

- (b) After n years,
 $t_n = \$1000 + \$1000 \times 0.045 + \dots + \$1000 \times 0.045$
 $= \$1000 + n \times \$1000 \times 0.045$
 $= \$1000 + \$45n$
 (c) $t_{10} = \$1000 + \$45(10) = \$1000 + \$450 = \$1450$

15. (a) $\{1, 8, 27, \dots\}$
 (b) 64, 125, 216
 (c) $t_n = n^3$
 (d) $t_{15} = 15^3 = 3375$

16. (a) $\{10, 5, 2.5, 1.25, 0.625, \dots\}$
 (b) $t_n = 20(0.5)^n$
 (c) $t_{10} = 20(0.5)^{10} = 0.0195 \text{ m}$
 (d) The ball will stop bouncing eventually, due to friction and so on. So, the sequence is finite. t_n is defined for all $n \in \mathbb{N}$ and the terms approach zero but will never reach it.

17. (a) $\{\$0.25, \$0.50, \$0.75, \$1.00, \$1.25, \dots\}$
 (b) $t_n = \$0.25n$
 (c) $t_{52} = \$0.25(52) = \$13.00$

18. (a) $t_7 = (2)^7 = 98$
 (b) $t_n = 2n^2$

19. (a)

Time (hours)	Bacteria
1	20
2	40
3	80
4	160
5	320
6	640
7	1280
8	2560

- (b) There are 2560 bacteria after 8 hours.
 (c) $b_n = 10(2^n)$ OR $b_n = 20(2^{n-1})$ OR $b_n = 5(2^{n+1})$
 (d) $b_{10} = 10(2^{10}) = 10(1024) = 10\,240$

20. The sequence given is $\{6, 12, 18, 24, \dots, 6n\}$. The numbers among 216, 575, 492, 750, 824, that are divisible by 6 are 216, 492, and 750.

21. (a) $\{6, 16, 30, \dots\}$
 (b) 48, 70, 96
 (c) There are n^2 surface squares with both top and bottom, and there are n surface squares $\times 4$ sides.
 $t_n = 4n + 2n^2 = 2n(n + 2)$
 (d) $t_{20} = 2(20)(20 + 2) = 40(22) = 880$

22. (a) $\{2, 8, 18, \dots\}$
 (b) 32, 50, 72
 (c) The length of each shape contains $2n$ cubes, the width contains n cubes, and the height of each shape is 1 unit. Thus, $t_n = 2n^2$.
 (d) $t_{20} = 2(20)^2 = 2(400) = 800$

23. (a) 24, 40
 (b) The number of toothpicks is $2h(h + 1)$, where h is the length of one side.
 (c) In stage 10, the number of toothpicks is $2(10)(11) = 220$.

Practice 1.2, page 26

- (a) 6, 8, 10, 12, 14, 16, 18, 20, 22, 24
 (b) 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000
 (c) $3, 2, \frac{5}{3}, \frac{3}{2}, \frac{7}{5}, \frac{4}{3}, \frac{9}{7}, \frac{5}{4}, \frac{11}{9}, \frac{6}{5}$
 (d) 1, 1.414 213 562, 1.732 050 808, 2, 2.236 067 977, 2.449 489 743, 2.645 751 311, 2.828 427 125, 3, 3.162 277 660
 (e) $-7, -12, -17, -22, -27, -32, -37, -42, -47, -52$
 (f) 0.5, 0.25, 0.125, 0.0625, 0.031 25, 0.015 625, 0.007 812 5, 0.003 906 25, 0.001 953 125, 0.000 976 562 5
 (g) 6, 18, 54, 162, 486, 1458, 4374, 13 122, 39 366, 118 098
 (h) 1, 4, 27, 256, 3125, 46 656, 823 543, 16 777 216, 387 420 489, 10 000 000 000

Practise, Apply, Solve 1.3U, page 32

1. (a) $\{3, 12, 21, 30, 39, \dots\}$
 (b) $\{6, 24, 96, 384, 1536, \dots\}$
 (c) $\{36, 18, 9, 4.5, 2.25, \dots\}$
 (d) $\{22, 20, 18, 16, 14, \dots\}$
 (e) $\{3, 4, 12, 48, 576, \dots\}$
 (f) $\{-5, 7, 2, 9, 11, \dots\}$
2. (a) $t_1 = 4, t_2 = (4) + 5 = 9, t_3 = (9) + 5 = 14, t_4 = (14) + 5 = 19, t_5 = (19) + 5 = 24$
 (b) $t_1 = -3, t_2 = 2(-3) - 2 = -8, t_3 = 2(-8) - 2 = -18, t_4 = 2(-18) - 2 = -38, t_5 = 2(-38) - 2 = -78$
 (c) $t_1 = 8, t_2 = 8 \div 2 = 4, t_3 = 4 \div 2 = 2, t_4 = 2 \div 2 = 1, t_5 = 1 \div 2 = 0.5$
 (d) $t_1 = 4, t_2 = (4)^2 = 16, t_3 = (16)^2 = 256, t_4 = (256)^2 = 65\,536, t_5 = (65\,536)^2 = 4\,294\,967\,296$
 (e) $t_1 = 5, t_2 = \frac{5}{2}, t_3 = \frac{5}{6}, t_4 = \frac{5}{24}, t_5 = \frac{5}{120} = \frac{1}{24}$
 (f) $t_1 = 1, t_2 = 2, t_3 = (2)(1) = 2, t_4 = (2)(2) = 4, t_5 = (4)(2) = 8$
 (g) $t_1 = 2, t_2 = -1, t_3 = -1 + 2 = 1, t_4 = 1 + (-1) = 0, t_5 = 0 + 1 = 1$
 (h) $t_1 = 1, t_2 = 1, t_3 = 2(1) - 3(1) = -1, t_4 = 2(-1) - 3(1) = -5, t_5 = 2(-5) - 3(-1) = -7$
3. (a) $t_1 = 2, t_{n+1} = t_n + 5$
 (b) $t_1 = -2, t_{n+1} = -2t_n$
 (c) $t_1 = 10, t_{n+1} = \frac{1}{2}t_n$
 (d) $t_1 = 2, t_{n+1} = (t_n)^2$
 (e) $t_1 = 243, t_{n+1} = \frac{1}{3}t_n$
 (f) $t_1 = 34, t_{n+1} = t_n - 9$
4. (a) $t_1 = 3, t_n = t_{n-1} + 9$
 (b) $t_1 = 6, t_n = 4(t_{n-1})$

(c) $t_1 = 36, t_n = \frac{1}{2}t_{n-1}$

(d) $t_1 = 22, t_n = t_{n-1} - 2$

(e) $t_1 = 3, t_2 = 4, t_n = (t_{n-1})(t_{n-2})$

(f) $t_1 = -5, t_2 = 7, t_n = (t_{n-1}) + (t_{n-2})$

5. (a) 85%

(b) \$29 750, \$25 287.50, \$21 494.38, \$18 270.22, \$15 529.69

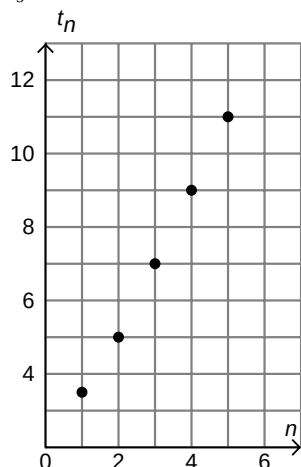
(c) $t_1 = \$35\ 000, t_n = 0.85(t_{n-1})$

6. (a) $t_1 = 4200, t_n = 0.8(t_{n-1}) + 850$

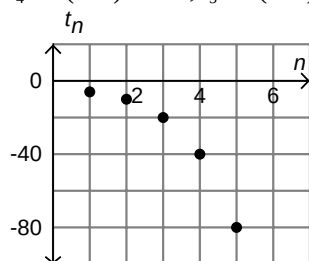
(b) Over time, the total number of trees will level off to 4250.

7. $t_1 = 6, t_2 = 4, t_3 = 2, t_4 = (6 + 4)(2) = 20,$
 $t_5 = (4 + 2)(20) = 120, t_6 = (2 + 20)(120) = 2\ 640,$
 $t_7 = (20 + 120)(2\ 640) = 369\ 600$

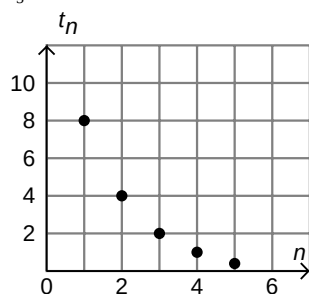
8. (a) $t_1 = 3, t_2 = 3 + 2 = 5, t_3 = 5 + 2 = 7, t_4 = 7 + 2 = 9,$
 $t_5 = 9 + 2 = 11$



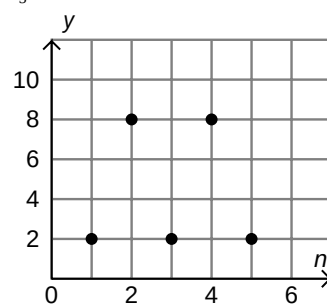
(b) $t_1 = -5, t_2 = 2(-5) = -10, t_3 = 2(-10) = -20,$
 $t_4 = 2(-20) = -40, t_5 = 2(-40) = -80$



(c) $t_1 = 8, t_2 = 8 \div 2 = 4, t_3 = 4 \div 2 = 2, t_4 = 2 \div 2 = 1,$
 $t_5 = 1 \div 2 = 0.5$



(d) $t_1 = 2, t_2 = 10 - 2 = 8, t_3 = 10 - 8 = 2, t_4 = 10 - 2 = 8,$
 $t_5 = 10 - 8 = 2$



9. (a) $t_1 = \$40\ 000, t_n = t_{n-1} + \$1250$

(b) \$40 000, \$41 250, \$42 500, \$43 750, \$45 000

(c) In ten years, $t_{11} = \$40\ 000 + \$1250(9) = \$51\ 250$

No, using the recursive formula is not convenient because all previous terms have to be calculated.

10. (a)

Stage	Number of Segments	Length of each Segment (cm)	Perimeter (cm)
1	3	18	54
2	12	6	72
3	48	2	96

(b) Number of Segments: $S_1 = 3, S_n = (3)4^{n-1} = (4)S_{n-1}$

Length of each segment: $L_1 = 18, L_n = (18)3^{1-n} = \frac{1}{3}L_{n-1}$

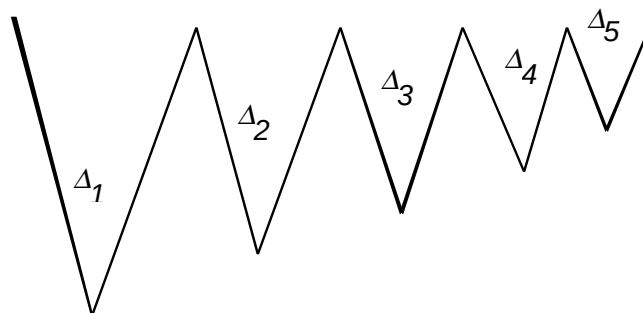
Perimeter: $P_1 = 3(18) = 54, P_n = 54\left(\frac{4}{3}\right)^{n-1} = \frac{4}{3}P_{n-1}$

Stage	Number of Segments	Length of each Segment (cm)	Perimeter (cm)
(c) 6	3072	0.0741	227.5556
(d) 7	12288	0.0247	303.4074
(e) 8	49152	0.0082	404.5432

11. (a) $t_1 = 50\text{ m}, t_n = 0.8t_{n-1}$

(b) 50 m, 40 m, 32 m, 25.6 m, 20.48 m

(c) Let l be the unstretched length of the chord.



Solution 1

Total distance = $2\Delta_1 + 2\Delta_2 + 2\Delta_3 + 2\Delta_4 + 2\Delta_5$
 $= 2\Delta_1 + 2(0.8)\Delta_1 + 2(0.8)^2\Delta_1 + 2(0.8)^3\Delta_1 + 2(0.8)^4\Delta_1$
 $= 2(50)(1 + 0.8 + 0.8^2 + 0.8^3 + 0.8^4)$
 $= 2(50)(3.3616)$
 $= 336.16\text{ m}$

Solution 2

$$\begin{aligned}\text{Total distance} &= 2\Delta_1 + 2\Delta_2 + 2\Delta_3 + 2\Delta_4 + 2\Delta_5 \\ &= 2(50) + 2(40) + 2(32) + 2(25.6) + 2(20.48) \\ &= 100 + 80 + 64 + 51.2 + 40.96 \\ &= 336.16 \text{ m}\end{aligned}$$

12. Answers will vary.

Example: $t_1 = 2$, $t_2 = 3$, $t_{n+2} = t_n(t_{n+1} - 2)$,
 $\{2, 3, 2, 0, -4, 0, 8, 0, -16, 0, \dots\}$.

13. $t_1 = 250$, $t_n = 0.3t_{n-1}$

If the amount levels off, then $t_n = t_{n-1}$ for large values of n .

$$t_n = 0.3t_{n-1} + 250$$

$$t_n = 0.3t_n + 250$$

$$t_n = \frac{250}{1 - 0.3}$$

$$t_n = 357.14 \text{ mg}$$

$\therefore$ the medicine remaining in Nadia's body will become consistent at approximately 357.14 mg.

14. (a) 2, 6, 12, 20, 30, 42

(b) $t_n = t_{n-1} + 2n$, $t_1 = 2$

15. $t_4 = 281$, $t_n = 3(t_{n-1}) + 5$

$$t_{n-1} = \frac{t_n - 5}{3}, t_3 = \frac{281 - 5}{3} = 92, t_2 = \frac{92 - 5}{3} = 29,$$

$$t_1 = \frac{29 - 5}{3} = 8$$

16. (a) $a_n = 1.05(a_{n-1}) - 50\,000$

(b) $a_0 = \$604\,266.05$, $a_1 = \$584\,479.35$, $a_2 = \$563\,703.32$,
 $a_3 = \$541\,888.49$, $a_4 = \$518\,982.91$, $a_5 = \$494\,932.06$,
 $a_6 = \$469\,678.66$, $a_7 = \$443\,162.60$, $a_8 = \$415\,320.73$,
 $a_9 = \$386\,086.77$, $a_{10} = \$355\,391.11$, $a_{11} = \$323\,160.67$,
 $a_{12} = \$289\,318.70$, $a_{13} = \$253\,784.64$, $a_{14} = \$216\,473.87$,
 $a_{15} = \$177\,297.56$, $a_{16} = \$136\,162.44$, $a_{17} = \$92\,970.56$,
 $a_{18} = \$47\,619.07$, $a_{19} = \$0.02$

(c) A prize of \$ 1 000 000 can be awarded even though only \$604 266.05 was invested.

17. (a) $t_n = 2t_{n-1}$, $t_1 = 3500$

(b) $t_5 = 3500(2)^5 = 112\,000$

18. (a) $t_n = 2t_{n-1}$, $t_1 = 2$

(b) The total number of ancestors is

$$2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 = 510.$$

19. $t_1 = 2$, $t_2 = 4$, $t_3 = 6$, $t_n = \frac{(t_{n-1})(t_{n-2})}{t_{n-3}}$

$$t_4 = \frac{(6)(4)}{2} = 12, t_5 = \frac{(12)(6)}{4} = 18, t_6 = \frac{(18)(12)}{6} = 36$$

20. $t_1 = 4$, $t_n = 0.48t_{n-1} = 4(0.48)^{n-1}$

$$t_1 = 4, t_2 = 1.92, t_3 = 0.9216, t_4 = 0.4424, t_5 = 0.2123,$$

$$t_6 = 0.1019, t_7 = 0.04892, t_8 = 0.02348, t_9 = 0.01127$$

$\therefore$ it takes 8 hours for the level to be less than 0.02 mg.

21. It would be much easier to use a general formula because you would only have to use the formula once. With a recursive formula, you would have to use it 88 times to find the 88th term of a sequence.

22. $\{5, 9, 18, 34, 59, \dots\}$, $t_n = t_{n-1} + n^2$, $t_1 = 5$

23. (a) $t_n = t_{n-1} + 2(n-1)$

(b)

n	t_n
1	20
2	22
3	26
4	32
5	40
6	50
7	62
8	76
9	92
10	110

$\therefore$ there are 110 seats in the last row.

(c) Number of seats $= t_1 + t_2 + t_3 + \dots + t_{10}$

$$= 20 + 22 + 26 + 32 + 40 + 50 + 62 + 76 + 92 + 110$$

$$= 530$$

24. Examine the pattern in the number of diagonals in the shapes shown and form a recursive formula. For the triangle, there are no diagonals to be drawn. Thus, $t_1 = 0$. The square has two diagonals; $t_2 = 2$. The pentagon (5 sides) has 5 diagonals; $t_3 = 5$. The hexagon (6 sides) has 9 diagonals; $t_4 = 9$. The recursive formula is $t_n = t_{n-1} + n$.

$$t_5 = (9) + 5 = 14 \text{ (heptagon - 7 sides)}$$

$$t_6 = (14) + 6 = 20 \text{ (octagon - 8 sides)}$$

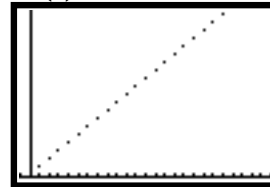
$$t_7 = (20) + 7 = 27 \text{ (nonagon - 9 sides)}$$

$$t_8 = (27) + 8 = 35 \text{ (decagon - 10 sides)}$$

$\therefore$ a decagon has 35 diagonals.

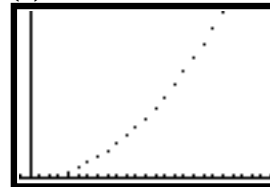
Practice 1.4, page 38

1. (a) i.



ii. $t_{10} = 63$, $t_{15} = 93$, $t_{20} = 123$

(b) i.



ii. $t_{10} = 100$, $t_{15} = 225$, $t_{20} = 400$