

Equation is $\frac{(x+1)^2}{9} - \frac{y^2}{7} = 1$.

$$7(x+1)^2 - 9y^2 = 63$$

$$7x^2 + 14x + 7 - 9y = 63$$

$$7x^2 - 9y^2 + 14x - 56 = 0$$

(c) This is an ellipse where $2a = 10$.

$a = 5, c = 4, b = 3$ Centre is $(-2, 3)$

Equation is $\frac{(x+2)^2}{25} + \frac{(y-3)^2}{9} = 1$.

$$9(x+2)^2 + 25(y-3)^2 = 225$$

$$9x^2 + 36x + 36 + 25y^2 - 150y - 225 = 225$$

$$9x^2 + 25y^2 + 36x - 150y + 36 = 0$$

(d) This is a hyperbola where $2a = 10$.

$a = 5, c = 6, b = \sqrt{11}$ Centre is $(-1, 4)$

Equation is $\frac{(x+1)^2}{25} - \frac{(y-4)^2}{11} = 1$.

$$11(x+1)^2 - 25(y-4)^2 = 275$$

$$11x^2 + 22x + 11 - 25y^2 + 200y - 400 = 275$$

$$11x^2 - 25y^2 + 22x + 200y - 664 = 0$$

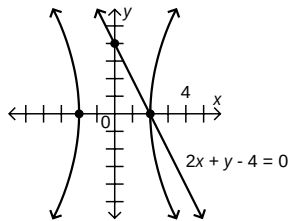
Practise, Apply, Solve 7.11U, page 626

1. (a) $(-3, 4)$ and $(4, -3)$

(b) $(-2, 1)$

(c) $(-1, -2)$ and $(1, -2)$

2.



$$4x^2 - y^2 = 16$$

$$\frac{x^2}{4} - \frac{y^2}{16} = 1$$

(a) The co-ordinates of the intersection point is $(2, 0)$.

(b) Substitute $(2, 0)$ into the equations:

$$\text{L.S.} = 4x^2 - y^2 \quad \text{R.S.} = 16$$

$$= 4(2)^2 - 0^2$$

$$= 4(4) - 0$$

$$= 16$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$$\text{L.S.} = 2x + y - 4 \quad \text{R.S.} = 0$$

$$= 2(2) + 0 - 4$$

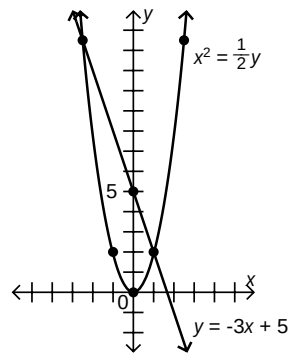
$$= 4 + 0 - 4$$

$$= 0$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$\therefore (2, 0)$ works in both equations.

3.



(a) Intersection points are $(1, 2)$ and $(-2.5, 12.5)$.

(b) Substitute $(1, 2)$ into the equations:

$$\text{L.S.} = x^2 \quad \text{R.S.} = \frac{1}{2}y$$

$$= 1^2 \quad = \frac{1}{2}(2)$$

$$= 1 \quad = 1$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$$\begin{aligned} \text{L.S.} &= y \\ &= 2 \end{aligned} \quad \begin{aligned} \text{R.S.} &= -3x + 5 \\ &= -3(1) + 5 \\ &= 2 \end{aligned}$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$\therefore (1, 2)$ works in both equations.

Substitute $(-2.5, 12.5)$ into the equations.

$$\text{L.S.} = x^2 \quad \text{R.S.} = \frac{1}{2}y$$

$$= (-2.5)^2 \quad = \frac{1}{2}(12.5)$$

$$= 6.25 \quad = 6.25$$

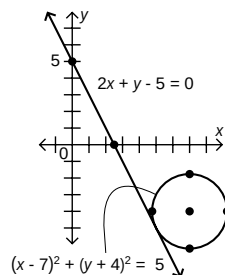
$$\therefore \text{L.S.} = \text{R.S.}$$

$$\begin{aligned} \text{L.S.} &= y \\ &= 12.5 \end{aligned} \quad \begin{aligned} \text{R.S.} &= -3x + 5 \\ &= -3(-2.5) + 5 \\ &= 7.5 + 5 \\ &= 12.5 \end{aligned}$$

$$\therefore \text{L.S.} = \text{R.S.}$$

$\therefore (-2.5, 12.5)$ also works in both equations.

4.



(a) The intersection is the point where $x = 5$ and $y = -5$.

(b) Substitute in both equations to check:

$$\begin{aligned} \text{L.S.} &= (x-7)^2 + (y+4)^2 & \text{R.S.} &= 5 \\ &= (5-7)^2 + (-5+4)^2 \\ &= (-2)^2 + (-1)^2 \\ &= 4 + 1 \\ &= 5 \end{aligned}$$

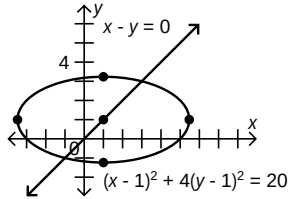
$\therefore \text{L.S.} = \text{R.S.}$

$$\begin{aligned} \text{L.S.} &= 2x + y - 5 & \text{R.S.} &= 0 \\ &= 2(5) + (-5) - 5 \\ &= 10 - 5 - 5 \\ &= 0 \end{aligned}$$

$\therefore \text{L.S.} = \text{R.S.}$

$\therefore$ the point $(5, -5)$ works in both equations.

5.



$$(x-1)^2 + 4(y-1)^2 = 20$$

$$\frac{(x-1)^2}{20} + \frac{(y-1)^2}{5} = 1$$

(a) The intersection points are $(-1, -1)$ and $(3, 3)$.

(b) Substitute the points into the equations:

Checking $(-1, -1)$:

$$\begin{aligned} \text{L.S.} &= (x-1)^2 + 4(y-1)^2 & \text{R.S.} &= 20 \\ &= (-1-1)^2 + 4(-1-1)^2 \\ &= (-2)^2 + 4(-2)^2 \\ &= 4 + 4(4) \\ &= 20 \end{aligned}$$

$\therefore \text{L.S.} = \text{R.S.}$

$$\begin{aligned} \text{L.S.} &= x - y & \text{R.S.} &= 0 \\ &= -1 - (-1) \\ &= -1 + 1 \\ &= 0 \end{aligned}$$

$\therefore \text{L.S.} = \text{R.S.}$

$\therefore (-1, -1)$ works in both equations.

Checking $(3, 3)$:

$$\begin{aligned} \text{L.S.} &= (x-1)^2 + 4(y-1)^2 & \text{R.S.} &= 20 \\ &= (3-1)^2 + 4(3-1)^2 \\ &= (2)^2 + 4(2)^2 \\ &= 4 + 16 \\ &= 20 \end{aligned}$$

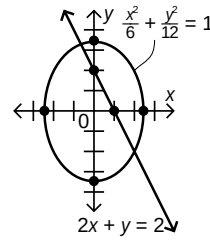
$\therefore \text{L.S.} = \text{R.S.}$

$$\begin{aligned} \text{L.S.} &= x - y & \text{R.S.} &= 0 \\ &= 3 - 3 \\ &= 0 \end{aligned}$$

$\therefore \text{L.S.} = \text{R.S.}$

$\therefore (3, 3)$ also works in both equations.

6.



(a) $(2, -2)$ appears to be one intersection point. The second intersection point is difficult to tell by the graph. It appears that $-1 < x < 0$.

(b) Algebraic solution:

$$\frac{x^2}{6} + \frac{y^2}{12} = 1 \text{-----} \textcircled{1}$$

Substitute $y = 2 - 2x$ into equation $\textcircled{1}$

$$\frac{x^2}{6} + \frac{(2-2x)^2}{12} = 1$$

$$2x^2 + (2-2x)^2 = 12$$

$$2x^2 + 4 - 8x + 4x^2 = 12$$

$$6x^2 - 8x - 8 = 0$$

$$3x^2 - 4x - 4 = 0$$

$$(3x+2)(x-2) = 0$$

$$x = -\frac{2}{3} \text{ or } x = 2$$

$$\text{If } x = 2, y = -2$$

$$\text{If } x = -\frac{2}{3},$$

$$y = 2 - 2x$$

$$y = 2 - 2\left(-\frac{2}{3}\right)$$

$$y = 2 + \frac{4}{3}$$

$$y = \frac{10}{3}$$

$\therefore \left(-\frac{2}{3}, \frac{10}{3}\right)$ and $(2, -2)$ are the solutions to the system.

7. (a)

$$x^2 + (x+2)^2 = 100$$

$$x^2 + x^2 + 4x + 4 = 100$$

$$2x^2 + 4x - 96 = 0$$

$$x^2 + 2x - 48 = 0$$

$$(x+8)(x-6) = 0$$

$$x = -8 \text{ or } x = 6$$

$$\text{If } x = -8, y = -6 \text{ and if } x = 6, y = 8.$$

$\therefore$ the points of intersection are $(-8, -6)$ and $(6, 8)$.

(b)

$$x^2 - 5^2 = -4$$

$$x^2 - 25 = -4$$

$$x^2 = 21$$

$$x = \pm\sqrt{21}$$

∴ the points of intersection are $(\sqrt{21}, 5)$ and $(-\sqrt{21}, 5)$.

(c)

$$x^2 = (2 - 3x) + 2$$

$$x^2 = 4 - 3x$$

$$x^2 + 3x - 4 = 0$$

$$(x - 1)(x + 4) = 0$$

$$x = 1 \text{ or } x = -4$$

If $x = 1$, $y = -1$ and if $x = -4$, $y = 14$.

∴ the points of intersection are $(1, -1)$ and $(-4, 14)$.

(d)

$$4x^2 + (2x - 1)^2 = 13$$

$$4x^2 + 4x^2 - 4x + 1 = 13$$

$$8x^2 - 4x - 12 = 0$$

$$2x^2 - x - 3 = 0$$

$$2x^2 + 2x - 3x - 3 = 0$$

$$2x(x + 1) - 3(x + 1) = 0$$

$$(2x - 3)(x + 1) = 0$$

$$x = \frac{3}{2} \text{ or } x = -1$$

If $x = \frac{3}{2}$, $y = 2$ and if $x = -1$, $y = -3$

∴ the points of intersection are $(\frac{3}{2}, 2)$ and $(-1, -3)$.

(e)

$$x^2 + (2 - x)^2 = 25$$

$$x^2 + 4 - 4x + x^2 = 25$$

$$2x^2 - 4x - 21 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 168}}{4}$$

$$x = \frac{4 \pm \sqrt{184}}{4}$$

$$x \doteq -2.4 \text{ or } x \doteq 4.4$$

If $x = -2.4$, $y = 4.4$ and if $x = 4.4$, $y = -2.4$.

∴ the intersection points are $(-2.4, 4.4)$ and $(4.4, -2.4)$.

8.

$$5x - 2y = 19$$

$$5x - 19 = 2y$$

$$\frac{5}{2}x - \frac{19}{2} = y$$

$$x^2 - \left(\frac{5}{2}x - \frac{19}{2}\right)^2 = -45$$

$$x^2 - \left(\frac{25}{4}x^2 - \frac{95}{2}x + \frac{361}{4}\right) = -45$$

$$4x^2 - 25x^2 + 190x - 361 = -180$$

$$-21x^2 + 190x - 181 = 0$$

$$x = \frac{-190 \pm \sqrt{190^2 - 4(-21)(-181)}}{2(-21)}$$

$$x = \frac{-190 \pm \sqrt{20\,896}}{-42}$$

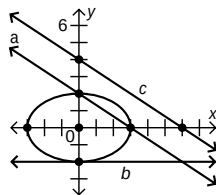
$$x \doteq \frac{-190 \pm 144.55}{-42}$$

$$x = 1.08 \text{ or } x = 7.97$$

If $x = 1.08$, $y = -6.8$ and if $x = 7.97$, $y = 10.43$.

∴ the sonic boom will be heard at these points: $(1.08, -6.8)$ and $(7.97, 10.43)$.

9. Many answers are possible. These are examples of possible answers.



(a) The line through $(0, 2)$ and $(3, 0)$ will intersect the ellipse in 2 places. This line has the equation $y = -\frac{2}{3}x + 2$.

(b) A horizontal line through $(0, -2)$ will intersect the ellipse in only 1 point. The line has the equation $y = -2$.

(c) The line through $(0, 4)$ and $(6, 0)$ will not intersect the ellipse. This line has the equation $y = -\frac{2}{3}x + 4$.

To verify the results algebraically:

(a)

$$4x^2 + 9\left(-\frac{2}{3}x + 2\right)^2 = 36$$

$$4x^2 + 9\left(\frac{4}{9}x^2 - \frac{8}{3}x + 4\right) = 36$$

$$4x^2 + 4x^2 - 24x + 36 = 36$$

$$8x^2 - 24x = 0$$

$$8x(x - 3) = 0$$

$$x = 0 \text{ or } x = 3$$

(yields 2 solutions)

(b)

$$4x^2 + 9(-2)^2 = 36$$

$$4x^2 + 36 = 36$$

$$4x^2 = 0$$

$$x = 0$$

(yields 1 solution)

(c)

$$4x^2 + 9\left(-\frac{2}{3}x + 4\right)^2 = 36$$

$$4x^2 + 9\left(\frac{4}{9}x^2 - \frac{16}{3}x + 16\right) = 36$$

$$4x^2 + 4x^2 - 48x + 144 = 36$$

$$8x^2 - 48x + 108 = 0$$

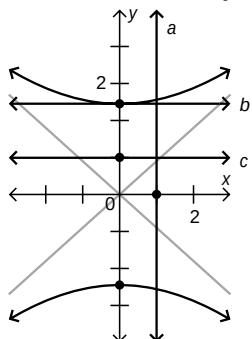
$$2x^2 - 12x + 27 = 0$$

$$x = \frac{12 \pm \sqrt{144 - 8(27)}}{4}$$

$$x = \frac{12 \pm \sqrt{-72}}{4}$$

(yields no solution)

10. Answers will vary. These are examples.



(a) A vertical line such as $x = 1$ will cross the graph at two points. Algebraic verification:

$$2(1)^2 - 3y^2 = -18$$

$$2 - 3y^2 = -18$$

$$-3y^2 = -20$$

$$y^2 = \frac{20}{3}$$

$$y = \pm \sqrt{\frac{20}{3}}$$

will yield 2 solutions

(b) A horizontal line through one of the vertices will intersect in 1 point. $\therefore y = \sqrt{6}$ will intersect in 1 point.

Algebraic verification:

$$2x^2 - 3(\sqrt{6})^2 = -18$$

$$2x^2 - 3(6) = -18$$

$$2x^2 = 0$$

$$x = 0$$

will yield 1 solution

(c) A horizontal line such as $y = 1$ will not intersect the hyperbola. Algebraic verification:

$$2x^2 - 3(1)^2 = -18$$

$$2x^2 - 3 = -18$$

$$2x^2 = -15$$

$$x^2 = -\frac{15}{2}$$

will yield no solution

11.

$$(x-1)^2 + (x+1+2)^2 = 9$$

$$x^2 - 2x + 1 + x^2 + 6x + 9 = 9$$

$$2x^2 + 4x + 1 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 8}}{4}$$

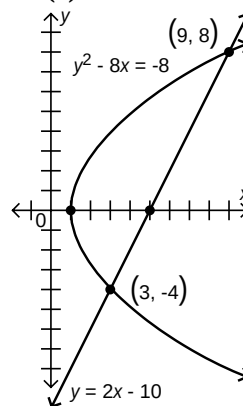
$$x = \frac{-4 \pm \sqrt{8}}{4}$$

$$x \doteq -0.29 \text{ or } x \doteq -1.71$$

If $x \doteq -0.29$, then $y \doteq 0.71$ and if $x \doteq -1.71$, then $y \doteq -0.71$.

$\therefore$ the intersection points are approximately $(-0.29, 0.71)$ and $(-1.71, -0.71)$.

12. (a)



Algebraic Solution:

$$x = \frac{y+10}{2}, \text{ substitute in the other equation}$$

$$y^2 - 8\left(\frac{y+10}{2}\right) = -8$$

$$y^2 - 4y - 40 = -8$$

$$y^2 - 4y - 32 = 0$$

$$(y-8)(y+4) = 0$$

$$y = 8 \text{ or } y = -4$$

Find x :

$$8 = 2x - 10$$

$$-4 = 2x - 10$$

$$x = 9$$

$$3 = x$$

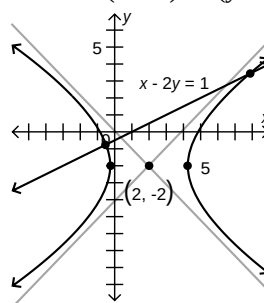
$\therefore$ the solutions are $(9, 8)$ and $(3, -4)$.

(b)

$$x^2 - 4x - y^2 - 5 - 4y = 0$$

$$x^2 - 4x + 4 - (y^2 + 4y + 4) = 5 + 4 - 4$$

$$(x-2)^2 - (y+2)^2 = 5$$



Algebraic solution:

Substitute $x = 1 + 2y$ in $(x - 2)^2 - (y + 2)^2 = 5$

$$(1 + 2y - 2)^2 - (y + 2)^2 = 5$$

$$(2y - 1)^2 - (y + 2)^2 = 5$$

$$4y^2 - 4y + 1 - y^2 - 4y - 4 = 5$$

$$3y^2 - 8y - 8 = 0$$

$$y = \frac{8 \pm \sqrt{64 + 96}}{6}$$

$$y = \frac{8 \pm \sqrt{160}}{6}$$

$$y \doteq 3.44 \text{ or } y \doteq -0.77$$

If $y \doteq 3.44$, $x \doteq 7.88$ and if $y \doteq -0.77$, $x \doteq -0.54$.

$\therefore$ the intersections are $(7.88, 3.44)$ and $(-0.54, -0.77)$.

13. Substitute $y = \frac{-1 - 4x}{3}$ in $(x - 2)^2 + (y + 3)^2 = 25$:

$$(x - 2)^2 + \left(\frac{-1 - 4x}{3} + 3\right)^2 = 25$$

$$x^2 - 4x + 4 + \frac{(-1 - 4x)^2}{9} + 2(-1 - 4x) + 9 = 25$$

$$9x^2 - 36x + 36 + (-1 - 4x)^2 + 18(-1 - 4x) + 81 = 225$$

$$9x^2 - 36x + 36 + 1 + 8x + 16x^2 - 18 - 72x + 81 = 225$$

$$25x^2 - 100x - 125 = 0$$

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0$$

$\therefore x = 5$ or $x = -1$

Substitute into $y = \frac{-1 - 4x}{3}$:

$$y = \frac{-1 - 4(5)}{3} = -7 \text{ and } y = \frac{-1 - 4(-1)}{3} = 1$$

The intersection points are $(5, -7)$ and $(-1, 1)$.

The midpoint of the segment joining these points is $(2, -3)$ which is the centre of the circle $(x - 2)^2 + (y + 3)^2 = 25$. $\therefore$ the intersection points are the endpoints of a diameter of the circle.

14. Answers may vary: The lines $y = 2$ and $y = -2$ are possible solutions.

15. Substitute $x = 22 + 2y$ in $x^2 + y^2 = 100$:

$$(22 + 2y)^2 + y^2 = 100$$

$$484 + 88y + 4y^2 + y^2 = 100$$

$$5y^2 + 88y + 384 = 0$$

$$y = \frac{-88 \pm \sqrt{7744 - 7680}}{10}$$

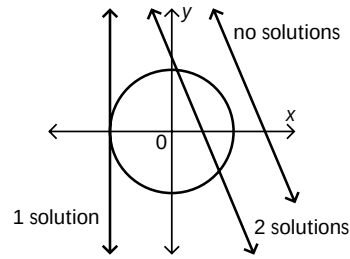
$$y = \frac{-88 \pm \sqrt{64}}{10}$$

$$y = \frac{-88 \pm 8}{10}$$

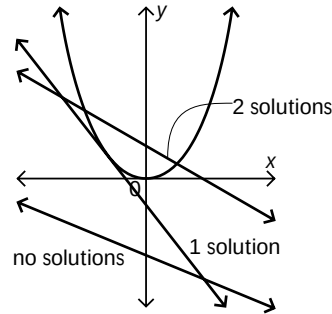
$$y = -9.6 \text{ or } y = -8$$

$\therefore$ the person will be detected by the motion sensor.

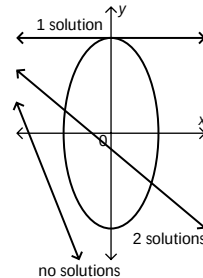
16. (a)



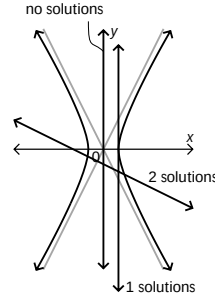
(b)



(c)



(d)



For each of the above scenarios, the algebraic solution for:

(i) one solution would have 1 real root

(ii) 2 solutions would have 2 real roots

(iii) no solution would have no real roots; not factorable; negative under root sign

17. Algebraic solution:

First, find the intersection points. Substitute $x = 2y + 4$ in

$$4x^2 + 16y^2 = 64:$$

$$4(2y + 4)^2 + 16y^2 = 64$$

$$4(4y^2 + 16y + 16) + 16y^2 = 64$$

$$16y^2 + 64y + 64 + 16y^2 = 64$$

$$32y^2 + 64y = 0$$

$$32y(y + 2) = 0$$

$$y = 0 \text{ or } y = -2$$

If $y = 0$, $x = 4$ and if $y = -2$, $x = 0$.

$\therefore$ the intersection points are $(4, 0)$ and $(0, -2)$.

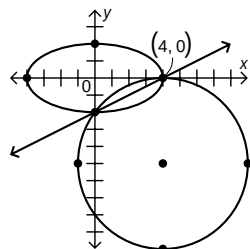
Now, substitute the points in $x^2 + y^2 - 8x + 10y + 16 = 0$ to see if the points lie on the conic. Substitute $(4, 0)$:

$$\begin{aligned} 4^2 + 0^2 - 8(4) + 10(0) + 16 &= 16 + 0 - 32 + 16 \\ &= 32 - 32 \\ &= 0 \end{aligned}$$

Substitute $(0, -2)$:

$$\begin{aligned} 0^2 + (-2)^2 - 8(0) + 10(-2) + 16 &= 0 + 4 - 0 - 20 + 16 \\ &= 0 \end{aligned}$$

$\therefore (4, 0)$ and $(0, -2)$ lie on the conic.



18. First find the intersection points of the line and the circle.

Substitute $y = 11 - 3x$ into $x^2 + y^2 - 10x - 10y + 25 = 0$:

$$\begin{aligned} x^2 + (11 - 3x)^2 - 10x - 10(11 - 3x) + 25 &= 0 \\ x^2 + 121 - 66x + 9x^2 - 10x - 110 + 30x + 25 &= 0 \\ 10x^2 - 46x + 36 &= 0 \\ 5x^2 - 23x + 18 &= 0 \end{aligned}$$

$$x = \frac{23 \pm \sqrt{23^2 - 4(5)(18)}}{2(5)}$$

$$x = \frac{23 \pm \sqrt{169}}{10}$$

$$x = \frac{23 \pm 13}{10}$$

$$x = \frac{18}{5} \text{ or } x = 1$$

If $x = \frac{18}{5}$, $y = \frac{1}{5}$ and if $x = 1$, $y = 8$.

$\therefore \left(\frac{18}{5}, \frac{1}{5}\right)$ and $(1, 8)$ are the intersection points.

Now find the line that is the perpendicular bisector of the

segment joining $\left(\frac{18}{5}, \frac{1}{5}\right)$ and $(1, 8)$. The midpoint is

$\left(\frac{23}{10}, \frac{41}{10}\right)$. The slope of the segment is -3 . Thus, the slope of

a perpendicular line is $\frac{1}{3}$. The equation of the line is:

$$y = \frac{1}{3}x + b$$

$$\frac{41}{10} = \frac{1}{3}\left(\frac{23}{10}\right) + b \quad \text{where } b = \frac{10}{3}$$

$\therefore$ the equation of the perpendicular bisector is $y = \frac{1}{3}x + \frac{10}{3}$.

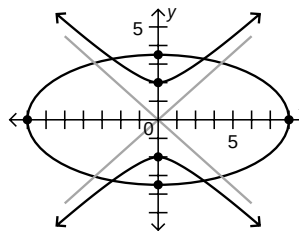
The centre of the given circle is $(5, 5)$. Now check to see if

$(5, 5)$ is on the line $y = \frac{1}{3}x + \frac{10}{3}$.

$$\text{L.S.} = 5$$

$$\text{R.S.} = \frac{1}{3}(5) + \frac{10}{3} = 5$$

19. (a)



It looks as though there will be 4 solutions.

(b)

$$x^2 = 49 - 4y^2$$

$$2(49 - 4y^2) - 3y^2 = -12$$

$$98 - 8y^2 - 3y^2 = -12$$

$$-11y^2 = -110$$

$$y^2 = 10$$

$$y = \pm\sqrt{10}$$

$$x^2 = 49 - 4(\sqrt{10})^2$$

$$x^2 = 49 - 40$$

$$x^2 = 9$$

$$x = \pm 3$$

$\therefore$ the solutions are $(3, \sqrt{10})$, $(-3, \sqrt{10})$, $(3, -\sqrt{10})$ and $(-3, -\sqrt{10})$.

(c) Yes, the answer for (b) gave 4 solutions which is also what the graph shows.

20. Set both equations equal to x^2 :

$$x^2 = 2 - y \quad x^2 = 25 - y^2$$

and set them equal:

$$2 - y = 25 - y^2$$

$$y^2 - y - 23 = 0$$

$$y = \frac{1 \pm \sqrt{1 + 92}}{2}$$

$$y = \frac{1 \pm \sqrt{93}}{2}$$

$$y \doteq \frac{1 \pm 9.64}{2}$$

$$y \doteq 5.32 \text{ or } y \doteq -4.32$$

If $y \doteq 5.32$, then there is no admissible value for x .

If $y \doteq -4.32$, $x \doteq 2.51$ or $x \doteq -2.51$.

$\therefore$ the intersection points are $(2.51, -4.32)$ and $(-2.51, -4.32)$.

Graphical solution:

