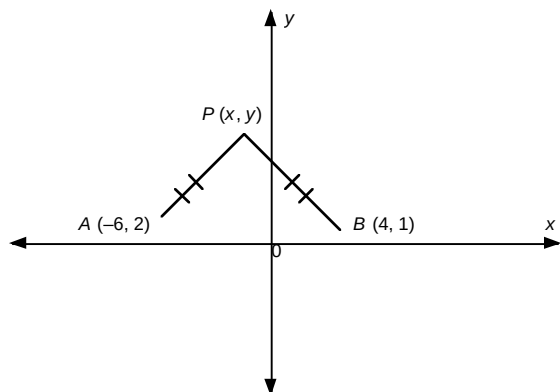


Chapter 7, Review and Practice, page 629

1. A mathematic object may be a table of values, a curve, a line, a geometric figure or a relation defined by an equation.
2. A circle is a locus of points equidistant from a fixed point, the centre of the circle.
3. The shape defined by the locus of points that are equidistant from two fixed points is a straight line defined as the perpendicular bisector of the line segment joining these points.
4. The shape defined by the locus of points that are equidistant from a fixed point and a fixed line is a parabola.
5. Draw a diagram.



Let $P(x, y)$ be any point on the locus.
Point P is same distance from A and B . So, $PA = PB$.
Using distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PA = \sqrt{(x+6)^2 + (y-2)^2}, \text{ and } PB = \sqrt{(x-4)^2 + (y-1)^2}$$

Since $PA = PB$,

$$\sqrt{(x+6)^2 + (y-2)^2} = \sqrt{(x-4)^2 + (y-1)^2}$$

$$(x+6)^2 + (y-2)^2 = (x-4)^2 + (y-1)^2$$

$$x^2 + 12x + 36 + y^2 - 4y + 4 = x^2 - 8x + 16 + y^2 - 2y + 1$$

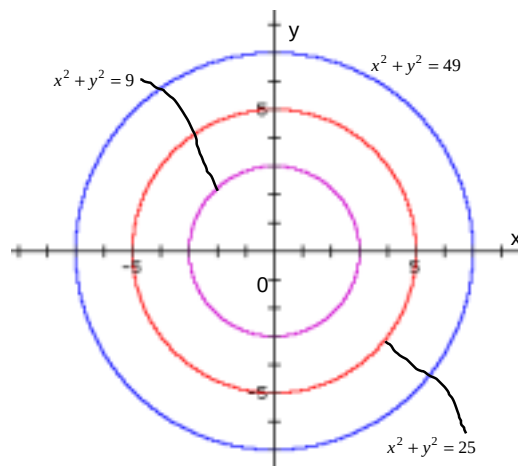
$$x^2 + 12x + 36 + y^2 - 4y + 4 - x^2 + 8x - 16 - y^2 + 2y - 1 = 0$$

$$20x - 2y + 23 = 0$$

$\therefore$ the equation for the locus of points is $20x - 2y + 23 = 0$.

6. $x^2 + y^2 = 25$ centre (0, 0) radius 5

There are 2 solutions – one circle inside the given circle, and one circle outside the given circle. The radius of the given circle is 5, while the radii of the other circles are 3 and 7. Therefore, the equations of the solutions are $x^2 + y^2 = 9$ and $x^2 + y^2 = 49$. The graphs of all 3 circles look like this:



7. (a)

$$4x^2 + 4y^2 = 1 \quad \text{centre (0, 0), radius} = \frac{1}{2}$$

$$x^2 + y^2 = \frac{1}{4}$$

(b)

$$x^2 + y^2 + 16x - 6y + 72 = 0 \quad \text{centre } (-8, 3), \text{ radius} = 1$$

$$x^2 + 16x + y^2 - 6y = -72$$

$$(x+8)^2 + (y-3)^2 = -72 + 64 + 9$$

$$(x+8)^2 + (y-3)^2 = 1$$

(c)

$$2x^2 + 2y^2 + 6x - 10y - 1 = 0 \quad \text{centre } \left(-\frac{3}{2}, \frac{5}{2}\right), \text{ radius} = 3$$

$$2x^2 + 6x + 2y^2 - 10y = 1$$

$$2(x^2 + 3x) + 2(y^2 - 5y) = 1$$

$$2\left(x + \frac{3}{2}\right)^2 + 2\left(y - \frac{5}{2}\right)^2 = 1 + \frac{9}{2} + \frac{25}{2}$$

$$\left(x + \frac{3}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = 9$$

8. $x^2 + y^2 = r^2$ is the equation of the circle. Substitute (8, -6) into the equation:

$$(8)^2 + (-6)^2 = r^2$$

$$r^2 = 100$$

$$r = 10$$

Thus, the equation is $x^2 + y^2 = 100$.

9. $(x-2)^2 + (y-1)^2 = r^2$ is the equation of the circle.

Substitute (5, 2) into the equation.

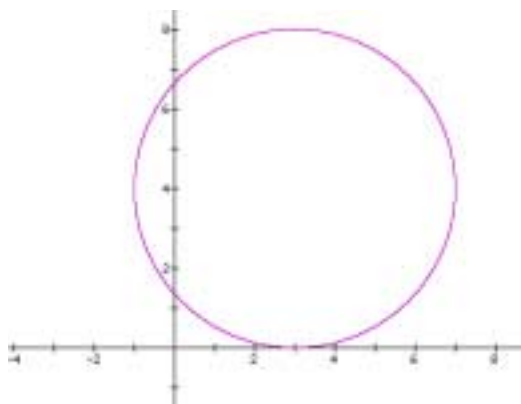
$$(5-2)^2 + (2-1)^2 = r^2$$

$$3^2 + 1^2 = r^2$$

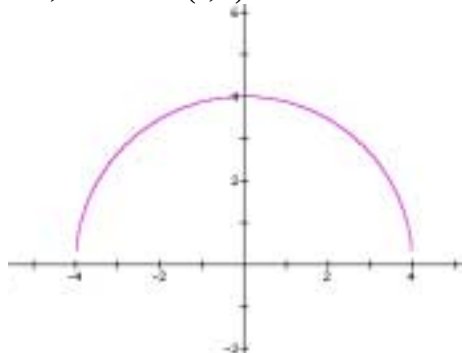
$$r = \sqrt{10}$$

Thus, the equation is $(x-2)^2 + (y-1)^2 = 10$.

10. The equation of the circle in standard form is $(x-h)^2 + (y-k)^2 = r^2$. Since the centre is (-3, 4), then the equation is $(x+3)^2 + (y-4)^2 = r^2$. To find r , consider that the circle is tangent to the x -axis as shown in the diagram. Then, the radius is 4 units, since the centre of the circle is 4 units from the x -axis. The equation is $(x+3)^2 + (y-4)^2 = 16$.



11. The graph of $y = \sqrt{16 - x^2}$ is a semi-circle above the x -axis, with centre $(0, 0)$ and radius 4.



12. An ellipse is the set or locus of points in a plane such that the sum of the distances from two fixed points to any other point is constant.

13. (a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$

(b) $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where $a > b$

14. (a) major axis $2a$ units, minor axis $2b$ units

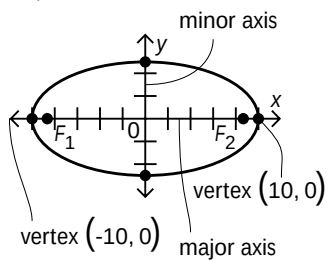
(b) major axis $2b$ units, minor axis $2a$ units

15. $a^2 = b^2 + c^2$

16. x -intercepts: $(-5, 0)$ and $(5, 0)$,

y -intercepts: $(0, -8)$ and $(0, 8)$

17.



$$x^2 + 4y^2 = 100$$

$$\frac{x^2}{100} + \frac{y^2}{25} = 1$$

For an ellipse:

$$a^2 = b^2 + c^2$$

$$10^2 = 5^2 + c^2$$

$$c^2 = 75$$

$$c = \sqrt{75}$$

$$= 5\sqrt{3}$$

So foci are $(-5\sqrt{3}, 0)$ and $(5\sqrt{3}, 0)$.

18. $b = 6, c = 5$

$$a^2 = b^2 + c^2$$

$$= 6^2 + 5^2$$

$$a = \sqrt{61}$$

Thus, $\frac{x^2}{61} + \frac{y^2}{36} = 1$ is the equation of the ellipse.

19. The equation of the ellipse is $\frac{x^2}{25} + \frac{y^2}{81} = 1$

Substitute $x = \pm 3$ (2 m from either wall) and solve for y , the height at $x = \pm 3$:

$$\frac{(\pm 3)^2}{25} + \frac{y^2}{81} = 1$$

$$\frac{9}{25} + \frac{y^2}{81} = 1$$

$$729 + 25y^2 = 2025$$

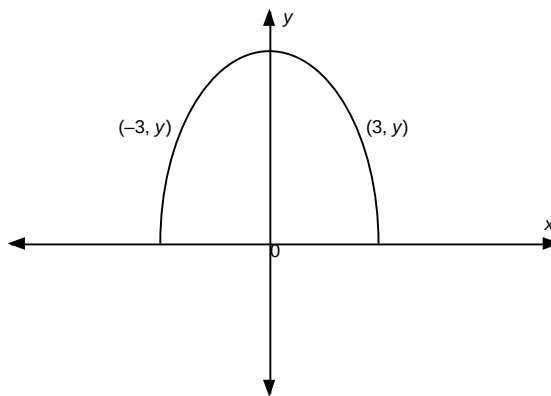
$$25y^2 = 1296$$

$$y^2 = \frac{1296}{25}$$

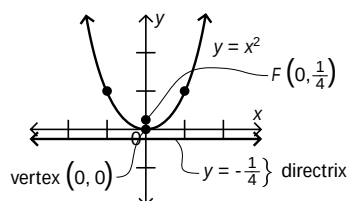
$$y = \frac{36}{5}$$

$$y = 7.2$$

$\therefore$ the height of the ceiling is $7.2 + 6 = 13.2$ m.



20.



Parabola:

$$y = x^2$$

$$4py = x^2$$

$$p = \frac{1}{4}$$

21. A parabola is the locus of points in a plane the same distance from a fixed point, called the focus, and a fixed line, called the directrix.

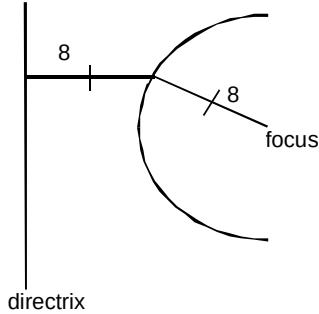
22. (a) $x^2 = 4py$

(b) $(x - h)^2 = 4p(y - k)$

(c) $y^2 = 4px$

(d) $(y - k)^2 = 4p(x - h)$

23. 8 units



24. (a) $y^2 = 12x$

$p = 3$

Vertex is $(0, 0)$

$\therefore$ Focus is $(3, 0)$

Directrix is $x = -3$

(b) $(x - 3)^2 = 8(y + 2)$

$p = 2$

Vertex is $(3, -2)$

$\therefore$ Focus is $(3, 0)$

Directrix is $y = -4$

25. (a) Vertex: $(0, 0)$, focus: $(0, 2)$

Thus, the equation in standard form is:

$$(x - h)^2 = 4p(y - k)$$

and $h = 0$, $k = 0$, $p = 2$

Thus, the equation is $x^2 = 8y$.

(b) Vertex: $(1, 2)$, directrix: $y = -2$

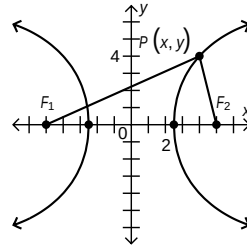
Thus, the equation in standard form is $(x - h)^2 = 4p(y - k)$ and

$h = 1$, $k = 2$, $p = 2$

Thus, the equation is $(x - 1)^2 = 8(y - 2)$

26. A hyperbola is a set or locus of points, in a plane, whose distances from two fixed points have a constant difference.

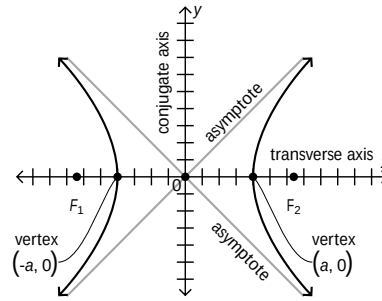
The two fixed points are called the foci.



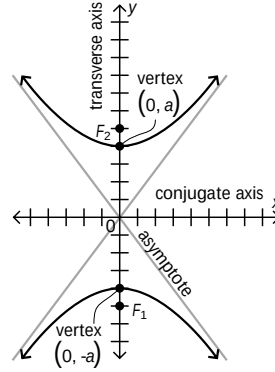
27. (a) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(b) $\frac{x^2}{b^2} - \frac{y^2}{a^2} = -1$

28. (a)



(b)



29. Since A and B both have the same y-coordinate, the transverse axis is horizontal. The distance between the focal points is 20. Therefore, the value of c is 10, a is 3. Use $c^2 = a^2 + b^2$ to find b :

$$c^2 = a^2 + b^2$$

$$100 = 9 + b^2$$

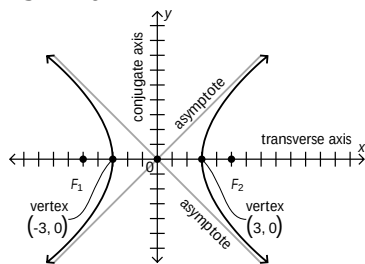
$$b^2 = 81$$

$$\therefore b = 9$$

$$\therefore \text{the equation of the hyperbola is } \frac{(x + 4)^2}{9} - \frac{(y - 6)^2}{81} = 1.$$

30. $16x^2 - 9y^2 = 144$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

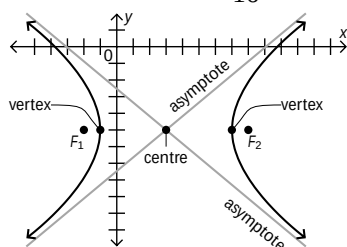


$$\begin{aligned} a &= 3, b = 4, \\ c^2 &= a^2 + b^2 \\ &= 9 + 16 \\ &= 25 \\ \therefore c &= 5 \end{aligned}$$

31. Since the centre is at $(3, -5)$ and the vertex $(7, -5)$ has the same y -coordinate as the centre, the form of the equation will be $\frac{(x-3)^2}{a^2} - \frac{(y+5)^2}{b^2} = 1$. Since the distance from the centre to the vertex is 4, the value of a is 4. Since the focus is at $(8, -5)$, the value of c is 5.

$$\begin{aligned} \text{Since } c^2 &= a^2 + b^2 \\ 5^2 &= 4^2 + b^2 \\ 25 &= 16 + b^2 \\ 3 &= b \end{aligned}$$

$$\therefore \text{the equation is } \frac{(x-3)^2}{16} - \frac{(y+5)^2}{9} = 1.$$



32. First, identify the type of conic by inspecting the coefficients, a and b , of the first two terms. Then, complete the squares to identify the vertices, major/minor axes, foci, or radii of the equations. Finally, determine if there are any transformations needed.

33. You can write any conic in the general form: $ax^2 + by^2 + 2gx + 2fy + c = 0$. To determine the type of conic, look at the sign of ab . If $a = b$, then the conic is a circle; if $ab > 0$, then the conic is an ellipse; if $ab < 0$, then the conic is a hyperbola.

Conic Section	Relationship
circle	$a = b$
parabola	a or $c = 0$
ellipse	$ab > 0, a \neq b$
hyperbola	$ab < 0, a \neq b$

34. (a) $4x^2 + 9y^2 + 8x - 36y + 4 = 0$

i. ellipse ($ab > 0$)

ii.

$$4x^2 + 9y^2 + 8x - 36y + 4 = 0$$

$$4x^2 + 8x + 9y^2 - 36y = -4$$

$$4(x^2 + 2x + 1 - 1) + 9(y^2 - 4y + 4 - 4) = -4$$

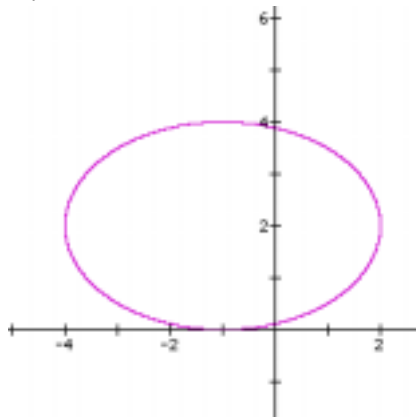
$$4(x+1)^2 + 9(y-2)^2 = -4 + 36 + 4$$

$$4(x+1)^2 + 9(y-2)^2 = 36$$

$$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$$

$$\therefore \text{the equation of the ellipse is } \frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1.$$

iii.



(b) $3x^2 + 3y^2 - 6x - 9 = 0$

i. circle ($a = b$)

ii.

$$3x^2 + 3y^2 - 6x - 9 = 0$$

$$3x^2 - 6x + 3y^2 - 9 = 0$$

$$3(x^2 - 2x + 1 - 1) + 3y^2 = 9$$

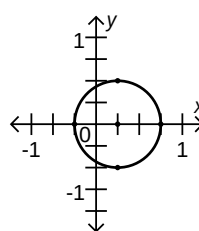
$$3(x-1)^2 - 3 + 3y^2 = 9$$

$$3(x-1)^2 + 3y^2 = 12$$

$$\frac{(x-1)^2}{4} + \frac{y^2}{4} = 1$$

$$\therefore \text{the equation of the circle is } \frac{(x-1)^2}{4} + \frac{y^2}{4} = 1.$$

iii.



(c) $3x^2 - 6y^2 - 2y + 4 = 0$

i. hyperbola ($ab < 0$)

ii.

$$3x^2 - 6y^2 - 2y + 4 = 0$$

$$3x^2 - 6y^2 - 2y = -4$$

$$3x^2 - 6\left(y^2 + \frac{1}{3}y\right) = -4$$

$$3x^2 - 6\left(y^2 + \frac{1}{3}y + \frac{1}{36} - \frac{1}{36}\right) = -4$$

$$3x^2 - 6\left(y + \frac{1}{6}\right)^2 = -4 - \frac{1}{6}$$

$$3x^2 - 6\left(y + \frac{1}{6}\right)^2 = -\frac{25}{6}$$

$$\frac{3x^2}{\left(\frac{25}{6}\right)} - \frac{6\left(y + \frac{1}{6}\right)^2}{\left(\frac{25}{6}\right)} = -1$$

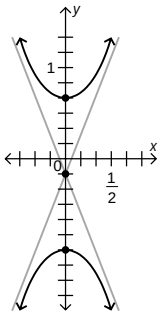
$$3\left(\frac{6}{25}\right)x^2 - 6\left(\frac{6}{25}\right)\left(y + \frac{1}{6}\right)^2 = -1$$

$$\frac{18}{25}x^2 - \frac{36}{25}\left(y + \frac{1}{6}\right)^2 = -1$$

$$\frac{x^2}{\left(\frac{25}{18}\right)} - \frac{\left(y + \frac{1}{6}\right)^2}{\left(\frac{25}{36}\right)} = -1$$

$\therefore$ the equation of the hyperbola is $\frac{x^2}{\left(\frac{25}{18}\right)} - \frac{\left(y + \frac{1}{6}\right)^2}{\left(\frac{25}{36}\right)} = -1$.

iii.



(d) $x^2 + 2y^2 - 12y + 2 = 0$

i. ellipse ($ab > 0$)

ii.

$$x^2 + 2y^2 - 12y + 2 = 0$$

$$2(y^2 - 6y) + x^2 = -2$$

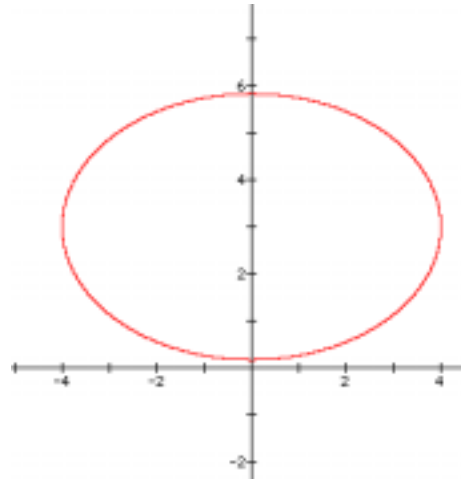
$$2(y^2 - 6y + 9 - 9) + x^2 = -2$$

$$2(y - 3)^2 + x^2 = 16$$

$$\frac{x^2}{16} + \frac{(y - 3)^2}{8} = 1$$

$\therefore$ the equation of the ellipse is $\frac{x^2}{16} + \frac{(y - 3)^2}{8} = 1$.

iii.



(e) $4x^2 - y^2 + 56x + 2y + 199 = 0$

i. hyperbola ($ab < 0$)

ii.

$$4x^2 - y^2 + 56x + 2y + 199 = 0$$

$$4x^2 + 56x - y^2 + 2y = -199$$

$$4(x^2 + 14x + 49 - 49) - (y^2 - 2y + 1 - 1) = -199$$

$$4(x + 7)^2 - 196 - (y - 1)^2 + 1 = -199$$

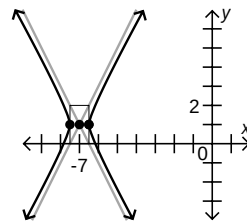
$$4(x + 7)^2 - (y - 1)^2 = -199 - 1 + 196$$

$$4(x + 7)^2 - (y - 1)^2 = -4$$

$$(x + 7)^2 - \frac{(y - 1)^2}{4} = -1$$

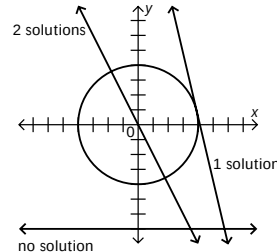
$\therefore$ the equation of the hyperbola is $(x + 7)^2 - \frac{(y - 1)^2}{4} = -1$.

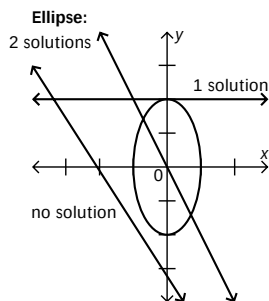
iii.



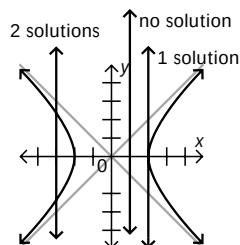
35.

Circle:

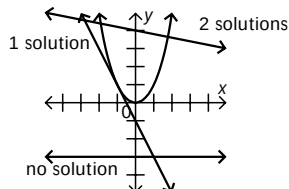




Hyperbola:



Parabola:



36. Graphing a system of equations is not always the best solution to find intersection points because the solutions are not always easily read from the graph.

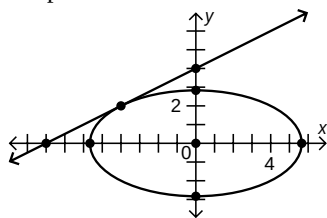
37. Algebraic Solution:

$$\begin{aligned}x - 2y &= -8 & x^2 + 4y^2 &= 32 \\x &= -8 + 2y \\(-8 + 2y)^2 + 4y^2 &= 32 \\64 - 32y + 4y^2 + 4y^2 &= 32 \\8y^2 - 32y + 32 &= 0 \\y^2 - 4y + 4 &= 0 \\(y - 2)^2 &= 0 \\y &= 2\end{aligned}$$

$$\therefore y = 2, x = -4$$

$\therefore$ the point of intersection is $(-4, 2)$.

Graphical Solution:



38. The standard form of a circle is $(x - h)^2 + (y - k)^2 = r^2$. We know the centre is on the y -axis and on the line $x - 5y - 15 = 0$. So, substitute $x = 0$ into the equation of the line to get:

$$\begin{aligned}0 - 5y - 15 &= 0 \\-5y &= 15 \\y &= -3\end{aligned}$$

Thus $(0, -3)$ is the centre of the circle.

Now we have $x^2 + (y + 3)^2 = r^2$.

Now substitute the point $(4, 2)$ to find r :

$$\begin{aligned}4^2 + (2 + 3)^2 &= r^2 \\4^2 + 5^2 &= r^2 \\r^2 &= 41\end{aligned}$$

$\therefore x^2 + (y + 3)^2 = 41$ is the equation of the circle.

39.

$$\begin{aligned}x + 2y &= 8 & x^2 + 4y^2 &= 40 \\x &= 8 - 2y \\(8 - 2y)^2 + 4y^2 &= 40 \\64 - 32y + 4y^2 + 4y^2 &= 40 \\8y^2 - 32y + 24 &= 0 \\y^2 - 4y + 3 &= 0 \\(y - 3)(y - 1) &= 0 \\y &= 3 \text{ or } y = 1\end{aligned}$$

If $y = 3$, $x = 2$ and if $y = 1$, $x = 6$.

So $(2, 3)$ and $(6, 1)$ are the points of intersection.

40.

$$\begin{aligned}x^2 + y^2 &= 5 & x + y - 1 &= 0 \\y &= 1 - x\end{aligned}$$

$$\begin{aligned}x^2 + (1 - x)^2 &= 5 \\x^2 + 1 - 2x + x^2 &= 5 \\2x^2 - 2x - 4 &= 0 \\x^2 - x - 2 &= 0 \\(x - 2)(x + 1) &= 0 \\x &= 2 \text{ or } x = -1\end{aligned}$$

If $x = 2$, $y = -1$ and if $x = -1$, $y = 2$.

So $(2, -1)$ and $(-1, 2)$ are the points of intersection.

Chapter 7U Review Test, page 633

1. (a) $\frac{x^2}{16} + \frac{y^2}{9} = 1$ ellipse

vertices: $(4, 0)$, $(-4, 0)$

foci:

$$a^2 = b^2 + c^2$$

$$16 = 9 + c^2$$

$$c^2 = 7$$

$$c = \sqrt{7}$$

$$(\sqrt{7}, 0), (-\sqrt{7}, 0)$$