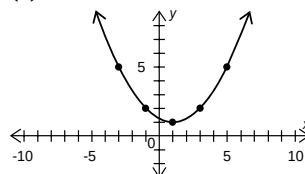
(b)



$$D = \{x \mid x \in \mathbf{R}\}$$

$$R = \{y \mid y \geq -2, y \in \mathbf{R}\}$$

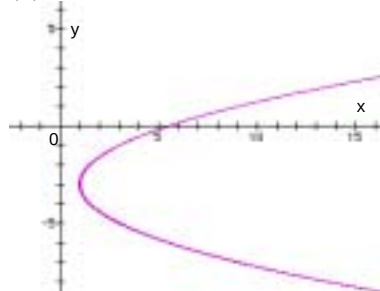
(c)



$$D = \{x \mid x \in \mathbf{R}\}$$

$$R = \{y \mid y \geq 1, y \in \mathbf{R}\}$$

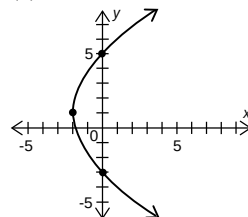
(d)



$$D = \{x \mid x \geq 1, x \in \mathbf{R}\}$$

$$R = \{y \mid y \in \mathbf{R}\}$$

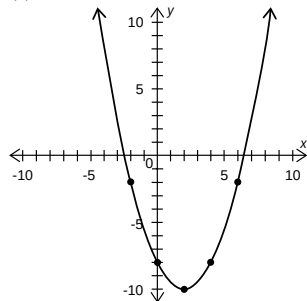
(e)



$$D = \{x \mid x \geq -1, x \in \mathbf{R}\}$$

$$R = \{y \mid y \in \mathbf{R}\}$$

(f)



$$D = \{x \mid x \in \mathbf{R}\}$$

$$R = \{y \mid y \geq -10, y \in \mathbf{R}\}$$

5. Answers will vary.

6. This would be a parabola with a directrix of $x = 4$ and a focus of $(-4, 0)$. The equation of the parabola is $y^2 = 16x$.

7. (a) $y^2 = 12x$

(b) $x^2 = -8y$

(c) $y^2 = -16x$

(d) $y^2 = -2x$

8. (a)

$$y^2 = 4px$$

$$(-4)^2 = 4p(-8)$$

$$16 = -32p$$

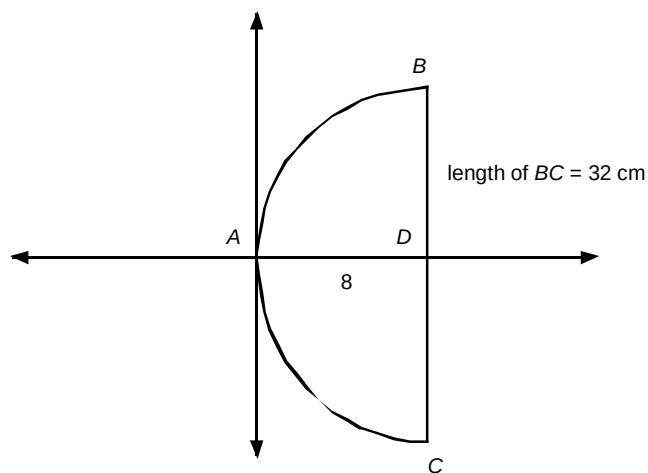
$$-\frac{1}{2} = p$$

$\therefore$ the equation is $y^2 = -2x$.

(b) The focus is $\left(-\frac{1}{2}, 0\right)$.

(c) The directrix is $x = \frac{1}{2}$.

9.



$$y^2 = 4px$$

Substitute the point $(8, 16)$:

$$16^2 = 4p(8)$$

$$256 = 32p$$

$$8 = p$$

$\therefore$ the bulb should be located at the focus $(8, 0)$.

10. (a)

$$\frac{f}{D} > 0.45$$

$$\frac{f}{5} > 0.45$$

$$f > 2.25$$

For a shallow dish, the focal length would be greater than 2.25 m.

(b)

$$0.3 \leq \frac{f}{5} \leq 0.45$$

$$1.5 \text{ m} \leq f \leq 2.25 \text{ m}$$

for an average dish

(c)

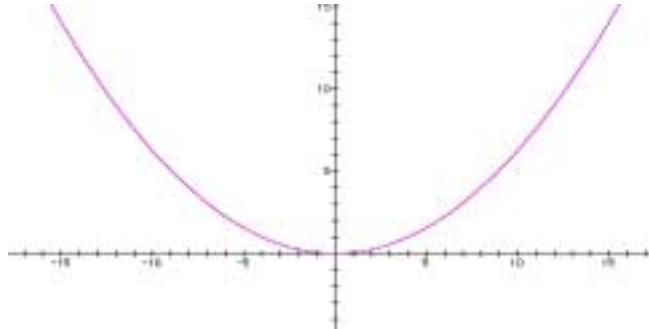
$$\frac{f}{D} < 0.3$$

$$\frac{f}{5} < 0.3$$

$$f < 1.5 \text{ m}$$

for a deep dish

11.



The focal length is 4.

$\therefore$ the equation is $x^2 = 16y$.

When $y = 6$,

$$x^2 = 96$$

$$x = \sqrt{96}$$

$\therefore$ the radius of the circular piece of glass is $\sqrt{96}$ cm. The

area of the piece of glass is:

$$A = \pi r^2$$

$$A = \pi(96)$$

$\therefore$ the area of the glass is 301.6 cm^2 .

12. $p = 80$

$\therefore$ the equation is $x^2 = 320y$.

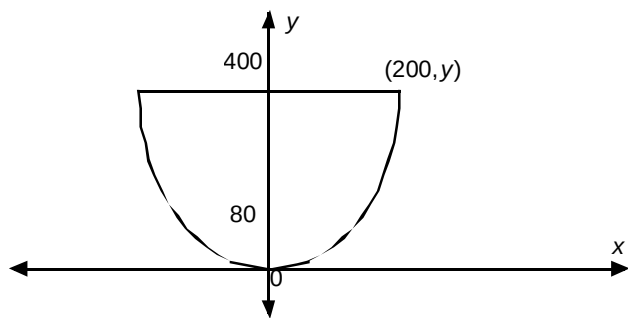
To find the depth, find y where $x = 200$.

$$200^2 = 320y$$

$$\frac{40\,000}{320} = y$$

$$125 = y$$

$\therefore$ the depth is 125 cm.



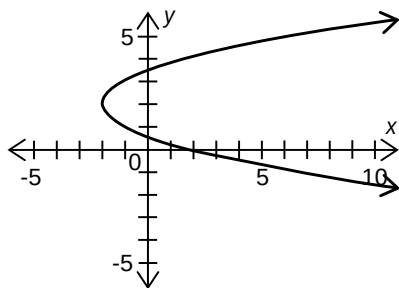
13.

$(y - 2)^2 = x + 2$ Take the square root of both sides.

$$y - 2 = \pm \sqrt{x + 2}$$

$$y = 2 \pm \sqrt{x + 2}$$

Two different equations need to be entered into the graphing calculator: $y = 2 + \sqrt{x + 2}$ (top) and $y = 2 - \sqrt{x + 2}$ (bottom).



14. (a) The value of p is 3. The equation is

$$y^2 = 12(x + 1).$$

(b) The value of p is 1.5. The equation is

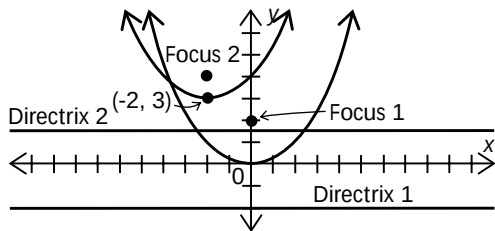
$$x^2 = 6(y - 2.5).$$

(c) The value of p is 1. The equation is

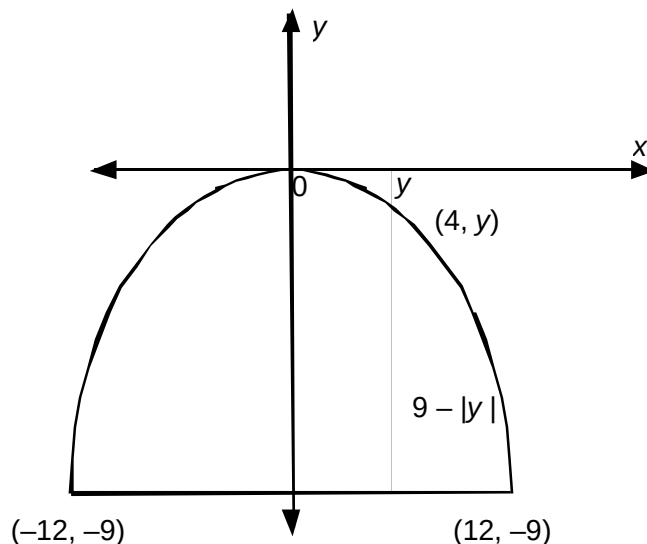
$$(x - 1)^2 = 4(y - 1).$$

15. Vertex is $(1, 2)$. If the directrix is 2 units to the left of the focus, then p is 1. Since the directrix is vertical and to the left of the focus, the parabola opens to the right. The equation is $(y - 2)^2 = 4(x - 1)$.

16.



17.



24 m

$$x^2 = 4py$$

Substitute $(12, -9)$ into the equation:

$$144 = 4p(-9)$$

$$p = -4$$

$\therefore x^2 = -16y$ is the equation. Now substitute $x = 4$ to solve the problem:

$$16 = -16y$$

$$y = -1$$

since $(9 - |y|)$ is the height of the girder. Then the height of the girder is 8 m.

18. The maximum value of $(4, 3)$ is the vertex of the parabola. The equation is of the form:

$$(x - 4)^2 = 4p(y - 3)$$

Now, substitute $(5, 2)$ into the equation.

$$(5 - 4)^2 = 4p(2 - 3)$$

$$1 = -4p$$

$$-\frac{1}{4} = p$$

$\therefore$ the equation is $(x - 4)^2 = -(y - 3)$.

19. (a) Both equations have h 's and k 's. In both equations there is an $(x - h)^2$. The differences are that the second equation has $4p$ and $(y - k)$ in brackets whereas k is on the other side of the first equation.

(b) If we start with the second form:

$$(x - h)^2 = 4p(y - k)$$

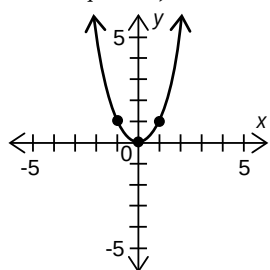
$$\frac{1}{4p}(x - h)^2 = y - k$$

$$\frac{1}{4p}(x - h)^2 + k = y$$

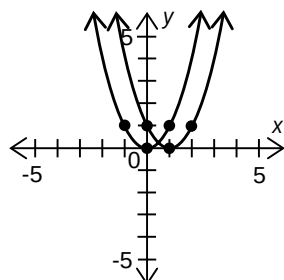
If $a = \frac{1}{4p}$ then we have $a(x - h)^2 + k = y$ which is the same as the first equation.

(c) Answers will vary.

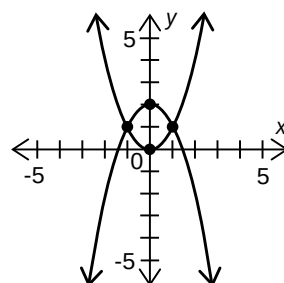
20. Infinite points of intersection: (both parabolas have the same equation)



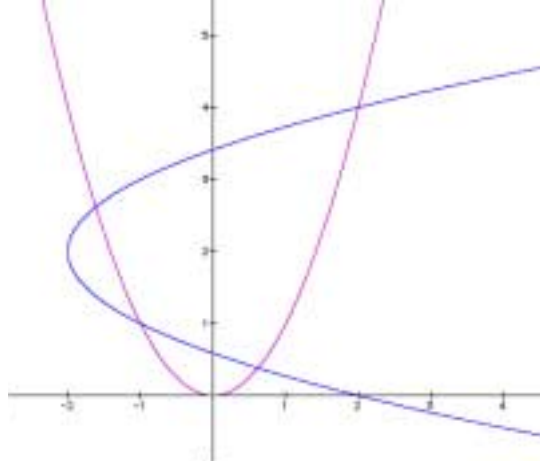
One point of intersection: (eg. $y = x^2$ and $y = (x - 1)^2$)



Two points of intersection: (eg. $y = x^2$ and $y = -x^2 + 2$)



Four points of intersection: (eg. $y = x^2$ and $(y - 2)^2 = (x + 2)$)



21. $y = x^2 - 5x - 4$ ①
 $y = -2x^2 + 3x - 1$ ②

Equate equations ① and ② and solve for x :

$$x^2 - 5x - 4 = -2x^2 + 3x - 1$$

$$3x^2 - 8x - 3 = 0$$

$$3x^2 - 9x + x - 3 = 0$$

$$3x(x - 3) + (x - 3) = 0$$

$$x = -\frac{1}{3}, x = 3$$

Now substitute into equation ① to solve for y :

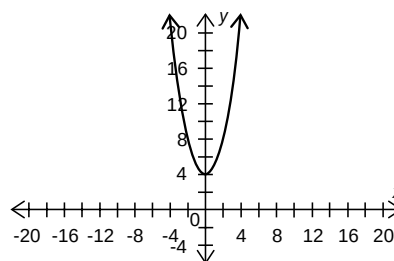
$$y = \left(-\frac{1}{3}\right)^2 + \frac{5}{3} - 4 = -\frac{20}{9}$$

$$y = (3)^2 - 5(3) - 4 = -10$$

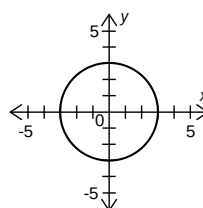
Thus the points of intersection are $\left(-\frac{1}{3}, -\frac{20}{9}\right)$ and $(3, -10)$.

Practice 7.8U, page 608

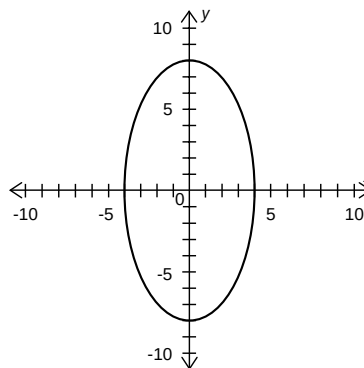
1. (a) i. parabola;
 ii. opens upward and is 4 units above the x -axis.
 iii.



- (b) i. circle; ii. centred at origin with radius 3 units
 iii.



- (c) i. ellipse; ii. centred at origin, with major vertical axis of 16 units and minor axis of 8 units
 iii.



- (d) i. ellipse; ii. centred at $(2, -1)$ with major vertical axis of 8 units, and minor axis of 6 units
 iii.

