

Solution 2

$$\begin{aligned}\text{Total distance} &= 2\Delta_1 + 2\Delta_2 + 2\Delta_3 + 2\Delta_4 + 2\Delta_5 \\ &= 2(50) + 2(40) + 2(32) + 2(25.6) + 2(20.48) \\ &= 100 + 80 + 64 + 51.2 + 40.96 \\ &= 336.16 \text{ m}\end{aligned}$$

12. Answers will vary.

Example: $t_1 = 2$, $t_2 = 3$, $t_{n+2} = t_n(t_{n+1} - 2)$,
 $\{2, 3, 2, 0, -4, 0, 8, 0, -16, 0, \dots\}$.

13. $t_1 = 250$, $t_n = 0.3t_{n-1}$

If the amount levels off, then $t_n = t_{n-1}$ for large values of n .

$$t_n = 0.3t_{n-1} + 250$$

$$t_n = 0.3t_n + 250$$

$$t_n = \frac{250}{1 - 0.3}$$

$$t_n = 357.14 \text{ mg}$$

$\therefore$ the medicine remaining in Nadia's body will become consistent at approximately 357.14 mg.

14. (a) 2, 6, 12, 20, 30, 42

(b) $t_n = t_{n-1} + 2n$, $t_1 = 2$

15. $t_4 = 281$, $t_n = 3(t_{n-1}) + 5$

$$t_{n-1} = \frac{t_n - 5}{3}, t_3 = \frac{281 - 5}{3} = 92, t_2 = \frac{92 - 5}{3} = 29,$$

$$t_1 = \frac{29 - 5}{3} = 8$$

16. (a) $a_n = 1.05(a_{n-1}) - 50\,000$

(b) $a_0 = \$604\,266.05$, $a_1 = \$584\,479.35$, $a_2 = \$563\,703.32$,
 $a_3 = \$541\,888.49$, $a_4 = \$518\,982.91$, $a_5 = \$494\,932.06$,
 $a_6 = \$469\,678.66$, $a_7 = \$443\,162.60$, $a_8 = \$415\,320.73$,
 $a_9 = \$386\,086.77$, $a_{10} = \$355\,391.11$, $a_{11} = \$323\,160.67$,
 $a_{12} = \$289\,318.70$, $a_{13} = \$253\,784.64$, $a_{14} = \$216\,473.87$,
 $a_{15} = \$177\,297.56$, $a_{16} = \$136\,162.44$, $a_{17} = \$92\,970.56$,
 $a_{18} = \$47\,619.07$, $a_{19} = \$0.02$

(c) A prize of \$ 1 000 000 can be awarded even though only \$604 266.05 was invested.

17. (a) $t_n = 2t_{n-1}$, $t_1 = 3500$

(b) $t_5 = 3500(2)^5 = 112\,000$

18. (a) $t_n = 2t_{n-1}$, $t_1 = 2$

(b) The total number of ancestors is

$$2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 = 510.$$

19. $t_1 = 2$, $t_2 = 4$, $t_3 = 6$, $t_n = \frac{(t_{n-1})(t_{n-2})}{t_{n-3}}$

$$t_4 = \frac{(6)(4)}{2} = 12, t_5 = \frac{(12)(6)}{4} = 18, t_6 = \frac{(18)(12)}{6} = 36$$

20. $t_1 = 4$, $t_n = 0.48t_{n-1} = 4(0.48)^{n-1}$

$$t_1 = 4, t_2 = 1.92, t_3 = 0.9216, t_4 = 0.4424, t_5 = 0.2123,$$

$$t_6 = 0.1019, t_7 = 0.04892, t_8 = 0.02348, t_9 = 0.01127$$

$\therefore$ it takes 8 hours for the level to be less than 0.02 mg.

21. It would be much easier to use a general formula because you would only have to use the formula once. With a recursive formula, you would have to use it 88 times to find the 88th term of a sequence.

22. $\{5, 9, 18, 34, 59, \dots\}$, $t_n = t_{n-1} + n^2$, $t_1 = 5$

23. (a) $t_n = t_{n-1} + 2(n-1)$

(b)

n	t_n
1	20
2	22
3	26
4	32
5	40
6	50
7	62
8	76
9	92
10	110

$\therefore$ there are 110 seats in the last row.

(c) Number of seats $= t_1 + t_2 + t_3 + \dots + t_{10}$

$$= 20 + 22 + 26 + 32 + 40 + 50 + 62 + 76 + 92 + 110$$

$$= 530$$

24. Examine the pattern in the number of diagonals in the shapes shown and form a recursive formula. For the triangle, there are no diagonals to be drawn. Thus, $t_1 = 0$. The square has two diagonals; $t_2 = 2$. The pentagon (5 sides) has 5 diagonals; $t_3 = 5$. The hexagon (6 sides) has 9 diagonals; $t_4 = 9$. The recursive formula is $t_n = t_{n-1} + n$.

$$t_5 = (9) + 5 = 14 \text{ (heptagon - 7 sides)}$$

$$t_6 = (14) + 6 = 20 \text{ (octagon - 8 sides)}$$

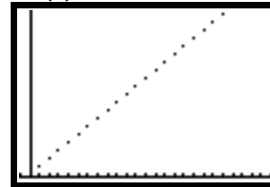
$$t_7 = (20) + 7 = 27 \text{ (nonagon - 9 sides)}$$

$$t_8 = (27) + 8 = 35 \text{ (decagon - 10 sides)}$$

$\therefore$ a decagon has 35 diagonals.

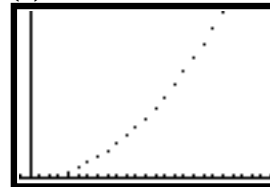
Practice 1.4, page 38

1. (a) i.



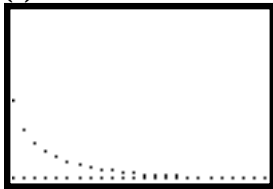
ii. $t_{10} = 63$, $t_{15} = 93$, $t_{20} = 123$

(b) i.



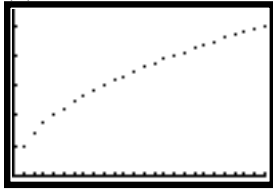
ii. $t_{10} = 100$, $t_{15} = 225$, $t_{20} = 400$

(c) i.



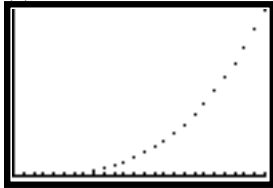
ii. $t_{10} = 0.1$, $t_{15} = 0.067$, $t_{20} = 0.05$

(d) i.



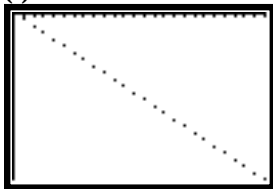
ii. $t_{10} = 3.162$, $t_{15} = 3.872$, $t_{20} = 4.472$

(e) i.



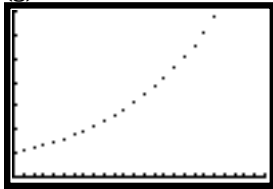
ii. $t_{10} = 1990$, $t_{15} = 6735$, $t_{20} = 15\,980$

(f) i.



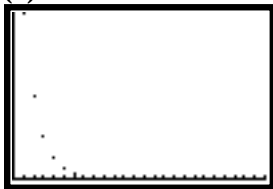
ii. $t_{10} = -10$, $t_{15} = -15$, $t_{20} = -20$

(g) i.



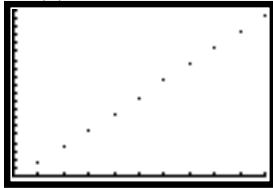
ii. $t_{10} = 25.937$, $t_{15} = 41.772$, $t_{20} = 67.274$

(h) i.



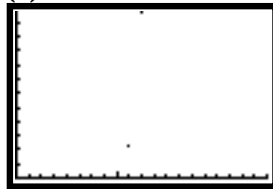
ii. $t_{10} = 0.009\,766$, $t_{15} = 0.000\,305\,2$, $t_{20} = 0.000\,009\,537$

2. (a) i.



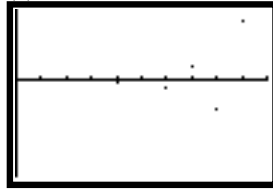
ii. $t_5 = 19$, $t_{10} = 39$

(b) i.



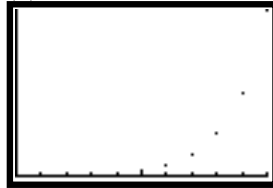
ii. $t_5 = 3750$, $t_{10} = 11\,718\,750$

(c) i.



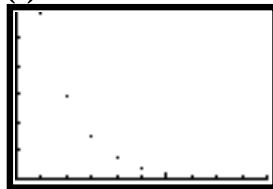
ii. $t_5 = 65$, $t_{10} = -2047$

(d) i.



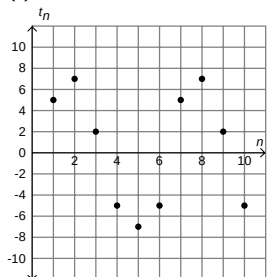
ii. $t_5 = 32$, $t_{10} = 1024$

(e) i.



ii. $t_5 = 6.25$, $t_{10} = 0.915\,312\,5$

(f) i.



ii. $t_5 = -7$, $t_{10} = -5$

Practise, Apply, Solve 1.6, page 47

1. (a) i. $d_1 = 11 - 4 = 7$, $d_2 = 15 - 11 = 4$, $d_1 \neq d_2$, not arithmetic

(b) i. arithmetic, ii. $d = 23 - 32 = -9$

(c) i. $d_1 = 6 - 5 = 1$, $d_2 = 5 - 6 = -1$, $d_1 \neq d_2$, not arithmetic

(d) i. $d_1 = 8 - 2 = 6$, $d_2 = 32 - 8 = 24$, $d_1 \neq d_2$, not arithmetic

(e) i. arithmetic, ii. $d = 33 - 22 = 11$

(f) i. arithmetic, ii. $d = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

2. (a) i. $t_n = 3n - 2 = 3n - 3 + 1 = 1 + (n - 1)(3)$, arithmetic, ii. $d = 3$

(b) i. $t_1 = 1$, $t_2 = 5$, $t_3 = 25$, $d_1 = 5 - 1 = 4$, $d_2 = 25 - 5 = 20$, $d_1 \neq d_2$, not arithmetic