

Chapter 1 Review Test, page 104

1. (a) $t_n = 6n + 5$ i. $t_1 = 6(1) + 5 = 11$, $t_2 = 6(2) + 5 = 17$,
 $t_3 = 6(3) + 5 = 23$, $t_4 = 6(4) + 5 = 29$ ii. $d_1 = 17 - 11 = 6$,
 $d_2 = 23 - 17 = 6$, $\therefore$ the sequence is arithmetic.

(b) $t_n = \frac{2}{n(n+1)}$ i. $t_1 = \frac{2}{1(1+1)} = \frac{2}{2} = 1$,

$t_2 = \frac{2}{2(2+1)} = \frac{2}{6} = \frac{1}{3}$, $t_3 = \frac{2}{3(3+1)} = \frac{2}{12} = \frac{1}{6}$,

$t_4 = \frac{2}{4(4+1)} = \frac{2}{20} = \frac{1}{10}$ ii. $d_1 = \frac{1}{3} - 1 = -\frac{2}{3}$,

$d_2 = \frac{1}{6} - \frac{1}{3} = -\frac{1}{6}$, $d_1 \neq d_2 \therefore$ the sequence is not arithmetic.

$r_1 = \frac{\frac{1}{3}}{1} = \frac{1}{3}$, $r_2 = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$, $r_1 \neq r_2 \therefore$ the sequence is not

geometric. $\therefore$ the sequence is neither.

(c) $t_n = 3(4)^{n-1}$ i. $t_1 = 3(4)^{1-1} = 3$, $t_2 = 3(4)^{2-1} = 12$,

$t_3 = 3(4)^{3-1} = 48$, $t_4 = 3(4)^{4-1} = 192$ ii. $r_1 = \frac{12}{3} = 4$,

$r_2 = \frac{48}{12} = 4$, $r_1 = r_2$, $\therefore$ the sequence is geometric

(d) $t_1 = 5$, $t_2 = 1$, $t_n = 3t_{n-1} - 2t_{n-2}$ i. $t_3 = 3(1) - 2(5) = -7$,
 $t_4 = 3(-7) - 2(1) = -23$ ii. $\therefore$ this recursive sequence is neither
arithmetic nor geometric

2. (a) $-6, -11, -16, \dots, -156$, $a = -6$, $d = -5$

$$t_n = a + (n-1)d$$

$$-156 = -6 + (n-1)(-5)$$

$$-156 = -6 - 5n + 5$$

$$-155 = -5n$$

$$n = 31$$

(b) $4, 12, 36, \dots, 972$, $a = 4$, $r = 3$

$$t_n = ar^{n-1}$$

$$972 = (4)(3)^{n-1}$$

$$243 = 3^{n-1}$$

$$3^5 = 3^{n-1}$$

$$5 = n - 1$$

$$n = 6$$

3. $t_{13} = -177$, $t_{22} = -207$

$$t_{22} - t_{13} = 9d$$

$$-207 - (-177) = 9d$$

$$-30 = 9d$$

$$d = \frac{-10}{3}$$

$$t_n = a + (n-1)d$$

$$-177 = a + (13-1)\left(\frac{-10}{3}\right)$$

$$-177 = a - 40$$

$$a = -137$$

$$t_n = -137 + (n-1)\left(\frac{-10}{3}\right) = \frac{-10}{3}n - \frac{401}{3}$$

4. (a) $\left(\frac{-8}{27}\right)^{\frac{-2}{3}} = \left(\frac{-2}{3}\right)^{-2} = \left(\frac{3}{-2}\right)^2 = \frac{9}{4}$

(b) $\left(36^{\frac{-1}{2}} + 8^{\frac{-2}{3}}\right) \div 12^{-2} = \left(\frac{1}{6} + \frac{1}{4}\right) \times 144 = 60$

$$5. (a) \frac{(16x^4y^2)^{\frac{1}{2}}(4y^6)^{\frac{1}{2}}}{(x^3y^6)^{\frac{1}{3}}} = \frac{(4x^2y)(2y^3)}{xy^2} = 8xy^2$$

$$(b) (a^{2m+3n})(a^{m-2n}) \div [(a^m)(a^{2m})] = \frac{a^{3m+n}}{a^{3m}} = a^n$$

6. In an arithmetic sequence, each term is the sum of the previous term plus a fixed amount. In a geometric sequence, each term is the product of the previous term and a fixed ratio.

Example #1: $2, 3, 4, 5, 6, \dots$ $a = 2$, $d_1 = 3 - 2 = 1$,
 $d_2 = 4 - 3 = 1$, $\therefore d_1 = d_2$, $\therefore$ the sequence is arithmetic.

OR $r_1 = \frac{3}{2}$, $r_2 = \frac{4}{3}$, $\therefore r_1 \neq r_2$, $\therefore$ the sequence is not
geometric.

Example #2: $2, 4, 8, 16, 32, \dots$, $a = 2$, $d_1 = 4 - 2 = 2$,
 $d_2 = 8 - 4 = 4$, $\therefore d_1 \neq d_2$, $\therefore$ the sequence is not arithmetic.

$r_1 = \frac{4}{2} = 2$, $r_2 = \frac{8}{4} = 2$, $\therefore r_1 = r_2$, $\therefore$ the sequence is
geometric.

$$7. (a) 2^{3(x-2)} = 64$$

$$2^{3(x-2)} = 2^6$$

$$3x - 6 = 6$$

$$x = 4$$

(b)

$$3^{x^2+4x} = \frac{1}{81}$$

$$3^{x^2+4x} = 3^{-4}$$

$$x^2 + 4x + 4 = 0$$

$$(x+2)^2 = 0$$

$$x = -2$$

$$8. t_n = 60\left(\frac{3}{5}\right)^n, t_{10} = 60\left(\frac{3}{5}\right)^{10} \doteq 0.36 \text{ m}$$

$\therefore$ the ball bounces 0.36 m after the tenth bounce.

$$9. i \frac{0.08}{2} = 0.04, n = 5 \times 2 = 10, P = \frac{\$20\,000}{(1.04)^{10}} = \$13\,511.28$$

$\therefore$ Jacques must invest \$13 511.28 now so that he will have
\$20 000 in five years.

10.

$$P_n = 12\,400(1.03)^n$$

$$13\,250 = 12\,400(1.03)^n$$

$$1.0685 = (1.03)^n$$

$$\ln(1.0685) = n \ln(1.03)$$

$$n = \frac{\ln(1.0685)}{\ln(1.03)}$$

$$n = 2.24 \text{ years}$$

$\therefore$ Lancaster will have a population of 12 400 in 2.24 years.

11. 14, 18, 22, 26, ..., t_{15}

$$t_n = 14 + 4(n - 1) = 4n + 10$$

$$t_{15} = 4(15) + 10 = 60 + 10 = 70$$

$\therefore$ the last row has 70 seats.

12. $i = \frac{0.0675}{2} = 0.03375$, $n = 3 \times 2 = 6$; $n = 2 \times 2 = 4$,

$$A = [\$5000(1.03375)^6 + \$2000](1.03375)^4 = \$9252.25$$

$\therefore$ Bernie will have a balance of \$9252.25 in the account.