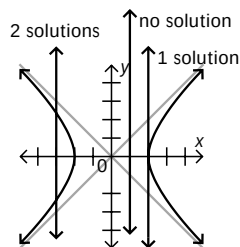
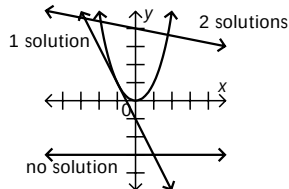


Hyperbola:



Parabola:



36. Graphing a system of equations is not always the best solution to find intersection points because the solutions are not always easily read from the graph.

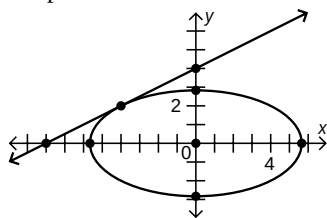
37. Algebraic Solution:

$$\begin{aligned}
 x - 2y &= -8 & x^2 + 4y^2 &= 32 \\
 x &= -8 + 2y \\
 (-8 + 2y)^2 + 4y^2 &= 32 \\
 64 - 32y + 4y^2 + 4y^2 &= 32 \\
 8y^2 - 32y + 32 &= 0 \\
 y^2 - 4y + 4 &= 0 \\
 (y - 2)^2 &= 0 \\
 y &= 2
 \end{aligned}$$

$$\therefore y = 2, x = -4$$

$\therefore$ the point of intersection is $(-4, 2)$.

Graphical Solution:



38. The standard form of a circle is $(x - h)^2 + (y - k)^2 = r^2$. We know the centre is on the y -axis and on the line $x - 5y - 15 = 0$. So, substitute $x = 0$ into the equation of the line to get:

$$\begin{aligned}
 0 - 5y - 15 &= 0 \\
 -5y &= 15 \\
 y &= -3
 \end{aligned}$$

Thus $(0, -3)$ is the centre of the circle.

Now we have $x^2 + (y + 3)^2 = r^2$.

Now substitute the point $(4, 2)$ to find r :

$$\begin{aligned}
 4^2 + (2 + 3)^2 &= r^2 \\
 4^2 + 5^2 &= r^2 \\
 r^2 &= 41
 \end{aligned}$$

$\therefore x^2 + (y + 3)^2 = 41$ is the equation of the circle.

39.

$$\begin{aligned}
 x + 2y &= 8 & x^2 + 4y^2 &= 40 \\
 x &= 8 - 2y \\
 (8 - 2y)^2 + 4y^2 &= 40 \\
 64 - 32y + 4y^2 + 4y^2 &= 40 \\
 8y^2 - 32y + 24 &= 0 \\
 y^2 - 4y + 3 &= 0 \\
 (y - 3)(y - 1) &= 0 \\
 y &= 3 \text{ or } y = 1
 \end{aligned}$$

If $y = 3$, $x = 2$ and if $y = 1$, $x = 6$.

So $(2, 3)$ and $(6, 1)$ are the points of intersection.

40.

$$\begin{aligned}
 x^2 + y^2 &= 5 & x + y - 1 &= 0 \\
 y &= 1 - x
 \end{aligned}$$

$$\begin{aligned}
 x^2 + (1 - x)^2 &= 5 \\
 x^2 + 1 - 2x + x^2 &= 5 \\
 2x^2 - 2x - 4 &= 0 \\
 x^2 - x - 2 &= 0 \\
 (x - 2)(x + 1) &= 0 \\
 x &= 2 \text{ or } x = -1
 \end{aligned}$$

If $x = 2$, $y = -1$ and if $x = -1$, $y = 2$.

So $(2, -1)$ and $(-1, 2)$ are the points of intersection.

Chapter 7U Review Test, page 633

1. (a) $\frac{x^2}{16} + \frac{y^2}{9} = 1$ ellipse

vertices: $(4, 0)$, $(-4, 0)$

foci:

$$a^2 = b^2 + c^2$$

$$16 = 9 + c^2$$

$$c^2 = 7$$

$$c = \sqrt{7}$$

$$(\sqrt{7}, 0), (-\sqrt{7}, 0)$$

(b) $x^2 - y^2 = 1$ hyperbola

vertices: $(1, 0), (-1, 0)$

foci:

$$c^2 = a^2 + b^2$$

$$c^2 = 1 + 1$$

$$c^2 = 2$$

$$c = \sqrt{2}$$

$$(\sqrt{2}, 0), (-\sqrt{2}, 0)$$

(c) $2x^2 + 2y^2 = 18$ circle

$$\frac{x^2}{9} + \frac{y^2}{9} = 1$$

vertices: $(3, 0), (-3, 0), (0, 3), (0, -3)$

foci:

$$a^2 = b^2 + c^2$$

$$c^2 = 0$$

$$(0, 0)$$

(d) $y^2 = 8(x - 3)$ parabola

$$p = 2$$

foci: $(2, 0), (-2, 0)$

vertex: $(3, 0)$

(e) $\frac{(x+2)^2}{25} + \frac{(y+1)^2}{9} = 1$ ellipse

centre: $(-2, -1)$

foci: $(-6, 0), (2, 0)$

vertices: $(-7, -1), (3, -1)$

2. **(a)** $x^2 + (y+2)^2 = 9$

(b) $\frac{x^2}{4} + \frac{y^2}{9} = -1$ $b = 3, \therefore 2b = 6$

(c) $x^2 = 24y$ $p = 6$ $(h, k) = (0, 0)$

3. $x^2 = -5y$ $p = -\frac{5}{4}$

$\therefore$ focus $= \left(0, -\frac{5}{4}\right)$, directrix: $y = \frac{5}{4}$

4. You can determine whether the transverse axis of a hyperbola is horizontal or vertical by the sign of the 1 on the right-hand side of the equation. In the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the transverse axis is horizontal (along the x -axis). In the equation $\frac{x^2}{b^2} - \frac{y^2}{a^2} = -1$, the transverse axis is vertical (along the y -axis).

5. $25x^2 - 9y^2 = -225$

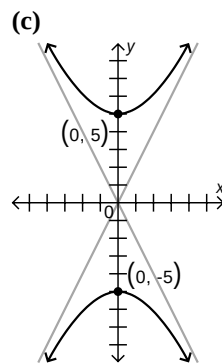
$$\frac{x^2}{9} - \frac{y^2}{25} = -1$$

(a) vertices: $(0, 5), (0, -5)$

(b) asymptotes:

$$y = \pm \frac{a}{b}x$$

$$y = \pm \frac{5}{3}x$$



6. $\frac{x^2}{25} + \frac{y^2}{64} = 1$

(a) x -intercepts: $(5, 0), (-5, 0)$

y -intercepts: $(0, 8), (0, -8)$

(b)

$$a^2 = b^2 + c^2$$

$$8^2 = 5^2 + c^2$$

$$c^2 = 39$$

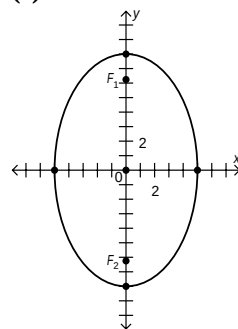
$$c = \sqrt{39}$$

foci: $(0, \sqrt{39}), (0, -\sqrt{39})$

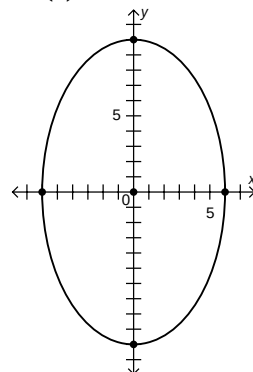
(c) length of major axis $= 2a$
 $= 2(8)$
 $= 16$

(d) vertices: $(0, 8)$ and $(0, -8)$

(e)



7. **(a)**



$$25x^2 + 9y^2 = 900$$

$$\frac{x^2}{36} + \frac{y^2}{100} = 1$$

$$a = 10, b = 6$$

$$a^2 = b^2 + c^2$$

$$100 = 36 + c^2$$

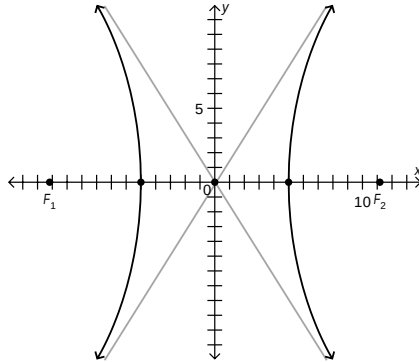
$$c^2 = 64$$

$$c = 8$$

vertices: $(0, 10), (0, -10)$

$\therefore$ foci are $(0, 8)$ and $(0, -8)$.

(b)



$$4x^2 - y^2 = 100$$

$$\frac{x^2}{25} - \frac{y^2}{100} = 1$$

$$a = 5, b = 10$$

$$c^2 = a^2 + b^2$$

$$c^2 = 5^2 + 10^2$$

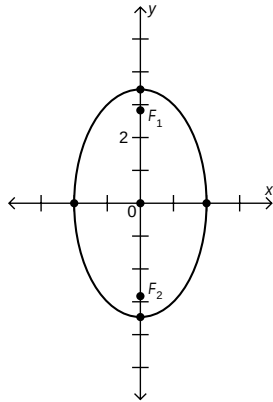
$$c^2 = 125$$

$$c = \sqrt{125} \text{ or } 5\sqrt{5}$$

vertices: $(5, 0), (-5, 0)$

$\therefore$ foci are $(5\sqrt{5}, 0)$ and $(-5\sqrt{5}, 0)$.

(c)



$$\frac{x^2}{4} + \frac{y^2}{12} = 1$$

vertices: $(0, \sqrt{12}), (0, -\sqrt{12})$

foci:

$$a^2 = b^2 + c^2$$

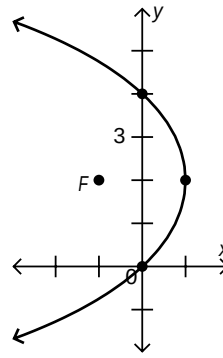
$$12 = 4 + c^2$$

$$c^2 = 8$$

$$c = \sqrt{8} \text{ or } 2\sqrt{2}$$

$\therefore$ foci are $(0, 2\sqrt{2})$ and $(0, -2\sqrt{2})$.

(d)



$$y^2 - 4y + 4x = 0$$

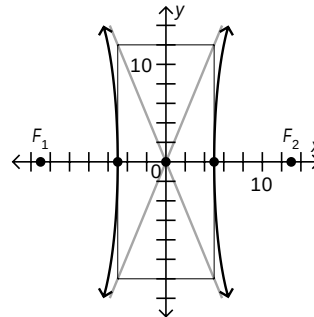
$$y^2 - 4y + 4 = -4x + 4$$

$$(y - 2)^2 = -4(x - 1)$$

$\therefore h = 1, k = 2$ and $p = -1$

vertex: $(1, 2)$ and focus: $(-1, 2)$

(e)



$$\frac{x^2}{25} - \frac{y^2}{144} = 1$$

$$a = 5, b = 12$$

$$c^2 = a^2 + b^2$$

$$= 25 + 144$$

$$= 169$$

$$c = 13$$

$\therefore$ vertices: $(5, 0)$ and $(-5, 0)$ and foci: $(13, 0)$ and $(-13, 0)$.

$$8. \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$2a = 4(2b)$$

$$a = 4b$$

$$\frac{x^2}{(4b)^2} + \frac{y^2}{b^2} = 1$$

Substitute $(-3, 2)$ into the equation and solve for b :

$$\frac{x^2}{16b^2} + \frac{y^2}{b^2} = 1$$

$$\frac{9}{16b^2} + \frac{4}{b^2} = 1$$

$$\frac{9}{16b^2} + \frac{64}{16b^2} = 1$$

$$\frac{73}{16b^2} = 1$$

$$16b^2 = 73$$

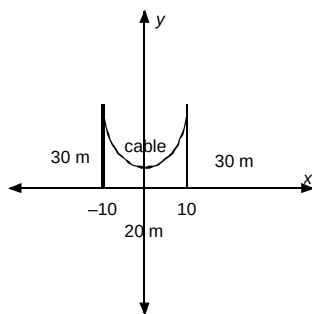
$$b^2 = \frac{73}{16}$$

$$b = \frac{\sqrt{73}}{4}$$

$$\text{Now, } a = 4\left(\frac{\sqrt{73}}{4}\right) = \sqrt{73}$$

So $\frac{x^2}{73} + \frac{16y^2}{73} = 1$ is the equation of the ellipse.

9.



Find the equation of the parabola and substitute for $x = 9$.

$$(x - h)^2 = 4p(y - k) \quad (h, k) = (0, 5)$$

$$x^2 = 4p(y - 5)$$

Substitute $(10, 30)$ into the equation to find p .

$$10^2 = 4p(30 - 5)$$

$$4 = 4p$$

$$p = 1$$

Thus, $x^2 = 4(y - 5)$ is the equation of the parabola.

Now, substitute $x = 9$ into the equation.

$$9^2 = 4(y - 5)$$

$$81 = 4(y - 5)$$

$$y = \frac{81}{4} + 5$$

$$\therefore y = \frac{101}{4}$$

$\therefore$ they are $\frac{101}{4}$ m from the ground.

10. (a) $a = b$, circle

(b) $ab < 0$, hyperbola

(c) $ab > 0$, ellipse

(d) $ab = 0$, parabola

(e) $ab < 0$, hyperbola

$$11. (y - k)^2 = 4p(x - h)$$

$(h, k) = (0, 0)$, p is positive

Since we know the focus is inside the parabola, if the focus is on the line $4x - 7y - 12 = 0$, then the line will intersect the parabola. So,

$$y^2 = 4px$$

$$7y = 4x - 12$$

$$y = \frac{4x - 12}{7}$$

$$\therefore y^2 - 12x = 0$$

$$12. x^2 + y^2 - 4x + 6y - 1 = 0$$

First put the equation of the circle in standard form by completing the squares:

$$x^2 - 4x + y^2 + 6y = 1$$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 1 + 4 + 9$$

$$(x - 2)^2 + (y + 3)^2 = 14$$

Thus, the centre is $(2, -3)$.

We know that for any circle, the diameter passes through centre $(2, -3)$ and $(0, -2)$.

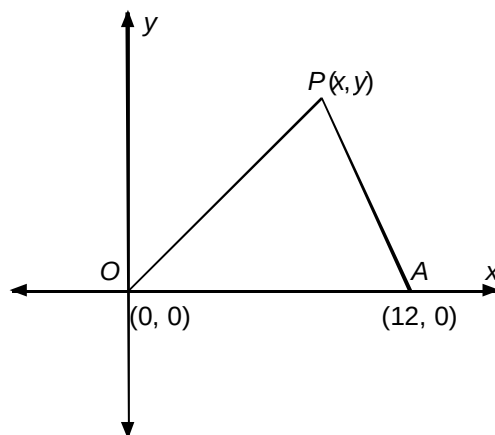
$$y + 3 = m(x - 2) \text{ where } m = -\frac{1}{2}$$

$$y + 3 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x - 2$$

$\therefore y = -\frac{1}{2}x - 2$ is the equation of the diameter.

13.



Although the size of $\triangle POA$ can vary, the tangents from P to the base angles can always be expressed as

$$\frac{y - 0}{x - 0} \text{ and } \frac{y - 0}{x - 12},$$

ie. the product of these tangents is:

$$\left(\frac{y}{x}\right)\left(\frac{y}{x-12}\right) = \frac{9}{16}$$

$$\frac{y^2}{x^2 - 12x} = \frac{9}{16}$$

$$16y^2 = 9(x^2 - 12x)$$

$$16y^2 = 9x^2 - 108x$$

$$9x^2 - 108x - 16y^2 = 0$$

$$9(x^2 - 12x + 36 - 36) - 16y^2 = 0$$

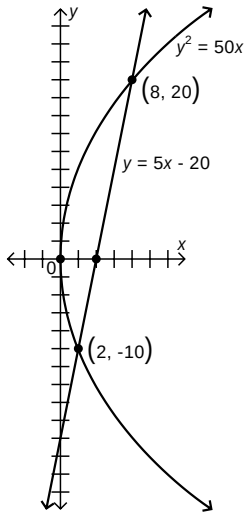
$$9(x-6)^2 - 16y^2 = 324$$

$$\therefore \frac{(x-6)^2}{36} - \frac{y^2}{\left(\frac{81}{4}\right)} = 1$$

$\therefore$ the equation for the locus of vertex P is

$$\frac{(x-6)^2}{36} - \frac{y^2}{\left(\frac{81}{4}\right)} = 1.$$

14. Graphical Solution:



Algebraic Solution:

$$5x - y - 20 = 0$$

$$y = 5x - 20$$

$$y^2 = 50x$$

$$(5x - 20)^2 = 50x$$

$$25x^2 - 200x + 400 - 50x = 0$$

$$25x^2 - 250x + 400 = 0$$

$$x^2 - 10x + 16 = 0$$

$$(x-8)(x-2) = 0$$

$$x = 8 \text{ or } x = 2$$

$$\text{If } x = 8, \text{ then } y = 5(8) - 20 = 40 - 20 = 20$$

$$\text{and if } x = 2, \text{ then } y = 5(2) - 20 = 10 - 20 = -10$$

$\therefore (8, 20)$ and $(2, -10)$ are the points of intersection.