

Cumulative Review Test 4

LocI and Conics, page 635

$$\begin{aligned} 1. (a) \text{ i. } t_n &= a + (n-1)d \\ &= -4 + (n-1)(9) \\ &= -4 + 9n - 9 \\ &= -13 + 9n \end{aligned}$$

$$\begin{aligned} \text{ii.} \\ t_{25} &= -13 + 9(25) \\ &= -13 + 225 \\ &= 212 \end{aligned}$$

$$\begin{aligned} \text{iii.} \\ S_n &= \frac{n}{2} [2a + (n-1)d] \\ &= \frac{20}{2} [2(-4) + (20-1)9] \\ &= 10[-8 + 171] \\ &= 1630 \end{aligned}$$

$$\begin{aligned} (b) \quad t_n &= ar^{n-1} \\ \text{i. } t_n &= 3(1.1)^{n-1} \\ \text{ii.} \\ t_{25} &= 3(1.1)^{25-1} \\ &= 3(1.1)^{24} \\ &= 29.55 \end{aligned}$$

$$\begin{aligned} \text{iii.} \\ S_n &= \frac{a(r^n - 1)}{r - 1} \\ S_{20} &= \frac{3[1.1^{20} - 1]}{1.1 - 1} \\ &= \frac{3[1.1^{20} - 1]}{0.1} \\ &= 171.82 \end{aligned}$$

$$\begin{aligned} 2. (a) \quad t_n &= ar^{n-1} \\ 201\,326\,592 &= 3(4)^{n-1} \\ \frac{201\,326\,592}{3} &= 4^{n-1} \\ 67\,108\,864 &= 4^{n-1} \\ \log(67\,108\,864) &= (n-1)\log 4 \\ n-1 &= \frac{\log(67\,108\,864)}{\log 4} \\ n-1 &= 13 \\ n &= 1+13 \\ n &= 14 \text{ terms} \end{aligned}$$

$$\begin{aligned} (b) \quad t_n &= a + (n-1)d \\ 162 &= 18 + (n-1)(4) \\ 144 &= 4n - 4 \\ 4n &= 148 \\ n &= \frac{148}{4} \\ n &= 37 \end{aligned}$$

$$\begin{aligned} 3. \quad a &= 55\,000, r = 1.025, n = 15 \\ A &= \frac{a(r^n - 1)}{r - 1} \\ &= \frac{55\,000(1.025^{15} - 1)}{1.025 - 1} \\ &= \$986\,225.97 \end{aligned}$$

$$\begin{aligned} 4. (a) \quad P &= 8500, r = \frac{0.08}{4} + 1 = 1.02, n = (10)(4) = 40 \\ A &= Pr^n \\ &= 8500(1.02)^{40} \\ &= \$18\,768.34 \end{aligned}$$

$$\begin{aligned} (b) \quad r &= 1.06, n = 5, A = 10\,000 \\ P &= \frac{A}{r^n} \\ &= \frac{10\,000}{(1.06)^5} \\ &= \$7472.58 \end{aligned}$$

$$\begin{aligned} (c) \quad a &= 500, r = \frac{0.07}{2} + 1 = 1.035, n = 8 \times 2 = 16 \\ A &= \frac{a(r^n - 1)}{r - 1} \\ &= \frac{500(1.035^{16} - 1)}{1.035 - 1} \\ &= \$10\,485.51 \end{aligned}$$

$$\begin{aligned} (d) \quad R &= 100, n = 10 \times 12 = 120, r = \frac{0.048}{12} + 1 = 1.004 \\ P &= \frac{R(r^n - 1)}{r^n(r - 1)} \\ &= \frac{100(1.004^{120} - 1)}{1.004^{120}(1.004 - 1)} \\ &= \$9515.60 \end{aligned}$$

$$\begin{aligned} 5. (a) \quad R &= 120, n = 24, r = \frac{0.18}{12} + 1 = 1.015 \\ P &= \frac{R(r^n - 1)}{r^n(r - 1)} \\ &= \frac{120(1.015^{24} - 1)}{1.015^{24}(1.015 - 1)} \\ &= \$2403.65 \\ \therefore \text{cash price} &= \$2403.65 + \$300 = \$2703.65 \end{aligned}$$

(b) cost:
 $= (\$120 \times 24)$
 $= \$2880 - \$2703.65$
 $= \$176.35$

6. $R = 420, P = 23\,000, r = \frac{0.054}{12} + 1 = 1.0045$

$$P = \frac{R(r^n - 1)}{r^n(r - 1)}$$

$$23\,000 = \frac{420(1.0045^n - 1)}{1.0045^n(1.0045 - 1)}$$

$$23\,000 = \frac{420(1.0045^n - 1)}{1.0045^n(0.0045)}$$

$$103.5 = \frac{420(1.0045^n - 1)}{1.0045^n}$$

$$0.246\,428\,571 = 1 - \frac{1}{1.0045^n}$$

$$\frac{1}{1.0045^n} = 0.753\,571\,429$$

$$1.0045^n = 1.327\,014\,218$$

$$n \log(1.0045) = \log(1.327\,014\,218)$$

$$n = \frac{\log(1.327\,014\,218)}{\log(1.0045)}$$

$$n = 63.02$$

$\therefore$ it will take her 64 months to repay her student loan.

7. $P = \$245\,000$

Down payment $= 0.25 \times 245\,000 = \$61\,250$

$P = \$245\,000 - \$61\,250 = \$183\,750$

$n = 25 \times 12 = 300$

$(1 + i)^{12} = (1.0395)^2$

$i = 0.006\,48$

$r = 1.006\,48$

(a)

$$P = \frac{R(r^n - 1)}{r^n(r - 1)}$$

$$183\,750 = \frac{R(1.006\,48^{300} - 1)}{1.006\,48^{300}(0.006\,48)}$$

$$R = \$1391.05$$

(b)

	A	B	C	D	E
1	Payment#	Payment	Interest Paid	Price Paid	Outstanding Balance
2	0	0			\$183 750.00
3	1	\$ 1391.05	= E2*0.00648	= B3 - C3	= E2 - D3
4	2	= B3	= E3*0.00648	= B4 - C4	= E3 - D4

Use the "fill down" (apple-D) command to complete the chart to payment #300.

(From spreadsheet) Outstanding balance at the end of:

Year 1- \$181 258.24 Year 2- \$178 565.66

Year 3- \$175 656.07 Year 4- \$172 512.00

Year 5- \$169 114.55

(c) (From spreadsheet)

Principal paid = \$14 635.45

Interest paid = \$68 827.55

(d) Total Interest Paid = \$233 569.25

Total Cost of Home = \$1391.05 $\times$ 300 + \$61 250
 $= \$478\,565.00$

8. (a) $\frac{16^{\frac{3}{4}} - 27^{\frac{-2}{3}}}{4^{\frac{-1}{2}}}$
 $= \frac{\left(\sqrt[4]{16}\right)^3 - \left(\sqrt[3]{27}\right)^{-2}}{(\sqrt{4})^{-1}}$
 $= \frac{\left(8 - \frac{1}{9}\right)}{\left(\frac{1}{2}\right)}$
 $= \frac{\left(\frac{71}{9}\right)}{\left(\frac{1}{2}\right)}$
 $= \frac{71}{9} \times \frac{2}{1}$
 $= \frac{142}{9}$

(b) $\left(\frac{27x^2y^{-5}}{64x^{-1}y^4}\right)^{\frac{2}{3}}$
 $= \frac{27^{\frac{2}{3}}x^{\frac{4}{3}}y^{-\frac{10}{3}}}{64^{\frac{2}{3}}x^{-\frac{2}{3}}y^{\frac{8}{3}}}$
 $= \frac{\left(\sqrt[3]{27}\right)^2 x^{\frac{4}{3}} \left(x^{\frac{2}{3}}\right)}{\left(\sqrt[3]{64}\right)^2 \left(y^{\frac{8}{3}}\right) \left(y^{\frac{10}{3}}\right)}$

$= \frac{9x^2}{16y^6}$
(c) $\left(\frac{4}{x}\right)^{n-m} (2x^2)^m$
 $= \frac{4^{n-m}}{x^{n-m}} (2^m)(x^{2m})$
 $= \left(\frac{4^n}{4^m}\right) (2^m)(x^{2m})$
 $= \left(\frac{x^n}{x^m}\right) (2^m)(x^{2m})$
 $= \frac{4^n}{4^m} \times \frac{x^m}{x^n} (2^m)(x^{2m})$
 $= \frac{2^{2n}}{2^{2m}} \times \frac{x^m}{x^n} (2^m)(x^{2m})$
 $= \frac{2^{2n} x^{3m}}{2^m x^n}$

9. (a) $2x^2 - 4x + 5 = 0$

$$\begin{aligned} x &= \frac{4 \pm \sqrt{(-4)^2 - 4(2)(5)}}{2(2)} \\ &= \frac{4 \pm \sqrt{16 - 40}}{4} \\ &= \frac{4 \pm \sqrt{-24}}{4} \\ &= \frac{4 \pm 2\sqrt{-6}}{4} \\ &= \frac{2 \pm \sqrt{6(-1)}}{2} \\ &= \frac{2 \pm \sqrt{6i^2}}{2} \\ &= \frac{2 \pm \sqrt{6}i}{2} \\ &= 1 \pm \frac{\sqrt{6}}{2}i \end{aligned}$$

(b)

$$\begin{aligned} \frac{90\,000}{x} &= \frac{90\,000}{x+6} + 2500 \\ \frac{90\,000}{x} &= \frac{90\,000 + 2500(x+6)}{x+6} \\ \frac{90\,000}{x} &= \frac{90\,000 + 2500x + 15\,000}{x+6} \end{aligned}$$

$$90\,000(x+6) = x(2500x + 15\,000)$$

$$90\,000x + 540\,000 = 2500x^2 + 15\,000x$$

$$2500x^2 + 15\,000 - 540\,000 = 0$$

$$\begin{aligned} x &= \frac{-15\,000 \pm \sqrt{15\,000^2 - 4(2500)(-540\,000)}}{2(2500)} \\ &= \frac{-15\,000 \pm \sqrt{5\,625\,000\,000}}{5000} \\ &= \frac{-15\,000 \pm 75\,000}{5000} \end{aligned}$$

$$x = -3 \pm 15$$

$$x = 12, x = -18$$

(c)

$$4^{x-2} = \frac{1}{64}$$

$$4^{x-2} = 4^{-3}$$

$$x - 2 = -3$$

$$x = -1$$

(d)

$$2^{x^2+20} = \left(\frac{1}{8}\right)^{3x}$$

$$2^{x^2+20} = (2^{-3})^{3x}$$

$$2^{x^2+20} = 2^{-9x}$$

$$x^2 + 20 = -9x$$

$$x^2 + 9x + 20 = 0$$

$$(x+4)(x+5) = 0$$

$$x = -4, x = -5$$

10. (a) i. domain = $\{x \mid x \leq 1, x \in R\}$, range = R

ii. It is not a function because it fails the vertical line test.

(b) i. domain = R , range = $\{y \mid y > 0, y \in R\}$

ii. It is a function because it passes the vertical line test.

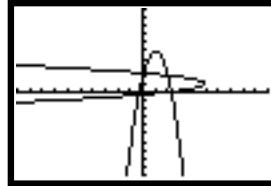
(c) i. domain = $\{x \mid x \geq 5, x \in R\}$, range = $\{y \mid y \geq 0, y \in R\}$

ii. It is a function because every x-value corresponds to a unique y-value.

(d) i. domain = R , range = $\{y \mid y \geq 4, y \in R\}$

ii. It is a function because every x-value corresponds to a unique y-value.

11.(a)



(b)

$$y = -3x^2 + 6x + 2$$

$$x = -3y^2 + 6y + 2$$

$$3y^2 - 6y = 2 - x$$

$$y^2 - 2y = \frac{2-x}{3}$$

$$y^2 - 2y + 1 - 1 = \frac{2-x}{3}$$

$$(y-1)^2 = \frac{2-x}{3} + 1$$

$$y-1 = \pm \sqrt{\frac{2-x}{3} + 1}$$

$$y = \pm \sqrt{\frac{2-x}{3} + 1} + 1$$

$$\therefore g^{-1}(x) = \pm \sqrt{\frac{2-x}{3} + 1} + 1$$

(c) restrictions: domain $[g(x)] = \{x \mid x \geq 1, x \in R\}$

or domain $[g(x)] = \{x \mid x \leq 1, x \in R\}$

(d) No, because the graph of g in that region fails the horizontal line test.

12. $g(x) = -2f(2x+6) + 3$

$$g(x) = -2f(2(x+3)) + 3$$

$$x \leftarrow +p \leftarrow \div k \leftarrow y = f(x) \rightarrow \times a \rightarrow +q \rightarrow y$$

$$x \leftarrow -3 \leftarrow \div 2 \leftarrow y = f(x) \rightarrow \times -2 \rightarrow +3 \rightarrow y$$

$$f(x) = a[f(k(x-p))] + q$$

(y) $a = -2$ $x \rightarrow -3, \div 2$

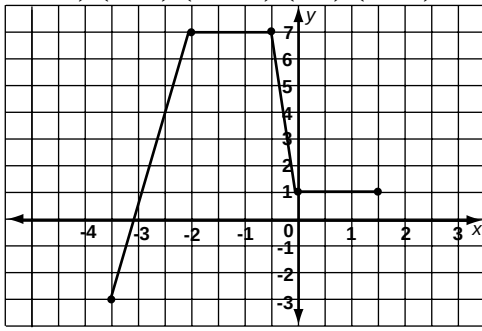
(x) $k = 2$ $y \rightarrow x - 2, +3$

(x) $p = -3$

(y) $q = 3$

$$f(x): (-4, 3), (-1, -2), (2, -2), (3, 1), (6, 1)$$

$$g(x): (-3.5, -3), (-2, 7), (-0.5, 7), (0, 1), (1.5, 1)$$



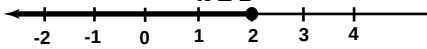
13. (a)

$$4(x - 3) + x \leq 6 - 2(x + 2)$$

$$4x - 12 + x \leq 6 - 2x - 4$$

$$7x \leq 14$$

$$x \leq 2$$



(b)

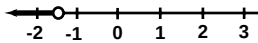
$$\frac{x}{3} - \frac{2x-3}{4} > \frac{5}{6}$$

$$\frac{4x}{12} - \frac{3(2x-3)}{12} > \frac{10}{12}$$

$$4x - 6x + 9 > 10$$

$$-2x > 1$$

$$x < -\frac{1}{2}$$



14. (a) First, complete the square to write the equation in standard form:

$$y = \frac{2}{3}x^2 - \frac{3}{4}x + 1$$

$$y = \frac{2}{3}\left(x^2 - \frac{9}{8}x + \frac{81}{256} - \frac{81}{256}\right) + 1$$

$$y = \frac{2}{3}\left(x - \frac{9}{16}\right)^2 - \frac{27}{128} + 1$$

$$y = \frac{2}{3}\left(x - \frac{9}{16}\right)^2 + \frac{101}{128}$$

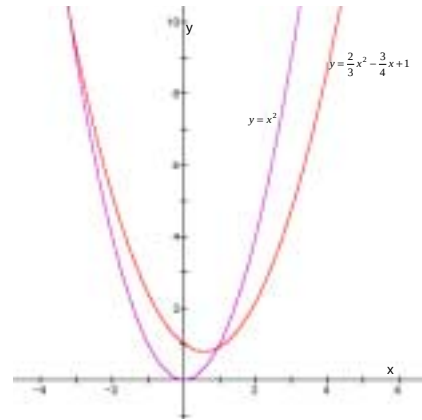
From the standard form, we can see what transformations must be applied to $y = x^2$;

horizontal shift $\frac{9}{16}$ units right, horizontal

stretch by a factor of $\frac{2}{3}$, and vertical

translation $\frac{101}{128}$ units up.

(b)



15. (a) $f(x) = kx^2 - 9x + 6$

i. To have two x -intercepts, the discriminant must be strictly greater than zero:

$$(-9)^2 - 4(k)(6) > 0$$

$$81 - 24k > 0$$

$$-24k > -81$$

$$k < \frac{81}{24}$$

$$k < \frac{27}{8}$$

ii. To have one x -intercept, the discriminant must be equal to zero:

$$(-9)^2 - 4k(6) = 0$$

$$81 - 24k = 0$$

$$24k = 81$$

$$k = \frac{27}{8}$$

iii. To have no x -intercepts, the discriminant must be strictly less than zero:

$$(-9)^2 - 4k(6) < 0$$

$$81 - 24k < 0$$

$$24k < 81$$

$$k > \frac{27}{8}$$

(b) $f(x) = 3x^2 + kx + 12$

i.

$$k^2 - 4(3)(12) > 0$$

$$k^2 - 144 > 0$$

$$k^2 > 144$$

$$|k| > 12$$

ii.

$$k^2 - 4(3)(12) = 0$$

$$k^2 - 144 = 0$$

$$k^2 = 144$$

$$k = \pm 12$$

iii.

$$k^2 - 4(3)(12) < 0$$

$$k^2 - 144 < 0$$

$$k^2 < 144$$

$$|k| < 12$$

(c) $f(x) = 9x^2 + 3x + k$

i.

$$32 - 4(9)(k) > 0$$

$$9 - 36k > 0$$

$$-36k > -9$$

$$k < \frac{9}{36}$$

$$k < \frac{1}{4}$$

ii.

$$32 - 4(9)(k) = 0$$

$$9 - 36k = 0$$

$$-36k = -9$$

$$k = \frac{9}{36}$$

$$k = \frac{1}{4}$$

iii.

$$32 - 4(9)(k) < 0$$

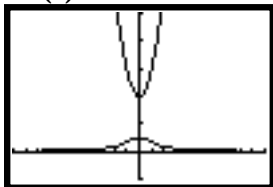
$$9 - 36k < 0$$

$$-36k < -9$$

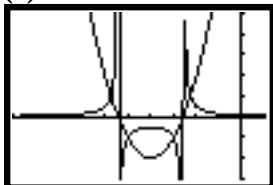
$$k > \frac{9}{36}$$

$$k > \frac{1}{4}$$

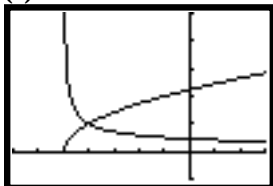
16. (a)



(b)



(c)



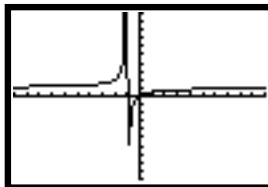
17. (a) i. zeros: $x = 0$

asymptotes: $x + 1 = 0$

$$x = -1$$

$$\text{domain} = \{x \mid x \in R, x \neq -1\}$$

ii.



$$\text{range} = \{y \mid y \neq 1, y \in R\}$$

(b) i. no zeros

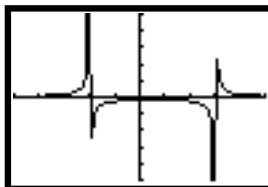
asymptotes: $x^2 - x - 6 = 0$

$$(x - 3)(x + 2) = 0$$

$$x = 3, x = -2$$

$$\text{domain} = \{x \mid x \neq 3, x \neq -2, x \in R\}$$

ii.



$$\text{range} = \{y \mid y \neq 0, y \in R\}$$

(c) i. zeros: $x^2 - 4 = 0$

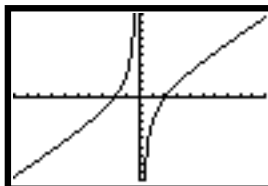
$$x^2 = 4$$

$$x = \pm 2$$

asymptotes: $x = 0$

$$\text{domain} = \{x \mid x \neq 0, x \in R\}$$

ii.



$$\text{range} = R$$

18. (a) $2x(3x - 1)(3x + 1) - (4x + 2)^3$

$$= (6x^2 - 2x)(3x + 1) - (4x + 2)(4x + 2)^2$$

$$= 18x^3 + 6x^2 - 6x^2 - 2x - (16x^2 + 16x + 4)$$

$$= 18x^3 - 2x - (64x^3 + 64x^2 + 16x + 32x^2 + 32x + 8)$$

$$= 18x^3 - 64x^3 - 96x^2 - 2x - 48x - 8$$

$$= -46x^3 - 96x^2 - 50x - 8$$

(b) $(6x - 2y)(x^2 + 4xy - 3y^2)$

$$= 6x^3 + 24x^2y - 18xy^2 - 2yx^2 - 8xy^2 + 6y^3$$

$$= 6x^3 + 22x^2y - 26xy^2 + 6y^3$$

19. (a) $\frac{5x^2 - 5}{x^2 - 4x - 5} = \frac{5x(x - 1)}{(x - 5)(x + 1)}, x \neq 5, -1$

(b) $\frac{4x^4y}{3x^2y^4} \times \frac{-6x^3y^2}{10x^4} = \frac{4x^2}{3y^3} \times \frac{-3y^2}{5x}, x \neq 0, y \neq 0$

$$= \frac{-4}{5} \times \frac{x}{y}$$

$$= \frac{-4x}{5y}, x \neq 0, y \neq 0$$

$$\begin{aligned}
 \text{(c)} \quad & \frac{2m^2 - m - 15}{m+2} \times \frac{m^2 - m - 6}{m^2 - m + 9} \\
 &= \frac{(2m+5)(m-3)}{(m+2)} \times \frac{(m-3)(m+2)}{(m^2 - m + 9)} \\
 &= \frac{(2m+5)(m-3)^2}{m^2 - m + 9}, m \neq -2, m \neq \frac{1 \pm \sqrt{35}i}{2}
 \end{aligned}$$

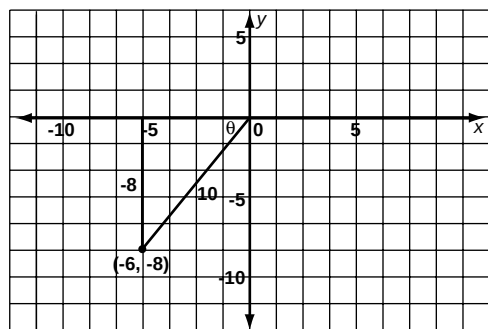
$$\begin{aligned}
 \text{(d)} \quad & \frac{x^2 - y^2}{2x^2 - 8x} \div \frac{(x-y)^2}{2xy} \\
 &= \frac{(x-y)(x+y)}{2x(x-4)} \times \frac{2xy}{(x-y)^2}, x \neq 0, 4, y; y \neq 0 \\
 &= \frac{y(x+y)}{(x-4)(x-y)}, x \neq 0, 4, y; y \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad & \frac{1}{x} - \frac{3}{y} + \frac{3x-y}{xy}, x \neq 0, y \neq 0 \\
 &= \frac{1}{x} \times \frac{y}{y} - \frac{3}{y} \times \frac{x}{x} + \frac{3x-y}{xy} \\
 &= \frac{y-3x+3x-y}{xy}
 \end{aligned}$$

$$= 0, x \neq 0, y \neq 0$$

$$\begin{aligned}
 \text{(f)} \quad & \frac{x+2}{x^2+5x+6} - \frac{x-3}{x^2-3x-10} \\
 &= \frac{x+2}{(x+3)(x+2)} - \frac{x-3}{(x-5)(x+2)}, x \neq -3, -2, 5 \\
 &= \frac{1}{x+3} - \frac{(x-3)}{(x-5)(x+2)} \\
 &= \frac{(x-5)(x+2) - (x-3)(x+3)}{(x+3)(x-5)(x+2)} \\
 &= \frac{x^2 - 3x - 10 - x^2 + 9}{(x+3)(x-5)(x+2)} \\
 &= \frac{-3x-1}{(x+3)(x-5)(x+2)}, x \neq -3, -2, 5
 \end{aligned}$$

20.



We can use the pythagorean theorem to solve for the unknown side:

$$\begin{aligned}
 c^2 &= (-6)^2 + (-8)^2 \\
 &= 36 + 64 \\
 &= 100 \\
 c &= 10
 \end{aligned}$$

Use trigonometric ratios to find the angle θ :

$$\cos \theta = \frac{-6}{10} \quad \sin \theta = \frac{-8}{10} \quad \tan \theta = \frac{8}{6}$$

$$\frac{\sin \theta + \cos \theta}{\tan \theta} = \frac{\sin\left(\frac{-8}{10}\right) + \cos\left(\frac{-6}{10}\right)}{\tan\left(\frac{8}{6}\right)} = 42.36^\circ$$

$$21. \text{(a)} \quad 35^\circ = 35 \left(\frac{\pi}{180} \right) = \frac{7\pi}{36} \text{ radians}$$

$$\text{(b)} \quad 320^\circ = 320 \left(\frac{\pi}{180} \right) = \frac{16\pi}{9} \text{ radians}$$

$$\text{(c)} \quad -180^\circ = -180 \left(\frac{\pi}{180} \right) = -\pi \text{ radians}$$

$$\text{(d)} \quad 225^\circ = 225 \left(\frac{\pi}{180} \right) = \frac{5\pi}{4} \text{ radians}$$

$$22. \text{(a)} \quad \frac{7\pi}{8} = \frac{7\pi}{8} \left(\frac{180^\circ}{\pi} \right) = 157.5^\circ$$

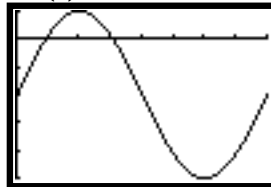
$$\text{(b)} \quad -\frac{\pi}{12} = -\frac{\pi}{12} \left(\frac{180^\circ}{\pi} \right) = -15^\circ$$

$$\text{(c)} \quad \frac{8\pi}{3} = \frac{8\pi}{3} \left(\frac{180^\circ}{\pi} \right) = 480^\circ$$

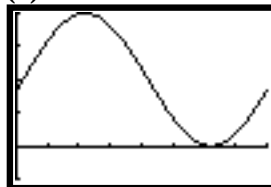
$$\text{(d)} \quad \frac{5\pi}{16} = \frac{5\pi}{16} \left(\frac{180^\circ}{\pi} \right) = 56.25^\circ$$

$$\begin{aligned}
 23. \quad \sin^2\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{6}\right) &= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 \\
 &= \frac{1}{4} + \frac{3}{4} \\
 &= 1
 \end{aligned}$$

24. (a)



(b)



$$25. (a) y = a \cos b(\theta - c) + d$$

$$a = \frac{\text{max} - \text{min}}{2} = \frac{0 - (-4)}{2} = \frac{4}{2} = 2$$

$$\text{period} = \frac{2\pi}{b}$$

$$360^\circ = \frac{2\pi}{b}$$

$$b = 1$$

$$c = \frac{\pi}{4}$$

$$d = \frac{\text{max} + \text{min}}{2} = \frac{0 + (-4)}{2} = -2$$

$$\therefore y = 2 \cos\left(\theta - \frac{\pi}{4}\right) - 2$$

$$(b) y = -a \cos b(\theta - c) + d$$

$$a = \frac{5 - 1.5}{2} = 1.75$$

$$\text{period} = \frac{2\pi}{b}$$

$$360^\circ = \frac{2\pi}{b}$$

$$b = 1$$

$$c = 0$$

$$d = \frac{5 + 1.5}{2} = 3.25$$

$$\therefore y = -1.75 \cos(\theta) + 3.25$$

26. (a) high tide = max value of function = 18 m

low tide = min value of function = 11.5 m

Choose the cosine function to model the depth of the water in the inlet because the maximum value (high tide) occurs at the beginning of the day (just after midnight).

$$y = a \cos b(t - c) + d$$

$$a = \frac{\text{max} - \text{min}}{2} = \frac{18 - 11.5}{2} = 3.25$$

$$d = \frac{\text{max} + \text{min}}{2} = \frac{18 + 11.5}{2} = 14.75$$

The period of the function is 24 hours. Low tide occurs 12 hours after high tide, and then high tide will occur again 12 hours later (at 12:06 a.m.).

$$\text{Period} = 24 = \frac{2\pi}{b}$$

$$\frac{\pi}{12} = b$$

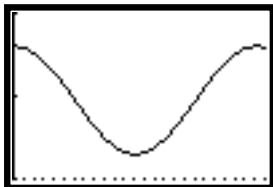
The horizontal phase shift is 6 minutes to the right, since high tide occurs at 12:06 a.m.

$$\frac{6 \text{ min}}{60 \text{ min/h}} = 0.1 \text{ hours}$$

$\therefore$ the depth of water in the inlet at time t is

$$d(t) = 3.25 \cos \frac{\pi}{12}(t - 0.1) + 14.75$$

(b)



(c) i)

$$d(t) = 15$$

$$15 = 3.25 \cos \frac{\pi}{12}(t - 0.1) + 14.75$$

$$0.25 = 3.25 \cos \frac{\pi}{12}(t - 0.1)$$

Examine $\cos x = 0.076923$

$$x = \cos^{-1}(0.076923)$$

$$x = 1.4938$$

$$\frac{\pi}{12}(t - 0.1) = 1.4938$$

$$(t - 0.1) = 5.7059$$

$$t = 5.8$$

There are two answers $0 \leq t \leq 24$

$$t = 5.8 \text{ hours,}$$

$$(0.8 \times 60 = 48 \text{ min})$$

$$t = 24 - 5.8$$

$$= 18.2 \text{ hours}$$

$$(0.2 \times 60 = 12 \text{ min})$$

$\therefore$ the water is at a depth of 15 m at approximately 5:48a.m. and 6:12p.m.

ii)

$$d(t) = 12$$

$$12 = 3.25 \cos \frac{\pi}{12}(t - 0.1) + 14.75$$

$$-2.75 = 3.25 \cos \frac{\pi}{12}(t - 0.1)$$

$$-0.8462 = \cos \frac{\pi}{12}(t - 0.1)$$

$$\text{examine } \cos x = -0.8462$$

$$x = \cos^{-1}(-0.8462)$$

$$x = 2.6$$

$$\frac{\pi}{12}(t - 0.1) = 2.6$$

$$(t - 0.1) = 9.9$$

$$t = 10$$

There are two answers $0 \leq t \leq 24$

$$t = 10 \text{ hours and } t = 24 - 10 = 14 \text{ hours}$$

$\therefore$ the water is at a depth of 12 m at approximately 10 a.m. and 2 p.m.

$$27. (a) 2 \sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

There are two answers for $0 \leq x \leq 2\pi$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$x = 1.0, 2.1$$

(b)

$$5 \cos x - 4 = 0$$

$$5 \cos x = 4$$

$$\cos x = \frac{4}{5}$$

There are two answers for $0 \leq x \leq 2\pi$

$$x = 36.9^\circ \text{ and } x = 360^\circ - 36.9^\circ = 323.1^\circ$$

$$(c) \sin 2x (2 \cos x + 1) = 0, 0 \leq x \leq 2\pi$$

$$1. \sin 2x = 0$$

$$2. 2 \cos x + 1 = 0$$

1. The period of $\sin 2x$ is π . Thus, there are five answers for $0 \leq x \leq 2\pi$
 $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ ($x = 0, 1.6, 3.1, 4.7, 6.3$)

2.

$$2 \cos x + 1 = 0$$

$$2 \cos x = -1$$

$$\cos x = -\frac{1}{2}$$

There are two answers: $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$$x = 0, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{3\pi}{2}, \frac{4\pi}{3}, 2\pi$$

(d)

$$3\sin^2 x - 11\sin x - 4 = 0$$

$$3\sin^2 x - 12\sin x - 4 = 0$$

$$3\sin x(\sin x - 4) + 1(\sin x - 4) = 0$$

$$(\sin x - 4)(3\sin x + 1) = 0$$

$$\therefore 1. \sin x - 4 = 0 \text{ or } 2. 3\sin x + 1 = 0$$

1.

$$\sin x - 4 = 0$$

$$\sin x = 4$$

The sine function has a maximum value of $\sin x = 1$ which is not possible.

2.

$$3\sin x + 1 = 0$$

$$3\sin x = -1$$

$$\sin x = -\frac{1}{3}$$

There are two possibilities for $0^\circ \leq x \leq 360^\circ$

$$x = \sin^{-1}\left(-\frac{1}{3}\right)$$

$$x = -19.5^\circ$$

$$x = 360^\circ - 19.5^\circ = 340.5^\circ \text{ or }$$

$$x = 180^\circ + 19.5^\circ = 199.5^\circ$$

$$28. (a) \frac{2}{\sin^2 \theta} = \frac{1}{1 - \cos \theta} + \frac{1}{1 + \cos \theta}$$

$$R.S = \frac{1 + \cos \theta + 1 - \cos \theta}{(1 - \cos \theta)(1 + \cos \theta)}$$

$$= \frac{2}{1 - \cos^2 \theta}$$

$$= \frac{2}{\sin^2 \theta}$$

$$\therefore L.S = R.S$$

$$(b) \tan^2 \theta + \cos^2 \theta + \sin^2 \theta = \frac{1}{\cos^2 \theta}$$

$$L.S = \frac{\sin^2 \theta}{\cos^2 \theta} + \cos^2 \theta + \sin^2 \theta$$

$$= \frac{\sin^2 \theta + \cos^4 \theta + \sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$$

$$= \frac{(1 - \cos^2 \theta) + \cos^4 \theta + (1 - \cos^2 \theta) \cos^2 \theta}{\cos^2 \theta}$$

$$= \frac{1 - \cos^2 \theta + \cos^4 \theta + \cos^2 \theta - \cos^4 \theta}{\cos^2 \theta}$$

$$= \frac{1}{\cos^2 \theta}$$

$$\therefore L.S = R.S$$

$$29. 20 \text{ km/h (1.5 h)} = 30 \text{ km}$$

$$18 \text{ km/h (1.5 h)} = 27 \text{ km}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$= 27^2 + 30^2 - 2(27)(30)\cos 102^\circ$$

$$= 729 + 900 - 1620 \cos 102^\circ$$

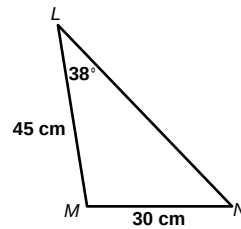
$$= 1629 - 1620 \cos 102^\circ$$

$$= 1965.82$$

$$c = 44.34$$

$\therefore$ they are 44.34 km apart after 1.5 hours.

30.



$$\frac{\sin N}{n} = \frac{\sin L}{l}$$

$$\frac{\sin N}{45} = \frac{\sin 38^\circ}{30}$$

$$\sin N = 0.923 \ 492 \ 213$$

$$N = \sin^{-1}(0.923 \ 492 \ 213)$$

$$N = 67.4^\circ$$

$$N = 67^\circ$$

$$M = 180^\circ - \angle N - \angle L$$

$$M = 180^\circ - 67^\circ - 38^\circ$$

$$M = 75^\circ$$

$$m^2 = l^2 + n^2 - 2 \ln(\cos m)$$

$$= 30^2 + 45^2 - 2(30)(45)\cos 75^\circ$$

$$= 900 + 2025 - 2700 \cos 75^\circ$$

$$= 2925 - 2700 \cos 75^\circ$$

$$= 2226.19$$

$$m = 47.18$$

$$m \doteq 47$$

31. Find the distance from campsite 1 to the tower:

$$\tan 78^\circ = \frac{d_1}{95}$$

$$d_1 = 95 \tan 78^\circ$$

$$= 446.94 \text{ m}$$

Find the distance from campsite 2 to the tower:

$$\tan 73^\circ = \frac{d_2}{95}$$

$$d_2 = 95 \tan 73^\circ$$

$$= 310.73 \text{ m}$$

$$d^2 = 446.94^2 + 310.73^2 - 2(446.94)(310.73) \cos 40^\circ$$

$$= 199\,755.4 + 96\,553.1 - 277\,755.3 \cos 40^\circ$$

$$= 296\,308.50 - 277\,755.3 \cos 40^\circ$$

$$= 83\,535.57$$

$$d = 289.03$$

$\therefore$ the campsites are 289.03 m apart.

32.U. $t_3 = 4t_2 + 2t_1$

$$= 4(3) + 2(5)$$

$$= 12 + 10$$

$$= 22$$

$t_4 = 4t_3 + 2t_2$

$$= 4(22) + 2(3)$$

$$= 88 + 6$$

$$= 94$$

$t_5 = 4t_4 + 2t_3$

$$= 4(94) + 2(22)$$

$$= 376 + 88$$

$$= 464$$

$t_6 = 4t_5 + 2t_4$

$$= 4(464) + 2(94)$$

$$= 1856 + 188$$

$$= 2044$$

First six terms are: 5, 3, 22, 94, 464, 2044

- 33 U. (a)

$$(i+3)^2 = i^2 + 6i + 9$$

$$= (-1) + 6i + 9$$

$$= 8 + 6i$$

- (b)

$$3i^4 = 3(i^2)^2$$

$$= 3(-1)^2$$

$$= 3(1)$$

$$= 3$$

- (c) $(4-5i)(3+2i)$

$$= 12 - 7i - 10i^2$$

$$= 12 - 7i - 10(-1)$$

$$= 12 - 7i + 10$$

$$= 22 - 7i$$

- (d) $(5-7i) + (6-4i) - (3-2i)$

$$= 5 - 7i + 6 - 4i - 3 + 2i$$

$$= 8 - 9i$$

- (e)

$$\frac{5+2i}{4+6i} \times \frac{4-6i}{4-6i}$$

$$= \frac{20-30i+8i-12i^2}{16-36i^2}$$

$$= \frac{20-22i-12(-1)}{16-36(-1)}$$

$$= \frac{32-22i}{52}$$

$$= -\frac{8}{5} + \frac{11}{10}i$$

(f) $[2i + (1-i)]^2$

$$= (2i + 1 - i)^2$$

$$= (i + 1)^2$$

$$= i^2 + 2i + 1$$

$$= (-1) + 2i + 1$$

$$= 2i$$

- (g)

$$\frac{(3+6i)}{(2-3i)(3+2i)}$$

$$= \frac{3+6i}{3+6i}$$

$$= \frac{6+4i-9i-6i^2}{3+6i}$$

$$= \frac{6-5i-6(-1)}{3+6i}$$

$$= \frac{3+6i}{12-5i} \times \frac{12+5i}{12+5i}$$

$$= \frac{36+87i+30(-1)}{144-25(-1)}$$

$$= \frac{6+87i}{169}$$

$$= \frac{6}{169} + \frac{87}{169}i$$

- (h)

$$\frac{(5+8i) + (-3+5i)}{(6+7i) - (5-4i)}$$

$$= \frac{2+13i}{1+11i} \times \frac{1-11i}{1-11i}$$

$$= \frac{2-22i+13i-143i^2}{1-121i^2}$$

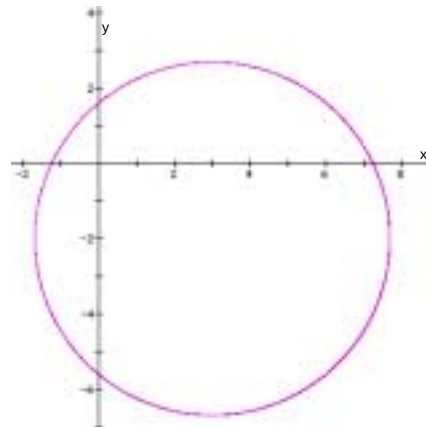
$$= \frac{2-9i-143(-1)}{1-121(-1)}$$

$$= \frac{2-9i+143}{1+121}$$

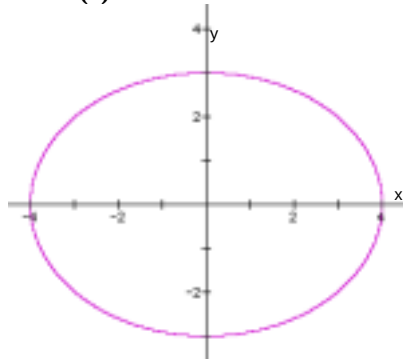
$$= \frac{145-9i}{122}$$

$$= \frac{145}{122} - \frac{9}{122}i$$

- 34.U.

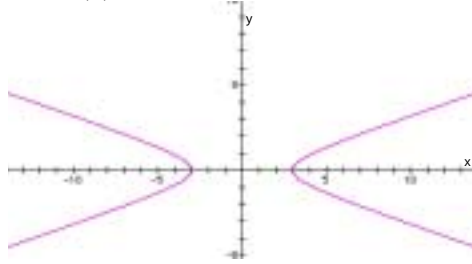


35.U. (a)



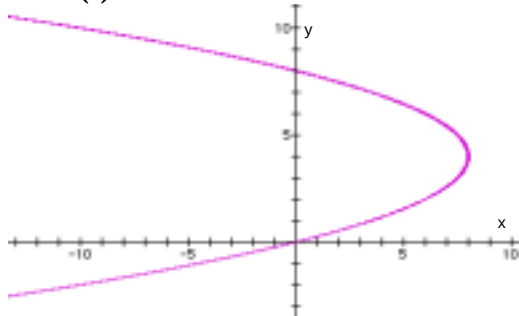
vertices: $(-4, 0)$, $(4, 0)$; foci: $(-2.65, 0)$, $(2.65, 0)$

(b)



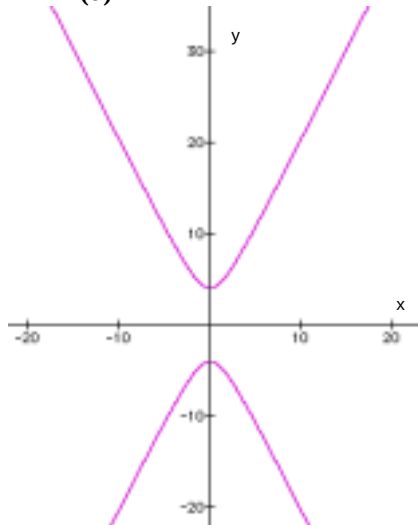
vertices: $(-3, 0)$, $(3, 0)$; foci: $(-3.16, 0)$, $(3.16, 0)$

(c)



vertex: $(8, 4)$; focus: $(7.5, 4)$

(d)



vertices: $(0, -4)$, $(0, 4)$; foci: $(0, -4.5)$, $(0, 4.5)$

36.U. (a) $\frac{(x-2)^2}{16} + \frac{(y-3)^2}{81} = 1$

(b)

$$y = \pm \frac{3}{5} \sqrt{x^2 - 25}$$

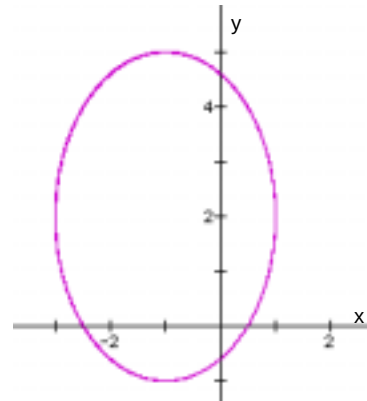
$$y^2 = \frac{9}{25} (x^2 - 25)$$

$$y^2 = \frac{9}{25} x^2 - 9$$

$$\frac{9}{25} x^2 - y^2 = 9$$

37.U. (a) i. ellipse

ii.



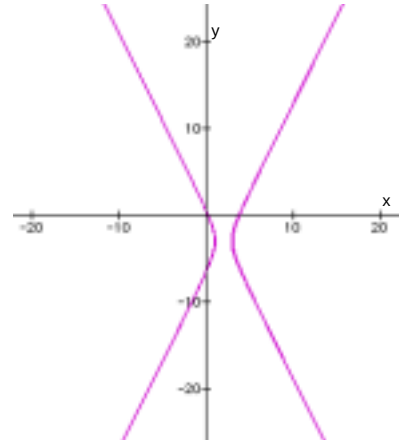
(b) i. parabola

ii.



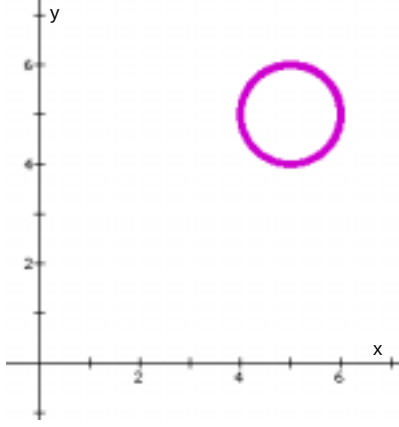
(c) i. hyperbola

ii.



(d) i. circle

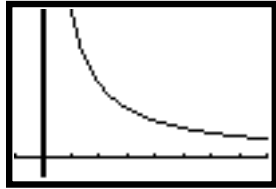
ii.



38.U. (a)

$$xy = 8$$

$$y = \frac{x}{8}$$



(b) Let $P(x, y)$ be any point on the locus.

$$PA = \sqrt{(x-0)^2 + (y-4)^2}, PB = \sqrt{(x-2)^2 + (y-6)^2}$$

$$\sqrt{(x-0)^2 + (y-4)^2} = \sqrt{(x-2)^2 + (y-6)^2}$$

$$x^2 + (y-4)^2 = (x-2)^2 + (y-6)^2$$

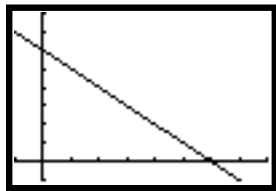
$$x^2 + y^2 - 8y + 16 = x^2 - 4x + 4 + y^2 - 12y + 36$$

$$-8y + 16 + 4x - 4 + 12y - 36 = 0$$

$$4y + 4x - 24 = 0$$

$$4y = -4x + 24$$

$$y = -x + 6$$



39.U. First isolate y in the second equation:

$$(x^2 - 6x + 9 - 9) + (y^2 + 8y + 16) = 0$$

$$(x-3)^2 - 9 + (y+4)^2 = 0$$

$$(y+4)^2 = -(x-3)^2 + 9$$

$$y+4 = \pm \sqrt{-(x-3)^2 + 9}$$

$$y = \pm \sqrt{-(x-3)^2 + 9} - 4$$

Now equate this new equation above with the first equation and solve for x :

$$2x - 4 = \pm \sqrt{-(x-3)^2 + 9} - 4$$

$$2x = \pm \sqrt{-(x-3)^2 + 9}$$

$$4x^2 = -(x-3)^2 + 9$$

$$4x^2 = -(x^2 - 6x + 9) + 9$$

$$4x^2 + x^2 - 6x = 0$$

$$5x^2 - 6x = 0$$

$$5x\left(x - \frac{6}{5}\right) = 0$$

$$x = 0 \text{ or } x = \frac{6}{5}$$

Now substitute $x = 0$ and $x = \frac{6}{5}$ into the first

equation to find the points of intersection:

when $x = 0$, $y = -4$

$$\begin{aligned} \text{when } x = \frac{6}{5}, y &= 2\left(\frac{6}{5}\right) - 4 \\ &= -\frac{8}{5} \end{aligned}$$

$\therefore$ the points of intersection are $(0, -4)$ and $\left(\frac{6}{5}, -\frac{8}{5}\right)$