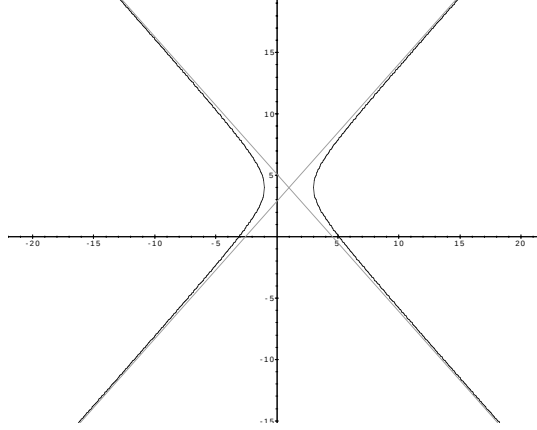


16. This problem can be solved visually using a graph.



Transverse axis = 4, ie. $2a = 4$

$$\therefore a = 2$$

Centre is (1, 4)

Foci are (4, 4) and (-2, 4)

$$c = \frac{4 - (-2)}{2}$$

$$c = \frac{6}{2}$$

$$\therefore c = 3$$

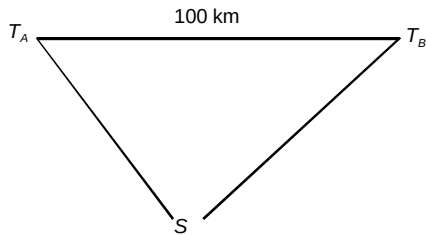
$$c^2 = a^2 + b^2$$

$$9 = 4 + b^2$$

$$b^2 = 5,$$

$$\therefore b = \sqrt{5}$$

$$\therefore \text{the equation is } \frac{(x-1)^2}{4} - \frac{(y-4)^2}{5} = 1.$$



17.

Think of points T_A and T_B as the foci of a hyperbola. Let point S be the location of the ship.

Given: $T_A S - T_B S = 30$

Since the difference between the distances is constant, define a hyperbola with T_A and T_B as the foci.

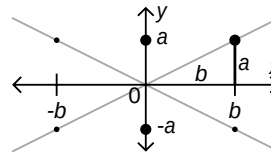
$$T_A S - T_B S = 2a \quad \text{and} \quad a = 15$$

Since $T_A S$ and $T_B S$ are 100 km apart, then $c = 50$.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (15)^2 + b^2 &= (50)^2 \\ 225 + b^2 &= 2500 \\ b^2 &= 2275 \end{aligned}$$

$$\therefore \text{the equation is } \frac{x^2}{225} - \frac{y^2}{2275} = 1.$$

18. To draw a graph of $\frac{x^2}{b^2} - \frac{y^2}{a^2} = -1$, the vertices would be at $(0, a)$ and $(0, -a)$. A diagram would look like this:



From the diagram, it is easy to see that the slope of the asymptotes would be $\frac{a}{b}$ and $-\frac{a}{b}$. $\therefore$ the equations of the asymptotes are $y = \frac{a}{b}x$ and $y = -\frac{a}{b}x$.

19. If the hyperbola were centred at the origin, the asymptotes would be $y = \frac{5}{3}x$ and $y = -\frac{5}{3}x$. However, the centre of the hyperbola is moved 2 to the right. Applying this translation to the asymptotes would yield $y = \frac{5}{3}(x - 2)$ and $y = -\frac{5}{3}(x - 2)$ as the asymptotes.

In general, the asymptotes for $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ would be $y - k = -\frac{b}{a}(x - h)$.

$$20. 3x^2 - y^2 = 12$$

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

$$a = 2 \quad b = \sqrt{12}$$

$$c^2 = 4 + 12$$

$$c = 4$$

Foci are (4, 0) and (-4, 0).

$$x^2 + 3y^2 = 24$$

$$\frac{x^2}{24} + \frac{y^2}{8} = 1$$

$$a = \sqrt{24} \quad b = \sqrt{8}$$

$$a^2 = b^2 + c^2$$

$$24 = 8 + c^2$$

$$c = 4$$

Foci are (4, 0) and (-4, 0).

Practise, Apply, Solve 7.10U, page 620

- | | |
|-----------------|----------------|
| 1. (a) Parabola | Vertex (-1, 2) |
| (b) Hyperbola | Centre (-1, 0) |
| (c) Ellipse | Centre (3, 8) |
| (d) Ellipse | Centre (0, -6) |
| (e) Parabola | Vertex (-1, 1) |
| (f) Hyperbola | Centre (2, -2) |

2. (a) Hyperbola
(b) Ellipse
(c) Parabola

- (d) Ellipse
(e) Circle
(f) Hyperbola
(g) Ellipse
(h) Parabola

3. (a) $x^2 - y^2 + 8x + 4y + 9 = 0$
 $x^2 + 8x - (y^2 - 4y) = -9$
 $x^2 + 8x + 16 - (y^2 - 4y + 4) = -9 + 16 - 4$

$$(x + 4)^2 - (y - 2)^2 = 3$$

$$\frac{(x + 4)^2}{3} - \frac{(y - 2)^2}{3} = 1$$

i. Hyperbola ii. Centre $(-4, 2)$

iii. Foci on horizontal axis

(b) $12x^2 + 4y^2 - 24x = 44$

$$3x^2 - 6x + y^2 = 11$$

$$3(x^2 - 2x + 1) + y^2 = 11 + 3$$

$$3(x - 1)^2 + y^2 = 14$$

$$\frac{(x - 1)^2}{\left(\frac{14}{3}\right)} + \frac{y^2}{14} = 1$$

i. Ellipse ii. Centre $(1, 0)$

iii. Foci on vertical axis

(c) $4x^2 - y^2 + 24x - 8y + 12 = 0$

$$4(x^2 + 6x + 9) - (y^2 + 8y + 16) = -12 + 36 - 16$$

$$4(x + 3)^2 - (y + 4)^2 = 8$$

$$\frac{(x + 3)^2}{2} - \frac{(y + 4)^2}{8} = 1$$

i. Hyperbola ii. Centre $(-3, -4)$

iii. Foci on horizontal axis

4. (a) Parabola

(b) $3x^2 - 6x - y + 1 = 0$

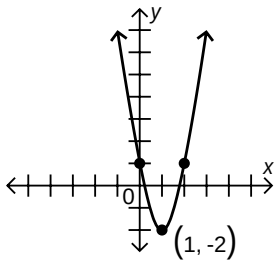
$$3(x^2 - 2x) = y - 1$$

$$3(x^2 - 2x + 1) = y - 1 + 3$$

$$3(x - 1)^2 = y + 2$$

$$(x - 1)^2 = \frac{1}{3}(y + 2)$$

(c)



5. (a) Hyperbola

(b) $y^2 - 3x^2 - 4y + 6x + 2 = 0$

$$-3x^2 + 6x + y^2 - 4y = -2$$

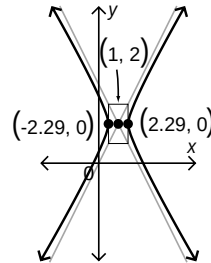
$$-3(x^2 - 2x + 1) + y^2 - 4y + 4 = -2 - 3 + 4$$

$$-3(x - 1)^2 + (y - 2)^2 = -1$$

$$3(x - 1)^2 - (y - 2)^2 = 1$$

$$\frac{(x - 1)^2}{\left(\frac{1}{3}\right)} - (y - 2)^2 = 1$$

(c)



6. (a) Circle

(b) $2x^2 + 2y^2 - 8x + 12y + 24 = 0$

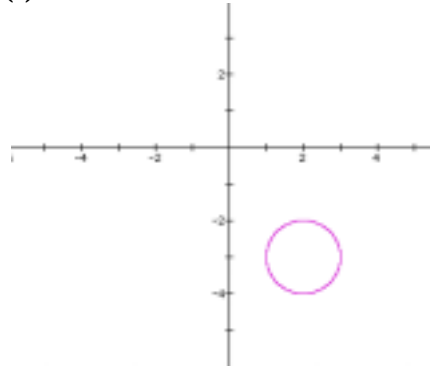
$$x^2 + y^2 - 4x + 6y + 12 = 0$$

$$x^2 - 4x + 4 + y^2 + 6y + 9 = -12 + 9 + 4$$

$$(x - 2)^2 + (y + 3)^2 = 1$$

Circle with centre $(2, -3)$ and radius 1.

(c)



7. (a) Ellipse

$$4x^2 + 9(y - 3)^2 = 36$$

$4x^2 + 9y^2 - 54y + 45 = 0$ is the general form.

(b) Ellipse

$$x^2 + 6x + 9 + y^2 - 2y + 1 = 9$$

$x^2 + y^2 + 6x - 2y + 1 = 0$ is the general form.

(c) Hyperbola

$$\frac{(x + 4)^2}{16} - \frac{(y - 3)^2}{4} = -1$$

$$(x + 4)^2 - 4(y - 3)^2 = -16$$

$$x^2 + 8x + 16 - 4(y^2 - 6y + 9) = -16$$

$$x^2 + 8x + 16 - 4y^2 + 24y - 36 = -16$$

$x^2 - 4y^2 + 8x + 24y - 4 = 0$ is the general form.

(d) Parabola

$$(x - 2)^2 = -6(y + 1)$$

$$x^2 - 4x + 4 = -6y - 6$$

$x^2 - 4x + 6y + 10 = 0$ is the general form.

(e) Hyperbola

$$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{5} = 1$$

$$5(x-1)^2 - 4(y+2)^2 = 20$$

$$5(x^2 - 2x + 1) - 4(y^2 + 4y + 4) = 20$$

$$5x^2 - 10x + 5 - 4y^2 - 16y - 16 = 20$$

$$5x^2 - 4y^2 - 10x - 16y - 31 = 0 \text{ is the general form.}$$

(f) Parabola

$$(y+3)^2 = 2(x-1)$$

$$y^2 + 6y + 9 = 2x - 2$$

$$y^2 + 6y - 2x + 11 = 0 \text{ is the general form.}$$

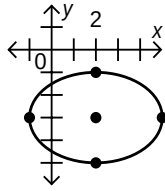
8. (a) $4x^2 + 9y^2 - 16x + 54y + 61 = 0$

$$4(x^2 - 4x) + 9(y^2 + 6y) = -61$$

$$4(x^2 - 4x + 4) + 9(y^2 + 6y + 9) = -61 + 16 + 81$$

$$4(x-2)^2 + 9(y+3)^2 = 36$$

$$\frac{(x-2)^2}{9} + \frac{(y+3)^2}{4} = 1$$

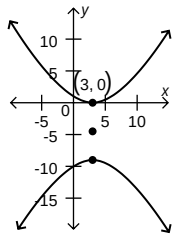


(b) $9x^2 - 4y^2 - 54x - 36y + 81 = 0$

$$9(x^2 - 6x + 9) - 4\left(y^2 + 9y + \frac{81}{4}\right) = -81 + 81 - 81$$

$$9(x-3)^2 - 4\left(y + \frac{9}{2}\right)^2 = -81$$

$$\frac{(x-3)^2}{9} - \frac{4\left(y + \frac{9}{2}\right)^2}{81} = -1$$

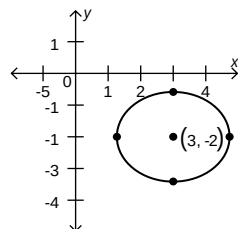


(c) $2x^2 + 3y^2 - 12x + 12y + 24 = 0$

$$2(x^2 - 6x + 9) + 3(y^2 + 4y + 4) = -24 + 18 + 12$$

$$2(x-3)^2 + 3(y+2)^2 = 6$$

$$\frac{(x-3)^2}{3} + \frac{(y+2)^2}{2} = 1$$



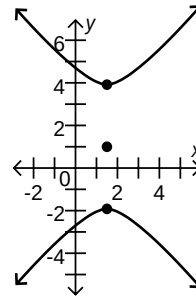
(d) $y^2 - 2x^2 - 2y + 6x - 12 = 0$

$$-2x^2 + 6x + y^2 - 2y = 12$$

$$-2\left(x^2 - 3x + \frac{9}{4}\right) + y^2 - 2y + 1 = 12 + 1 - \frac{9}{2}$$

$$-2\left(x - \frac{3}{2}\right)^2 + (y-1)^2 = \frac{17}{2}$$

$$\frac{-4\left(x - \frac{3}{2}\right)^2}{17} + \frac{2(y-1)^2}{17} = 1$$

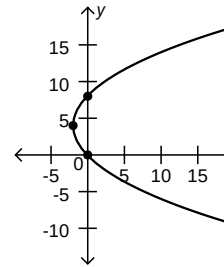


(e) $y^2 - 8x - 8y = 0$

$$y^2 - 8y = 8x$$

$$y^2 - 8y + 16 = 8x + 16$$

$$(y-4)^2 = 8(x+2)$$



9. At first it looks as though this could be the equation of an ellipse since a and b are not equal to 1. However $a = b$ and therefore this is the equation of a circle.

10. (a) $y^2 - 5x^2 + 20x = 50$

$$-5(x^2 - 4x + 4) + y^2 = 50 - 20$$

$$-5(x-2)^2 + y^2 = 30$$

$$\frac{(x-2)^2}{6} - \frac{y^2}{30} = -1$$

$\therefore$ The centre is $(2, 0)$.

(b) If the hyperbola were in standard position, then the vertices would be $(0, \pm\sqrt{30})$. Since the centre of the hyperbola is $(2, 0)$, the vertices will be $(2, \sqrt{30})$ and $(2, -\sqrt{30})$.

To find the foci, use $c^2 = a^2 + b^2$

$$a = \sqrt{30} \quad b = \sqrt{6}$$

$$c^2 = 30 + 6$$

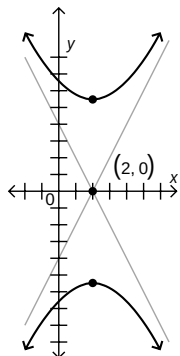
$$c = 6$$

$\therefore$ the foci are $(2, 6)$ and $(2, -6)$.

(c) The length of the transverse axis is $2a$ or $2\sqrt{30}$. The length of the conjugate axis is $2b$ or $2\sqrt{6}$.

(d) The equations of the asymptotes are given by $y = \pm \frac{a}{b}x$ when centred at the origin. $\therefore$ the equations of the asymptotes are $y = \pm \frac{\sqrt{30}}{\sqrt{6}}x$ or $y = \pm \sqrt{5}x$ if the conic was centred at the origin. However, since it is centred at $(2, 0)$, the asymptotes will be $y = \pm \sqrt{5}(x - 2)$.

(e)



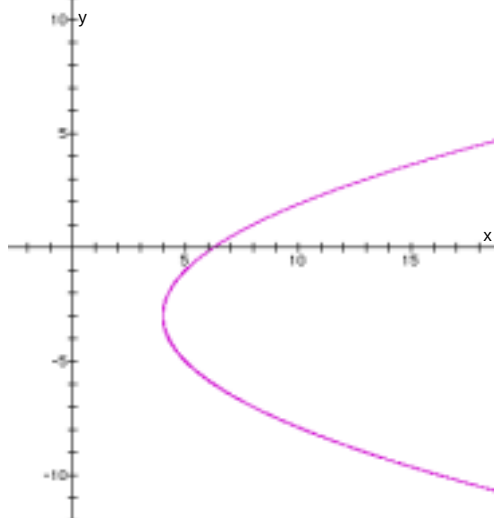
11. A quick sketch of the graph shows that the graph is horizontal and is thus of the form $(y - k)^2 = 4p(x - h)$. The focal length is 3 and the vertex is $(4, 2)$. Therefore, $p = -3$, $h = 4$, $k = 2$ and the equation is $(y - 2)^2 = -12(x - 4)$ in standard form. So,
 $y^2 - 4y + 4 + 12x - 48 = 0$
 $y^2 - 4y + 12x - 44 = 0$ is the equation in general form.

12. A circle with centre $(2, 3)$ and radius of 5 would have a standard form of $(x - 2)^2 + (y - 3)^2 = 25$. Expanding this:
 $x^2 - 4x + 4 + y^2 - 6y + 9 = 25$
 $x^2 + y^2 - 4x - 6y - 12 = 0$ is the general form of the equation.

13. (a) Parabola

$$y^2 + 6y + 9 = 4x - 25 + 9$$

$$(y + 3)^2 = 4(x - 4)$$



(b) Hyperbola

$$9x^2 - 16y^2 - 36x + 96y + 36 = 0$$

$$9x^2 - 36x - 16y^2 + 96y = -36$$

$$9(x^2 - 4x) - 16(y^2 - 6y) = -36$$

$$9(x^2 - 4x + 4) - 16(y^2 - 6y + 9) = -36 + 36 - 144$$

$$9(x - 2)^2 - 16(y - 3)^2 = -144$$

$$\frac{(x - 2)^2}{16} - \frac{(y - 3)^2}{9} = -1$$

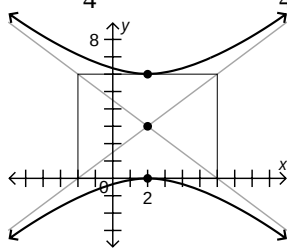
Centre: $(2, 3)$

The asymptotes have slope $\pm \frac{3}{4}$ and contain $(2, 3)$. Thus,

their equations are:

$$\frac{y - 3}{x - 2} = \frac{3}{4} \text{ and } \frac{y - 3}{x - 2} = -\frac{3}{4};$$

$$y = \frac{3x + 6}{4} \text{ and } y = -\frac{3x + 18}{4}.$$



(c) Ellipse

$$4x^2 - 8x + y^2 - 12 = 0$$

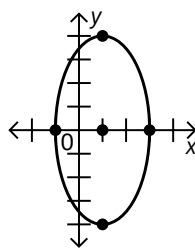
$$4x^2 - 8x + y^2 = 12$$

$$4(x^2 - 2x) + y^2 = 12$$

$$4(x^2 - 2x + 1) + y^2 = 12 + 4$$

$$4(x - 1)^2 + y^2 = 16$$

$$\frac{(x - 1)^2}{4} + \frac{y^2}{16} = 1$$

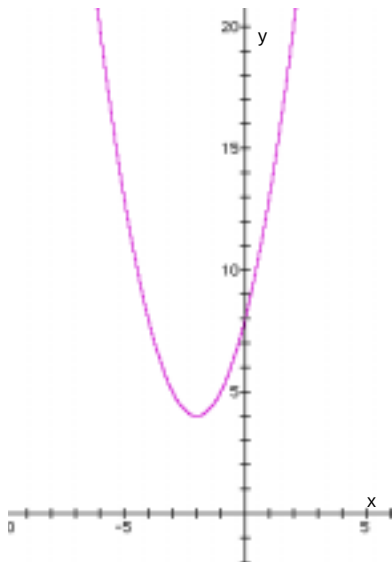


(d) Parabola

$$x^2 + 4x - y + 8 = 0$$

$$x^2 + 4x + 4 = y - 8$$

$$(x + 2)^2 = y - 8$$



(e) Ellipse

$$25x^2 + 4y^2 + 100x - 16y + 16 = 0$$

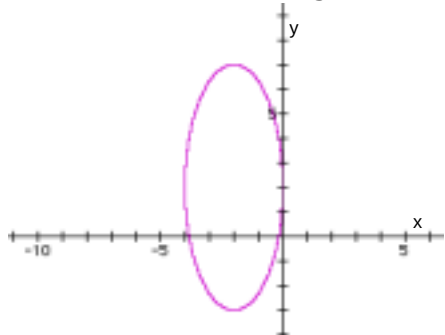
$$25x^2 + 100x + 4y^2 - 16y = -16$$

$$25(x^2 + 4x) + 4(y^2 - 4y) = -16$$

$$25(x^2 + 4x + 4) + 4(y^2 - 4y + 4) = -16 + 100 + 16$$

$$25(x+2)^2 + 4(y-2)^2 = 100$$

$$\frac{(x+2)^2}{4} + \frac{(y-2)^2}{25} = 1$$



14. The equation is of the form $(y-h)^2 = 4p(x-k)^2$ where (k, h) is the vertex and p is the focal length. In this case,

$$p = \frac{1}{2}, k = 1, h = 4.$$

$$(y-1)^2 = 4p(x-4)^2$$

$$(y-1)^2 = 2(x-4)^2$$

$$y^2 - 2y + 1 = 2x^2 - 16x + 32$$

$$2x^2 - y^2 - 16x + 2y + 31 = 0$$

15. $x^2 + y^2 + 6x + 6y + 2 = 0$

$$x^2 + 6x + 9 + y^2 + 6y + 9 = -2 + 9 + 9$$

$$(x+3)^2 + (y+3)^2 = 16$$

This circle has a centre at $(-3, -3)$. The translation to move the centre back to the origin would be a translation of 3 units right and 3 units up, or $(x, y) \rightarrow (x+3, y+3)$.

16. Since the vertex is $(5, 0)$, then $a = 5$. Since the focus is $(6, 0)$, $c = 6$. Since the vertices and foci are on the x -axis, the hyperbola has a horizontal orientation. To find b , use:

$$c^2 = a^2 + b^2$$

$$36 = 25 + b^2$$

$$\therefore b = \sqrt{11}$$

The standard form of the equation is $\frac{x^2}{25} - \frac{y^2}{11} = 1$. Changing this to general form would give $11x^2 - 25y^2 - 275 = 0$.

17. The centre of the circle will be the midpoint of the diameter (AB) . Therefore, the centre is $C(2, 3)$. The radius is the distance between the centre and either A or B .

$$AC = \sqrt{(-2-2)^2 + (4-3)^2}$$

$$= \sqrt{16+1}$$

$$= \sqrt{17}$$

$\therefore$ the standard form is $(x-2)^2 + (y-3)^2 = 17$. To find the general form, expand:

$$x^2 - 4x + 4 + y^2 - 6y + 9 = 17$$

$$x^2 + y^2 - 4x - 6y - 4 = 0$$

18. Answers may vary.

19. First, find the vertices of the ellipse.

$$7x^2 + 11y^2 = 77$$

$$\frac{x^2}{11} + \frac{y^2}{7} = 1$$

Vertices are $(\sqrt{11}, 0)$ and $(-\sqrt{11}, 0)$.

Now, find the foci of the ellipse.

$$a^2 = b^2 + c^2$$

$$11 = 7 + c^2$$

$$c = 2$$

$\therefore$ the foci are $(2, 0)$ and $(-2, 0)$.

Next, find the equation of the hyperbola with vertices of

$(2, 0)$ and $(-2, 0)$ and foci of $(\sqrt{11}, 0)$ and $(-\sqrt{11}, 0)$.

In a hyperbola,

$$c^2 = a^2 + b^2$$

$$11 = 4 + b^2$$

$$b = \sqrt{7}$$

The hyperbola is of the form $\frac{x^2}{4} - \frac{y^2}{7} = 1$.

20. (a) This is an ellipse where $2a = 10$.

$a = 5, c = 3, b = 4$ Centre is $(1, 0)$

Equation is $\frac{(x-1)^2}{25} + \frac{y^2}{16} = 1$.

$$16(x-1)^2 + 25y^2 = 400$$

$$16(x^2 - 2x + 1) + 25y^2 = 400$$

$$16x^2 - 32x + 16 + 25y^2 = 400$$

$$16x^2 + 25y^2 - 32x - 384 = 0$$

(b) This is a hyperbola where $2a = 6$.

$a = 3, c = 4, b = \sqrt{7}$ Centre is $(-1, 0)$

Equation is $\frac{(x+1)^2}{9} - \frac{y^2}{7} = 1$.

$$7(x+1)^2 - 9y^2 = 63$$

$$7x^2 + 14x + 7 - 9y = 63$$

$$7x^2 - 9y^2 + 14x - 56 = 0$$

(c) This is an ellipse where $2a = 10$.

$a = 5, c = 4, b = 3$ Centre is $(-2, 3)$

Equation is $\frac{(x+2)^2}{25} + \frac{(y-3)^2}{9} = 1$.

$$9(x+2)^2 + 25(y-3)^2 = 225$$

$$9x^2 + 36x + 36 + 25y^2 - 150y - 225 = 225$$

$$9x^2 + 25y^2 + 36x - 150y + 36 = 0$$

(d) This is a hyperbola where $2a = 10$.

$a = 5, c = 6, b = \sqrt{11}$ Centre is $(-1, 4)$

Equation is $\frac{(x+1)^2}{25} - \frac{(y-4)^2}{11} = 1$.

$$11(x+1)^2 - 25(y-4)^2 = 275$$

$$11x^2 + 22x + 11 - 25y^2 + 200y - 400 = 275$$

$$11x^2 - 25y^2 + 22x + 200y - 664 = 0$$

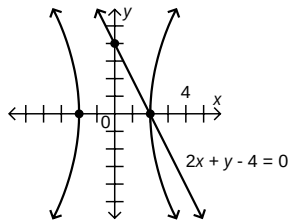
Practise, Apply, Solve 7.11U, page 626

1. (a) $(-3, 4)$ and $(4, -3)$

(b) $(-2, 1)$

(c) $(-1, -2)$ and $(1, -2)$

2.



$$4x^2 - y^2 = 16$$

$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$

(a) The co-ordinates of the intersection point is $(2, 0)$.

(b) Substitute $(2, 0)$ into the equations:

$$\text{L.S.} = 4x^2 - y^2 \quad \text{R.S.} = 16$$

$$= 4(2)^2 - 0^2$$

$$= 4(4) - 0$$

$$= 16$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$$\text{L.S.} = 2x + y - 4 \quad \text{R.S.} = 0$$

$$= 2(2) + 0 - 4$$

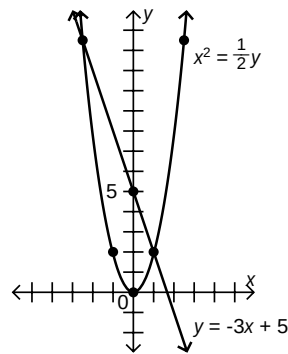
$$= 4 + 0 - 4$$

$$= 0$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$\therefore (2, 0)$ works in both equations.

3.



(a) Intersection points are $(1, 2)$ and $(-2.5, 12.5)$.

(b) Substitute $(1, 2)$ into the equations:

$$\text{L.S.} = x^2$$

$$= 1^2$$

$$= 1$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$$\text{R.S.} = \frac{1}{2}y$$

$$= \frac{1}{2}(2)$$

$$= 1$$

$$\text{L.S.} = y$$

$$= 2$$

$$\text{R.S.} = -3x + 5$$

$$= -3(1) + 5$$

$$= 2$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$\therefore (1, 2)$ works in both equations.

Substitute $(-2.5, 12.5)$ into the equations.

$$\text{L.S.} = x^2$$

$$= (-2.5)^2$$

$$= 6.25$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$$\text{R.S.} = \frac{1}{2}y$$

$$= \frac{1}{2}(12.5)$$

$$= 6.25$$

$$\text{L.S.} = y$$

$$= 12.5$$

$$\text{R.S.} = -3x + 5$$

$$= -3(-2.5) + 5$$

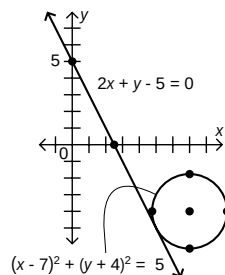
$$= 7.5 + 5$$

$$= 12.5$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$\therefore (-2.5, 12.5)$ also works in both equations.

4.



(a) The intersection is the point where $x = 5$ and $y = -5$.