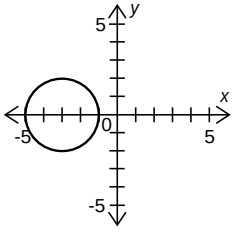
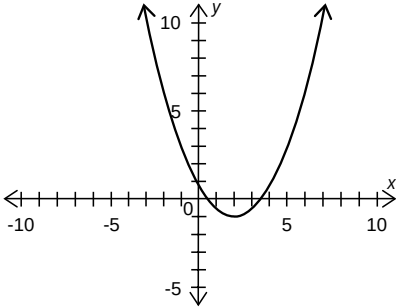


- (e) i. circle; ii. centred at $(-3, 0)$ with radius 2 units
iii.

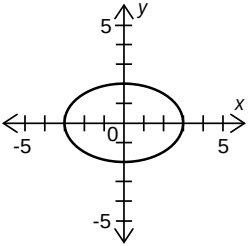


- (f) i. parabola; ii. opens upward with a vertex of $(2, -1)$
iii.



2. (a) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

(b)



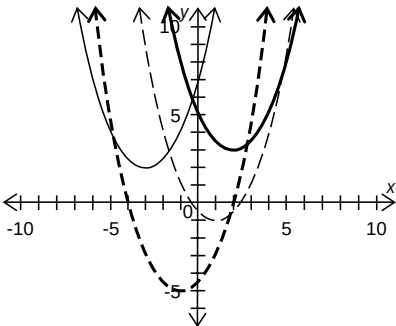
3. (a) i. $(x + 3)^2 = 2(y - 2)$

(b) i. $(x - 1)^2 = 2(y + 1)$

(c) i. $(x - 2)^2 = 2(y - 3)$

(d) i. $(x + 1)^2 = 2(y + 5)$

ii.



Practise, Apply, Solve 7.9U, page 615

1. (a) Let $y = 0$

$$4x^2 - 9(0)^2 = 36$$

$$4x^2 = 36$$

$$x^2 = 9$$

x -intercepts are ± 3 .

(b) Let $x = 0$

$$2(0)^2 - 3y^2 = -12$$

$$-3y^2 = -12$$

$$y^2 = 4$$

y -intercepts are ± 2 .

(c) Let $y = 0$

$$x^2 - (0)^2 = 18$$

$$x^2 = 18$$

x -intercepts are $\pm \sqrt{18}$ or $\pm 3\sqrt{2}$.

(d) Let $x = 0$

$$3y^2 - (0)^2 = 36$$

$$3y^2 = 36$$

$$y^2 = 12$$

y -intercepts are $\pm \sqrt{12}$ or $\pm 2\sqrt{3}$.

(e) Let $y = 0$

$$6x^2 - 10(0)^2 = 120$$

$$6x^2 = 120$$

$$x^2 = 20$$

x -intercepts are $\pm \sqrt{20}$ or $\pm 2\sqrt{5}$.

(f) Let $y = 0$

$$5x^2 - 10(0)^2 = 50$$

$$5x^2 = 50$$

$$x^2 = 10$$

x -intercepts are $\pm \sqrt{10}$.

2. (a) $\frac{x^2}{5} - \frac{y^2}{20} = 1$

$$a = \sqrt{5} \quad b = \sqrt{20}$$

$$c^2 = a^2 + b^2$$

$$c^2 = (\sqrt{5})^2 + (\sqrt{20})^2$$

$$c^2 = 5 + 20$$

$$c = 5$$

$$\therefore a = \sqrt{5} \quad b = \sqrt{20} \quad c = 5$$

(b) $\frac{x^2}{36} - \frac{y^2}{9} = 1$

$$a = 6 \quad b = 3$$

$$c^2 = a^2 + b^2$$

$$c^2 = 6^2 + 3^2$$

$$c^2 = 36 + 9$$

$$c^2 = 45$$

$$c = \sqrt{45}$$

$$\therefore a = 6 \quad b = 3 \quad c = \sqrt{45}$$

(c) $\frac{x^2}{5} - \frac{y^2}{20} = -1$

$$a = \sqrt{20} \quad b = \sqrt{5}$$

$$c^2 = a^2 + b^2$$

$$c^2 = 20 + 5$$

$$c^2 = 25$$

$$c = 5$$

$$\therefore a = \sqrt{20} \text{ or } 2\sqrt{5} \quad b = \sqrt{5} \quad c = 5$$

$$(d) \frac{x^2}{4} - \frac{y^2}{16} = 1$$

$$a = 2 \quad b = 4$$

$$c^2 = a^2 + b^2$$

$$c^2 = 4 + 16$$

$$c^2 = 20$$

$$c = \sqrt{20}$$

$$\therefore a = 2 \quad b = 4 \quad c = \sqrt{20} \text{ or } 2\sqrt{5}$$

$$(e) \frac{x^2}{8} - \frac{y^2}{12} = -1$$

$$a = \sqrt{12} \quad b = \sqrt{8}$$

$$c^2 = a^2 + b^2$$

$$c^2 = 12 + 8$$

$$c^2 = 20$$

$$c = \sqrt{20}$$

$$\therefore a = \sqrt{12} \text{ or } 2\sqrt{3} \quad b = \sqrt{8} \text{ or } 2\sqrt{2} \quad c = \sqrt{20} \text{ or } 2\sqrt{5}$$

$$(f) \frac{x^2}{144} - \frac{y^2}{81} = 1$$

$$a = 12 \quad b = 9$$

$$c^2 = a^2 + b^2$$

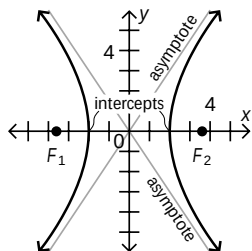
$$c^2 = 144 + 81$$

$$c^2 = 225$$

$$c = 15$$

$$\therefore a = 12 \quad b = 9 \quad c = 15$$

$$3. (a) \frac{x^2}{4} - \frac{y^2}{9} = 1$$



i. intercepts: $(-2, 0), (2, 0)$

ii. To find foci (F_1 and F_2)

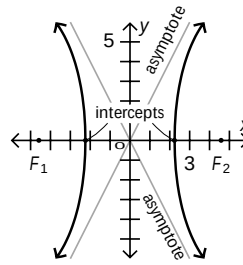
$$c^2 = 4 + 9$$

$$c = \sqrt{13}$$

$$F_1 (-\sqrt{13}, 0), F_2 (\sqrt{13}, 0)$$

iii. asymptotes: $y = \frac{3}{2}x$ and $y = -\frac{3}{2}x$

$$(b) \frac{x^2}{5} - \frac{y^2}{16} = 1$$



i. intercepts: $(-\sqrt{5}, 0), (\sqrt{5}, 0)$

ii. To find foci (F_1 and F_2)

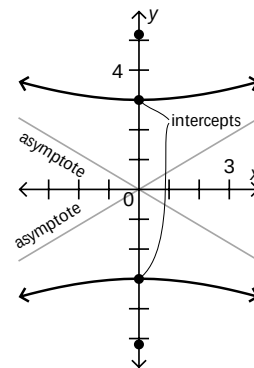
$$c^2 = 5 + 16$$

$$c = \sqrt{21}$$

$$F_1 (-\sqrt{21}, 0), F_2 (\sqrt{21}, 0)$$

iii. asymptotes: $y = \frac{4}{\sqrt{5}}x$ and $y = -\frac{4}{\sqrt{5}}x$

$$(c) \frac{x^2}{25} - \frac{y^2}{81} = -1$$



i. intercepts: $(0, 9), (0, -9)$

ii. To find foci (F_1 and F_2)

$$c^2 = 25 + 81$$

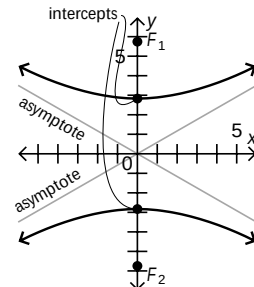
$$c^2 = 106$$

$$c = \sqrt{106}$$

$$F_1 (0, \sqrt{106}), F_2 (0, -\sqrt{106})$$

iii. asymptotes: $y = \frac{9}{5}x$ and $y = -\frac{9}{5}x$

$$(d) \frac{y^2}{8} - \frac{x^2}{25} = 1$$



i. intercepts: $(0, 2\sqrt{2}), (0, -2\sqrt{2})$

ii. To find foci (F_1 and F_2)

$$c^2 = 8 + 25$$

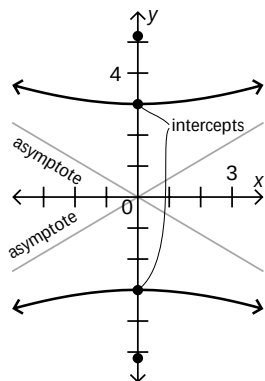
$$c^2 = 33$$

$$c = \sqrt{33}$$

$$F_1 (0, \sqrt{33}), F_2 (0, -\sqrt{33})$$

iii. asymptotes: $y = \frac{2\sqrt{2}}{5}x$ and $y = -\frac{2\sqrt{2}}{5}x$

(e) $\frac{x^2}{18} - \frac{y^2}{9} = -1$



i. intercepts: (0, 3), (0, -3)

ii. To find foci (F_1 and F_2)

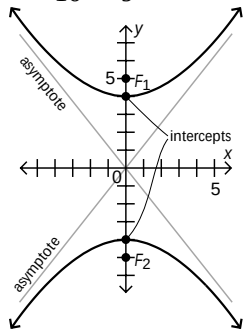
$$c^2 = 18 + 9$$

$$c = \sqrt{27}$$

$$F_1(0, \sqrt{27}), F_2(0, -\sqrt{27})$$

iii. asymptotes: $y = \frac{1}{\sqrt{2}}x$ and $y = -\frac{1}{\sqrt{2}}x$

(f) $\frac{y^2}{16} - \frac{x^2}{9} = 1$



i. intercepts: (0, 4), (0, -4)

ii. To find foci (F_1 and F_2)

$$c^2 = 16 + 9$$

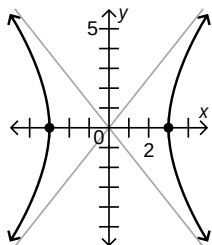
$$c = 5$$

$$F_1(0, 5), F_2(0, -5)$$

iii. asymptotes: $y = \frac{4}{3}x$ and $y = -\frac{4}{3}x$

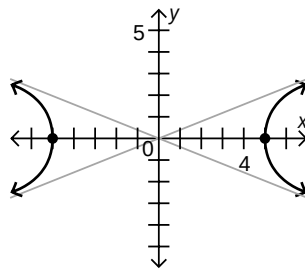
4. (a) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Centre is (0, 0); Vertices are (3, 0) and (-3, 0).



(b) $\frac{x^2}{25} - \frac{y^2}{4} = 1$

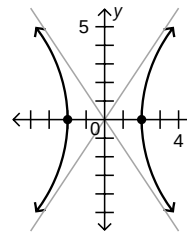
Centre is (0, 0); Vertices are (5, 0) and (-5, 0).



(c) $2x^2 - y^2 = 8$

$$\frac{x^2}{4} - \frac{y^2}{8} = 1$$

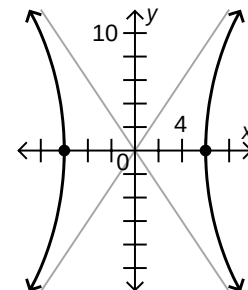
Centre is (0, 0); Vertices are (2, 0) and (-2, 0).



(d) $81x^2 - 36y^2 = 2916$

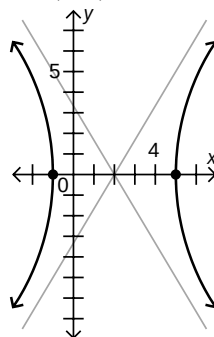
$$\frac{x^2}{36} - \frac{y^2}{81} = 1$$

Centre is (0, 0); Vertices are (6, 0) and (-6, 0).



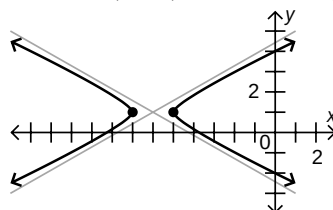
(e) $\frac{(x-2)^2}{9} - \frac{y^2}{25} = 1$

Centre is (2, 0); Vertices are (5, 0) and (-1, 0).



(f) $(x+6)^2 - 4(y-1)^2 = 1$

Centre is (-6, 1); Vertices are (-7, 1) and (-5, 1).



5. (a) $x^2 - y^2 = 1$

Asymptotes are $y = x$ and $y = -x$.

(b) $\frac{x^2}{4} - \frac{y^2}{16} = 1$

Asymptotes are $y = 2x$ and $y = -2x$.

(c) $\frac{x^2}{9} - \frac{y^2}{25} = -1$

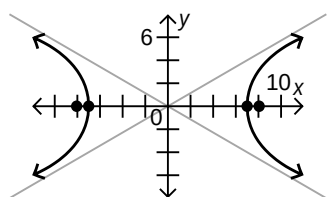
Asymptotes are $y = \frac{5}{3}x$ and $y = -\frac{5}{3}x$.

(d) $4x^2 - y^2 = 64$

$\frac{x^2}{16} - \frac{y^2}{64} = 1$

Asymptotes are $y = 2x$ and $y = -2x$.

6.



(a) Intercepts are $(7, 0)$ and $(-7, 0)$.

(b) The length of the transverse axis is 14 units.

(c) $c^2 = 49 + 16$, $c = \sqrt{65}$

The foci are $(\sqrt{65}, 0)$ and $(-\sqrt{65}, 0)$.

(d) The asymptotes are $y = \frac{4}{7}x$ and $y = -\frac{4}{7}x$.

7. (a) $a = 6$ $c = 8$

$c^2 = a^2 + b^2$

$64 = 36 + b^2$

$28 = b^2$

$b = \sqrt{28}$

$\therefore$ the equation is $\frac{x^2}{28} - \frac{y^2}{36} = -1$.

(b) $a = 3$ $c = 6$

$c^2 = a^2 + b^2$

$36 = 9 + b^2$

$27 = b^2$

$\sqrt{27} = b$

$\therefore \frac{x^2}{27} - \frac{y^2}{9} = -1$ is the equation.

(c) $a = 4$ $b = 2$

$\therefore \frac{x^2}{4} - \frac{y^2}{16} = -1$ is the equation.

(d) $a = 3$ $c = 5$

$c^2 = a^2 + b^2$

$25 = 9 + b^2$

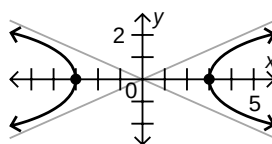
$16 = b^2$

$b = 4$

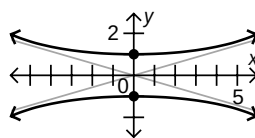
$\therefore \frac{x^2}{9} - \frac{y^2}{16} = 1$ is the equation.

8. (a) $x^2 - 9y^2 = 9$

$\frac{x^2}{9} - y^2 = 1$

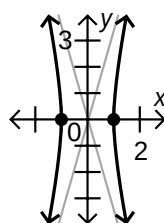


(b) $\frac{x^2}{9} - y^2 = -1$



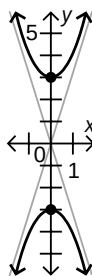
(c) $9x^2 - y^2 = 9$

$x^2 - \frac{y^2}{9} = 1$



(d) $9x^2 - y^2 = -9$

$x^2 - \frac{y^2}{9} = -1$



Comments: The asymptotes of the first 2 graphs are the same but the graphs are oriented different ways. The asymptotes of the second 2 graphs are the same but the graphs are oriented different ways.

9. Some of the properties of an equilateral hyperbola include:

- The asymptotes are $y = x$ and $y = -x$.
- The conjugate and transverse axes will have the same length.
- The value of c will be $\sqrt{2}a$.

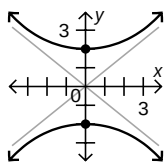
10. $c = 3$ $a = 2$

$c^2 = a^2 + b^2$

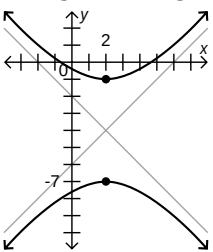
$9 = 4 + b^2$

$\sqrt{5} = b$

$\therefore$ the equation is $\frac{x^2}{5} - \frac{y^2}{4} = -1$.



11. $\frac{(x-2)^2}{9} - \frac{(y+4)^2}{9} = -1$



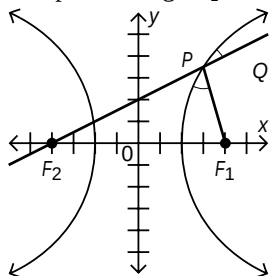
12. (a) $\frac{(x+3)^2}{9} - \frac{(y-2)^2}{25} = 1$

(b) $\frac{(x-4)^2}{9} - \frac{(y+3)^2}{25} = 1$

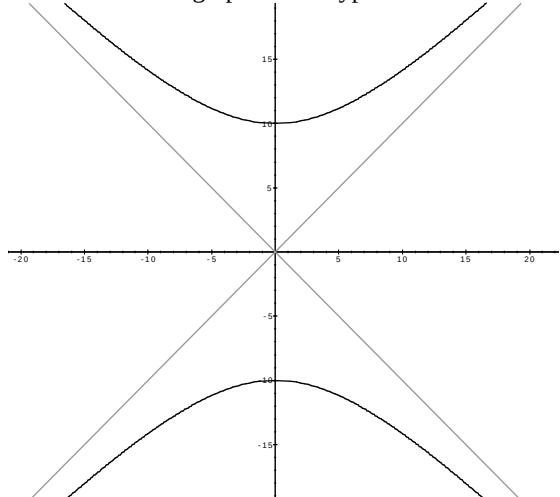
(c) $\frac{(x-2)^2}{9} - \frac{y^2}{25} = 1$

(d) $\frac{(x+8)^2}{9} - \frac{(y+4)^2}{25} = 1$

13. Answers will vary but should include the reflector properties of hyperbolas. A ray originating at one focal point, F_1 , will reflect at P and "follow" to Q . An extension of PQ will pass through F_2 .



14. Examine the graph of the hyperbola.



Rearrange the equation of the hyperbola.

$$x^2 - y^2 = -100$$

Divide by 100

$$-\frac{x^2}{100} + \frac{y^2}{100} = 1$$

$$-\left(\frac{x}{10}\right)^2 + \left(\frac{y}{10}\right)^2 = 1$$

The hyperbola crosses the y -axis at 10 and -10 . The hyperbola which crosses the y -axis at -10 is the part which is supporting the arch of the bridge. If the base has a width of 35 m, then the x -coordinates on either side of the y -axis are 17.5 and -17.5 ($35 \div 2$).

When $x = 17.5$,

$$(17.5)^2 - y^2 = -100$$

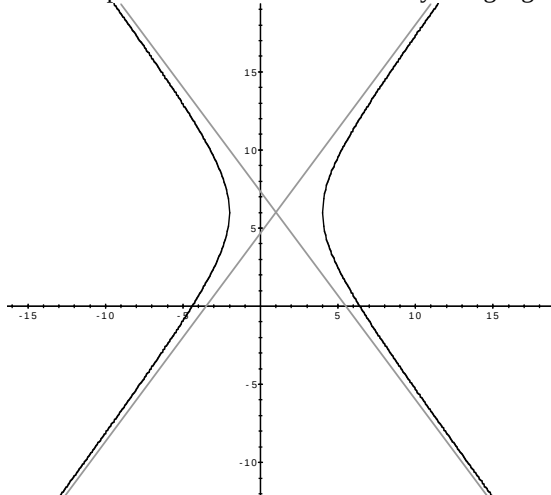
$$306.25 - y^2 = -100$$

$$406.25 = y^2$$

$$\pm 20.16 = y$$

The negative answer does not apply in the context of the question. The graph is translated down 10 units, so the actual height of the arch at its centre is $20.16 - 10 = 10.16$ m.

15. This problem can be solved visually using a graph.



$$2a = 4 - (-2)$$

$$2a = 6$$

$$\therefore a = 3$$

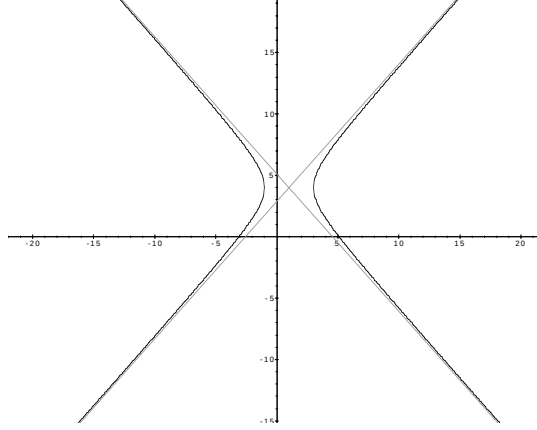
$$\text{Conjugate axis} = 8, \text{ ie. } 2b = 8,$$

$$\therefore b = 4$$

$$\text{Centre } (h, k) = (1, 6)$$

$$\therefore \text{the equation is } \frac{(x-1)^2}{9} - \frac{(y-6)^2}{16} = 1.$$

16. This problem can be solved visually using a graph.



Transverse axis = 4, ie. $2a = 4$

$$\therefore a = 2$$

Centre is (1, 4)

Foci are (4, 4) and (-2, 4)

$$c = \frac{4 - (-2)}{2}$$

$$c = \frac{6}{2}$$

$$\therefore c = 3$$

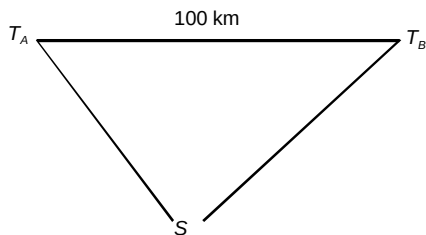
$$c^2 = a^2 + b^2$$

$$9 = 4 + b^2$$

$$b^2 = 5,$$

$$\therefore b = \sqrt{5}$$

$$\therefore \text{the equation is } \frac{(x-1)^2}{4} - \frac{(y-4)^2}{5} = 1.$$



17.

Think of points T_A and T_B as the foci of a hyperbola. Let point S be the location of the ship.

Given: $T_A S - T_B S = 30$

Since the difference between the distances is constant, define a hyperbola with T_A and T_B as the foci.

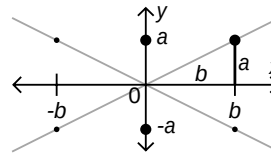
$$T_A S - T_B S = 2a \quad \text{and} \quad a = 15$$

Since $T_A S$ and $T_B S$ are 100 km apart, then $c = 50$.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (15)^2 + b^2 &= (50)^2 \\ 225 + b^2 &= 2500 \\ b^2 &= 2275 \end{aligned}$$

$$\therefore \text{the equation is } \frac{x^2}{225} - \frac{y^2}{2275} = 1.$$

18. To draw a graph of $\frac{x^2}{b^2} - \frac{y^2}{a^2} = -1$, the vertices would be at $(0, a)$ and $(0, -a)$. A diagram would look like this:



From the diagram, it is easy to see that the slope of the asymptotes would be $\frac{a}{b}$ and $-\frac{a}{b}$. $\therefore$ the equations of the asymptotes are $y = \frac{a}{b}x$ and $y = -\frac{a}{b}x$.

19. If the hyperbola were centred at the origin, the asymptotes would be $y = \frac{5}{3}x$ and $y = -\frac{5}{3}x$. However, the centre of the hyperbola is moved 2 to the right. Applying this translation to the asymptotes would yield $y = \frac{5}{3}(x - 2)$ and $y = -\frac{5}{3}(x - 2)$ as the asymptotes.

In general, the asymptotes for $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ would be $y - k = -\frac{b}{a}(x - h)$.

$$20. 3x^2 - y^2 = 12$$

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

$$a = 2 \quad b = \sqrt{12}$$

$$c^2 = 4 + 12$$

$$c = 4$$

Foci are (4, 0) and (-4, 0).

$$x^2 + 3y^2 = 24$$

$$\frac{x^2}{24} + \frac{y^2}{8} = 1$$

$$a = \sqrt{24} \quad b = \sqrt{8}$$

$$a^2 = b^2 + c^2$$

$$24 = 8 + c^2$$

$$c = 4$$

Foci are (4, 0) and (-4, 0).

Practise, Apply, Solve 7.10U, page 620

- | | |
|-----------------|----------------|
| 1. (a) Parabola | Vertex (-1, 2) |
| (b) Hyperbola | Centre (-1, 0) |
| (c) Ellipse | Centre (3, 8) |
| (d) Ellipse | Centre (0, -6) |
| (e) Parabola | Vertex (-1, 1) |
| (f) Hyperbola | Centre (2, -2) |

2. (a) Hyperbola
(b) Ellipse
(c) Parabola