

Chapter 2

Practise, Apply, Solve 2.1 page 114

1. (a) yes

(b) no

(c) yes

(d) no

$$2. t_n = a + (n-1)d, S_n = \frac{n(t_1 + t_n)}{2}$$

$$(a) t_{10} = 2 + 9(8) = 2 + 72 = 74$$

$$S_{10} = \frac{10(2 + 74)}{2}$$

$$= 5(76)$$

$$= 380$$

$$(b) t_{10} = 100 + 9(-15) = 100 - 135 = -35$$

$$S_{10} = \frac{10(100 - 35)}{2}$$

$$= 5(65)$$

$$= 325$$

$$(c) t_{10} = -18 + 9(7) = -18 + 63 = 45$$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$S_{10} = \frac{10(-18 + 45)}{2}$$

$$= 5(27)$$

$$= 135$$

$$(d) t_{10} = \frac{1}{6} + 9\left(\frac{1}{3}\right) = \frac{1}{6} + 3 = 3\frac{1}{6} \text{ or } \frac{19}{6}$$

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$S_{10} = \frac{10\left(\frac{1}{6} + \frac{19}{6}\right)}{2}$$

$$= 5\left(\frac{20}{6}\right)$$

$$= \frac{100}{6} \text{ or } \frac{50}{3}$$

$$3. \text{ The sum of the numbers from 1 to 200 is given by } \frac{200 \times 201}{2}.$$

$$4. (a) \text{ sum} = \frac{30 \times 31}{2} = 465$$

$$(b) \text{ sum} = \frac{60 \times 61}{2} = 1830$$

$$(c) \text{ sum} = \frac{1000 \times 1001}{2} = 500\,500$$

$$(d) \text{ sum} = \frac{24 \times 25}{2} = 300$$

5. (a)

$$S_{12} = \frac{12(t_1 + t_{12})}{2}$$

$$= 6(7 + 51)$$

$$= 348$$

(b)

$$S_{15} = \frac{15(t_1 + t_{15})}{2}$$

$$= \frac{15(88 + 4)}{2}$$

$$= \frac{15(92)}{2}$$

$$= 690$$

(c)

$$S_{20} = \frac{20(t_1 + t_{20})}{2}$$

$$= 10(-784 + 869)$$

$$= 850$$

(d)

$$S_{30} = \frac{30(t_1 + t_{30})}{2}$$

$$= \frac{30(-2 + 85)}{2}$$

$$= 15(83)$$

$$= 1245$$

$$6. S_n = \frac{n}{2}[2a + (n-1)d]$$

(a)

$$S_{15} = \frac{15}{2}[2(3) + 14(4)]$$

$$= \frac{15}{2}(62)$$

$$= 465$$

(b)

$$S_{20} = \frac{20}{2}[2(11) + (19)11]$$

$$= 10(231)$$

$$= 2310$$

(c)

$$S_{27} = \frac{27}{2}[2(4) + 26(-5)]$$

$$= \frac{27}{2}(-122)$$

$$= -1647$$

(d)

$$\begin{aligned}
S_{14} &= \frac{14}{2} \left[2 \left(\frac{1}{2} \right) + 13 \left(\frac{1}{8} \right) \right] \\
&= 7 \left(1 + \frac{13}{8} \right) \\
&= 7 \left(\frac{21}{8} \right) \\
&= \frac{147}{8}
\end{aligned}$$

7.

$$\begin{aligned}
S_n &= \frac{n}{2} [2a + (n-1)d] \\
S_{21} &= \frac{21}{2} [2(2.8) + 20(0.4)] \\
&= \frac{21}{2} (5.6 + 8) \\
&= \frac{21}{2} (13.6) \\
&= 142.8 \text{ or } \frac{714}{5}
\end{aligned}$$

8. $t_n = a + (n-1)d$, $S_n = \frac{n}{2} [2a + (n-1)d]$

(a) $156 = 13 + (n-1)(11)$
 $156 = 13 + 11n - 11$
 $156 - 2 = 11n$
 $154 = 11n$
 $n = 14$
 $S_{11} = \frac{14}{2} [2(13) + 13(11)]$
 $= 7(169)$
 $= 1183$

(b) $-37 = 15 + (n-1)(-4)$
 $-37 = 15 - 4n + 4$
 $-56 = -4n$
 $n = 14$
 $S_{14} = \frac{14}{2} [2(15) + 13(-4)]$
 $= 7(-22)$
 $= -154$

(c) $252 = -77 + (n-1)(7)$
 $252 = -77 + 7n - 7$
 $336 = 7n$
 $n = 48$
Using $S_n = \frac{n(t_1 + t_n)}{2}$
 $S_{48} = \frac{48(-77 + 252)}{2}$
 $= 24(175)$
 $= 4200$

(d)

$$\begin{aligned}
\frac{5}{3} &= \frac{1}{6} + (n-1) \left(\frac{1}{6} \right) \\
\frac{5}{3} &= \frac{1}{6} + \frac{1}{6}n - \frac{1}{6} \\
\frac{5}{3} &= \frac{n}{6} \\
\frac{10}{6} &= \frac{n}{6} \\
n &= 10 \\
S_{10} &= \frac{10 \left(\frac{1}{6} + \frac{5}{6} \right)}{2} \\
&= 5 \left(\frac{1}{6} + \frac{10}{6} \right) \\
&= 5 \left(\frac{11}{6} \right) \\
&= \frac{55}{6}
\end{aligned}$$

9. $t_{17} = 53$ and $t_{28} = 86$

$53 = a + 16d$ ① and $86 = a + 27d$ ②

Solve using elimination.

$$\begin{array}{rcl}
53 & = & a + 16d \quad \text{①} \\
86 & = & a + 27d \quad \text{②} \\
\hline
\text{①} - \text{②} & & -33 = -11d \quad \text{subtract ② from ①} \\
d & = & 3
\end{array}$$

Substitute $d = 3$ into ①.

$53 = a + 16(3)$

$53 = a + 48$

$a = 5$

$S_n = \frac{n}{2} [2a + (n-1)d]$

$S_{50} = \frac{50}{2} [2(5) + 49(3)]$
 $= 25(10 + 147)$
 $= 3925$

10. $t_7 = 34$ and $t_9 = 48$

$34 = a + 6d$ ① and $48 = a + 8d$ ②

Solve using elimination by subtracting ① from ②.

$$\begin{array}{rcl}
48 & = & a + 8d \quad \text{②} \\
34 & = & a + 6d \quad \text{①} \\
\hline
14 & = & 2d \\
d & = & 7
\end{array}$$

Substitute $d = 7$ into ①.

$34 = a + 6(7)$

$34 = a + 42$

$34 - 42 = a$

$a = -8$

$S_n = \frac{n}{2} [2a + (n-1)d]$

$S_{20} = \frac{20}{2} [2(-8) + 19(7)]$
 $= 10(-16 + 133)$
 $= 1170$

11. $t_{15} = 43$

$$43 = a + 14d \quad \textcircled{1}$$

$$S_{15} = 120$$

$$120 = \frac{15}{2}(2a + 14d)$$

$$120 = 15a + 105d \quad \textcircled{2}$$

Solve the system of equations.

$$43 = a + 14d \quad \textcircled{1}$$

$$120 = 15a + 105d \quad \textcircled{2}$$

$$15 \times \textcircled{1} \quad 645 = 15a + 210d \quad \textcircled{3}$$

$$\underline{120 = 15a + 105d} \quad \textcircled{4}$$

$$525 = 105d$$

$$\textcircled{3} - \textcircled{4} \quad d = 5$$

Substitute $d = 5$ into $\textcircled{1}$.

$$43 = a + 14(5)$$

$$43 = a + 70$$

$$43 - 70 = a$$

$$a = -27$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2(-27) + 19(5)]$$

$$= 10(-54 + 95)$$

$$= 410$$

12. $t_{12} = 15$ and $S_{15} = 105$

$$15 = a + 11d \quad \textcircled{1}$$

$$105 = \frac{15}{2}(2a + 14d)$$

$$105 = 15a + 105d \quad \textcircled{2}$$

Solve the system of two equations.

$$15 = a + 11d \quad \textcircled{1}$$

$$105 = 15a + 105d \quad \textcircled{2}$$

$$15 \times \textcircled{1} \quad 225 = 15a + 165d \quad \textcircled{3}$$

$$\underline{105 = 15a + 105d} \quad \textcircled{4}$$

$$120 = 60d$$

$$\textcircled{3} - \textcircled{4} \quad d = 2$$

Substitute $d = 2$ into $\textcircled{1}$.

$$15 = a + 11(2)$$

$$15 - 22 = a$$

$$a = -7$$

The first term is -7 .

The second term is $-7 + 2$ or -5 .

The third term is $-5 + 2$ or -3 .

Sum of first three terms: $(-7) + (-5) + (-3) = -15$

13. The total number of seats can be found using an arithmetic series with $a = 59$, $d = -2$, and $n = 25$.

$$S_{25} = \frac{25}{2}[2(59) + 24(-2)]$$

$$= \frac{25}{2}(118 - 48)$$

$$= \frac{25}{2}(70)$$

$$= 875$$

There are 875 seats in the auditorium.

14. $a = 1$, $d = 2$, and $S_n = 400$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$400 = \frac{n}{2}[2(1) + (n-1)2]$$

$$400 = \frac{n}{2}(2 + 2n - 2)$$

$$400 = \frac{n}{2}(2n)$$

$$400 = n^2$$

$$n = \sqrt{400}$$

$$n = 20$$

The value of n is 20.

15. $a = 5$, $d = 4$, and $S_n = 945$

$$945 = \frac{n}{2}[2(5) + (n-1)(4)]$$

$$945 = \frac{n}{2}[10 + 4n - 4]$$

$$945 = \frac{n}{2}(4n + 6)$$

$$945 = 2n^2 + 3n$$

$$2n^2 + 3n - 945 = 0$$

$$(2n + 45)(n - 21) = 0$$

$$n = -22.5, \quad n = 21$$

Since n cannot be negative, the series has 21 terms.

16. We use $a = 13$, $d = 2$, and $n = 7$ to find the total number of seats in the first 7 rows.

$$S_7 = \frac{7}{2}[2(13) + 6(2)]$$

$$= \frac{7}{2}(26 + 12)$$

$$= \frac{7}{2}(38)$$

$$= 133$$

The number of seats in the 7th row is given by

$$t_7 = 13 + 6(2)$$

$$= 25$$

The eighth row will have $25 + 3$ or 28 seats. We use $a = 28$, $d = 3$, and $n = 5$ to find the total number of seats in the 8th to 12th rows.

$$S_5 = \frac{5}{2}[2(28) + 4(3)]$$

$$= \frac{5}{2}(56 + 12)$$

$$= \frac{5}{2}(68)$$

$$= 170$$

The lecture hall has a total of 303 ($133 + 170$) seats.

17. We first find how far she fell over the first 8 seconds. The distances she falls each subsequent second forms an arithmetic series with $a = 4.9$ and $d = 9.8$. We find S_8 .

$$S_8 = \frac{8}{2}[2(4.9) + 7(9.8)]$$

$$= 4(9.8 + 68.6)$$

$$= 313.6$$

We now find S_{13} , the total distance fallen over the first 13 seconds.

$$S_{13} = \frac{13}{2}[2(4.9) + 12(9.8)]$$

$$= \frac{13}{2}(127.4)$$

$$= 828.1$$

Between the 8th and 13th second she fell $(828.1 - 313.6)$ 514.5 m.

18. It is better to use $S_n = \frac{n(t_1 + t_n)}{2}$ when n and the first and last term of the series are known. In both cases, the values of a (the first term) and n must be known.

19. The depreciation of the car once driven off the lot is 20% of \$17 000. We find S_6 with $a = -450$ to determine the depreciation of the car over 6 years.

$$S_6 = \frac{6}{2}[2(-450) + 5(-450)]$$

$$= 3[7(-450)]$$

$$= -9450$$

The car has depreciated $(3400 + 9450)$ or \$12 850 over 6 years.

20. (a) The number of seats in the arena can be represented by an arithmetic series with $a = 220$, $d = 16$, and $S_n = 10\,920$.

$$10\,920 = \frac{n}{2}[2(220) + (n-1)(16)]$$

$$10\,920 = \frac{n}{2}(440 + 16n - 16)$$

$$10\,920 = \frac{n}{2}(424 + 16n)$$

$$10\,920 = 212n + 8n^2$$

$$8n^2 + 212n - 10\,920 = 0$$

$$2n^2 + 53n - 2730 = 0$$

$$(2n + 105)(n - 26) = 0$$

$$n = -52.5 \text{ and } n = 26$$

Since n must be positive, the arena has 26 rows of seats.

(b) t_{27} represents the number of seats in the first new row.

$$t_{27} = 220 + 26(16) = 220 + 416 = 636$$

$$t_{28} = 636 + 16 = 652$$

$$t_{29} = 652 + 16 = 668$$

$$t_{30} = 668 + 16 = 684$$

The 4 new rows add a total of 2640 $(636 + 652 + 668 + 684)$ new seats. The new capacity will be 13 560 $(10\,920 + 2640)$ seats.

21. Five new squares must be added to the 2×2 square to produce a 3×3 square. Let t_n represent the number of squares added to the previous square in order to obtain an $n \times n$ square. We have $t_1 = 1$, $t_2 = 3$, and $t_3 = 5$. For a 4×4 square we must add 7 squares to the 3×3 square, therefore $t_4 = 7$. To determine the total number of squares in an $n \times n$ square, we find S_n with $a = 1$ and $d = 2$.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2(1) + (n-1)(2)]$$

$$= \frac{n}{2}(2 + 2n - 2)$$

$$= \frac{n}{2}(2n)$$

$$= n^2$$

$$22. S_n = a + (a + d) + (a + 2d) + (a + 3d) + \dots + a + (n-2)d + a + (n-1)d$$

$$S_n = a + (n-1)d + a + (n-2)d + a + (n-3)d + \dots + (a + 3d) + (a + 2d) + (a + d) + a$$

Add the above 2 equations.

$$2S_n = 2a + (n-1)d + 2a + (n-1)d + 2a + (n-1)d + \dots + 2a + (n-1)d + 2a + (n-1)d$$

The above has n terms.

$$2S_n = n \times [2a + (n-1)d]$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

23. First, we find the sum of $1 + 5 + 9 + 13 + \dots + 201$.

We find n using $a = 1$, $d = 4$, and $t_n = 201$.

$$201 = 1 + (n-1)(4)$$

$$201 = 1 + 4n - 4$$

$$204 = 4n$$

$$n = 51$$

$$S_{51} = \frac{51}{2}[2(1) + 50(4)]$$

$$= \frac{51}{2}(202)$$

$$= 5151$$

We now find the sum of $3 + 7 + 11 + 15 + \dots + 203$. We can find n using $a = 3$, $d = 4$, and $t_n = 203$ or we can observe that this sum will have an equal number of terms as found above since each term is 2 greater than the terms in the first sum.

$$S_{51} = \frac{51}{2}[2(3) + 50(4)]$$

$$= \frac{51}{2}(206)$$

$$= 5253$$

The value of $(1 + 5 + 9 + \dots + 201) - (3 + 7 + 11 + \dots + 203)$ is therefore $5151 - 5253 = -102$.

An alternative solution is to use the observation that both series have 51 terms with each term in the first series being two less than the terms in the second series. Therefore, the difference between the two series is $51 \times (-2) = -102$.

24. $300 - 299 + 298 - 297 + 296 - \dots + 100 - 99$ can be rewritten as:
 $(300 + 298 + 296 + \dots + 100) - (299 + 297 + 295 + \dots + 99)$.
 We find the number of terms in either series as they both have the same number of terms. We solve for n with $a = 300$,
 $d = -2$, and $t_n = 100$.
 $t_n = a + (n - 1)d$
 $100 = 300 + (n - 1)(-2)$
 $100 = 300 - 2n + 2$
 $100 - 302 = -2n$
 $-202 = -2n$
 $n = 101$
 Since every term in the first series is one greater than each term in the second series, the total sum is $101 \times 1 = 101$.

Practise, Apply, Solve 2.3, page 123

1. (a) i. geometric, **ii.** $r = \frac{1}{2}$

(b) i. geometric, **ii.** $r = \frac{4}{5}$

(c) i. neither

(d) i. geometric, **ii.** $r = -\frac{3}{2}$

(e) i. geometric, **ii.** $r = -\frac{1}{2}$

(f) i. neither

2. $t_n = ar^{n-1}$, $S_n = \frac{a(r^n - 1)}{r - 1}$

(a) i. $t_n = 2(3)^{n-1}$

ii.

$$S_n = \frac{2(3^n - 1)}{3 - 1}$$

$$= 2(3^n - 1)$$

iii. $S_8 = 2(3^8 - 1) = 2(6560) = 13\,120$

(b) i. $t_n = 1\left(-\frac{2}{3}\right)^{n-1} = \left(-\frac{2}{3}\right)^{n-1}$

ii.

$$S_n = \frac{1\left[\left(-\frac{2}{3}\right)^n - 1\right]}{-\frac{2}{3} - 1}$$

$$= \frac{\left(-\frac{2}{3}\right)^n - 1}{-\frac{5}{3}}$$

$$= -\frac{3}{5}\left[\left(-\frac{2}{3}\right)^n - 1\right]$$

iii.

$$S_8 = -\frac{3}{5}\left[\left(-\frac{2}{3}\right)^8 - 1\right]$$

$$= -\frac{3}{5}\left(\frac{256}{6561} - \frac{6561}{6561}\right)$$

$$= -\frac{3}{5}\left(-\frac{6305}{6561}\right)$$

$$= \frac{18\,915}{32\,805}$$

$$= \frac{1261}{2187}$$

$$\doteq 0.58$$

(c) i. $t_n = 6(-2)^{n-1}$

ii.

$$S_n = \frac{6[(-2)^n - 1]}{-2 - 1}$$

$$= -2[(-2)^n - 1]$$

iii. $S_8 = -2[(-2)^8 - 1] = -2(256 - 1) = -510$

(d) i. $t_n = 81\left(\frac{1}{3}\right)^{n-1}$

ii.

$$S_n = \frac{81\left[\left(\frac{1}{3}\right)^n - 1\right]}{\frac{1}{3} - 1}$$

$$= 81\left[\left(\frac{1}{3}\right)^n - 1\right] \times \left(-\frac{3}{2}\right)$$

$$= -\frac{243}{2}\left[\left(\frac{1}{3}\right)^n - 1\right]$$

iii.

$$S_8 = -\frac{243}{2}\left[\left(\frac{1}{3}\right)^8 - 1\right]$$

$$= -\frac{243}{2}\left(\frac{1}{6561} - \frac{6561}{6561}\right)$$

$$= -\frac{243}{2}\left(-\frac{6560}{6561}\right)$$

$$= \frac{3280}{27}$$

$$= 121.48$$

(e) i. $t_n = 0.4\left(\frac{1}{10}\right)^{n-1}$