

Practice 2.6, page 144

1. (a) $6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 + 51 = 285$
 (b) $1 + 8 + 27 + 64 + 125 + 216 + 343 + 512 + 729 + 1000 = 3025$
 (c) $3 + 6 + 12 + 24 + 48 + 96 + 192 + 384 + 768 + 1536 = 3069$
 (d) $-2 + (-7) + (-12) + (-22) + (-27) + (-32) + (-37) + (-42) + (-47) = -245$
 (e) $1100 + 1210 + 1331 + 1464.1 + 1610.51 + 1771.56 + 1948.72 + 2143.59 + 2357.95 + 2593.74 = 17\,531.17$
 (f) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$
 (g) $105 + 110.25 + 115.76 + 121.55 + 127.63 + 134.01 + 140.71 + 147.75 + 155.13 + 162.89 = 1320.68$
 (h) $25 + 29 + 33 + 37 + 41 + 45 + 49 + 53 + 57 + 61 = 430$
2. (a) 6, 17, 33, 54, 80, 111, 147, 188, 234, 285
 (b) 1, 9, 36, 100, 225, 441, 784, 1296, 2025, 3025
 (c) 3, 9, 21, 45, 93, 189, 381, 765, 1533, 3069
 (d) -2, -9, -21, -38, -60, -87, -119, -156, -198, -245
 (e) 1100, 2310, 3641, 5105.1, 6715.61, 8487.17, 10 435.89, 12 579.48, 14 937.43, 17 531.17
 (f) 1, 3, 6, 10, 15, 21, 28, 36, 45, 55
 (g) 105, 215.25, 331.01, 452.56, 580.19, 714.20, 854.91, 1002.66, 1157.79, 1320.68
 (h) 25, 54, 87, 124, 165, 210, 259, 312, 369, 430

Practise, Apply, Solve 2.7, page 151

1. (a) 2.158 925
 (b) 1.973 823
 (c) 2.032 794
 (d) 1.348 850
 (e) 4.247 851
 (f) 10.401 270
2. (a) 53 175.25
 (b) 11 433.50
 (c) 47 898.65
 (d) 1 247 260.00
3. (a) $i = \frac{0.12}{4} = 0.03 = 3\%, n = 3 \times 4 = 12$
 (b) $i = \frac{0.065}{1} = 0.065 = 6.5\%, n = 10 \times 1 = 10$
 (c) $i = \frac{0.18}{12} = 0.015 = 1.5\%, n = 7 \times 12 = 84$
 (d) $i = \frac{0.13}{52} = 0.0025 = 0.25\%, n = 5 \times 52 = 260$

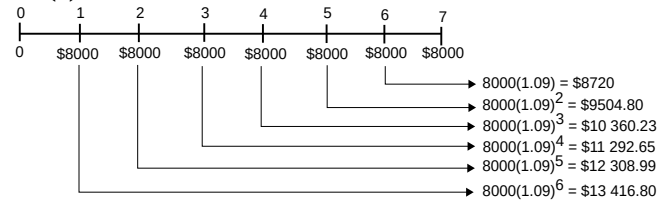
4. (a) $S_{13} = \frac{100(1.07^{13} - 1)}{1.07 - 1} = 2014.06$

(b) $S_{25} = \frac{3200(1.055^{25} - 1)}{1.055 - 1} = 163\,688.28$

(c) $S_{41} = \frac{300(1.02^{41} - 1)}{1.02 - 1} = 18\,783.01$

5. Same as question 4.

6. (a) i.



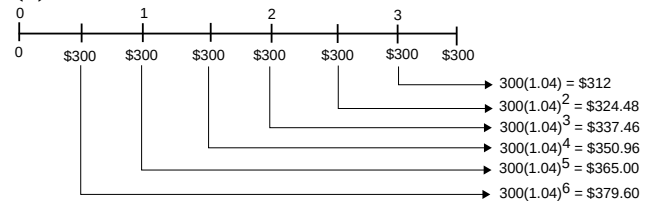
\$13 416.80, \$12 308.99, \$11 292.65, \$10 360.23, \$9504.80, \$8720, \$8000

ii. $FV = 8000 + 8000(1.09) + 8000(1.09)^2 + 8000(1.09)^3 + 8000(1.09)^4 + 8000(1.09)^5 + 8000(1.09)^6$

iii.

$$S_7 = \frac{a(r^7 - 1)}{r - 1} = \frac{8000(1.09^7 - 1)}{1.09 - 1} = \$73\,603.48$$

(b) i.



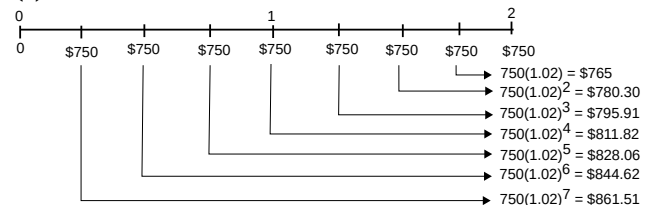
\$379.60, \$365, \$350.96, \$337.46, \$324.48, \$312, \$300

ii. $FV = 300 + 300(1.04) + 300(1.04)^2 + 300(1.04)^3 + 300(1.04)^4 + 300(1.04)^5 + 300(1.04)^6$

iii.

$$S_7 = \frac{a(r^7 - 1)}{r - 1} = \frac{300(1.04^7 - 1)}{1.04 - 1} = \$2369.49$$

(c) i.



\$861.51, \$844.62, \$828.06, \$811.82, \$795.91, \$780.30, \$765, \$750

ii. $FV = 750 + 750(1.02) + 750(1.02)^2 + 750(1.02)^3 + 750(1.02)^4 + 750(1.02)^5 + 750(1.02)^6 + 750(1.02)^7$

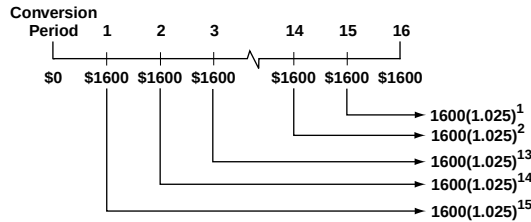
iii.

$$S_8 = \frac{a(r^8 - 1)}{r - 1}$$

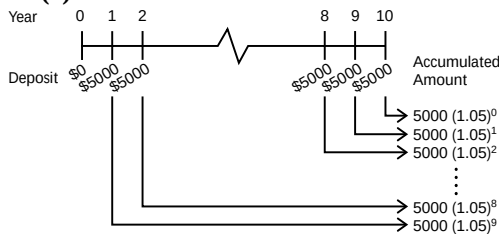
$$= \frac{750(1.02^8 - 1)}{1.02 - 1}$$

$$= \$6437.23$$

7.



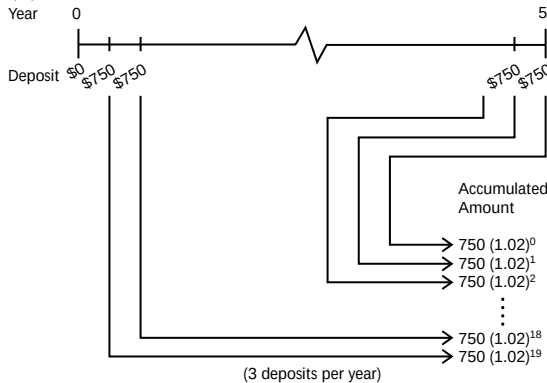
8. (a) i.



ii. $S_{10} = 5000 + 5000(1.05) + 5000(1.05)^2 + \dots + 5000(1.05)^9$,

iii. $S_{10} = \frac{5000(1.05^{10} - 1)}{1.05 - 1} = \$62\,889.46$

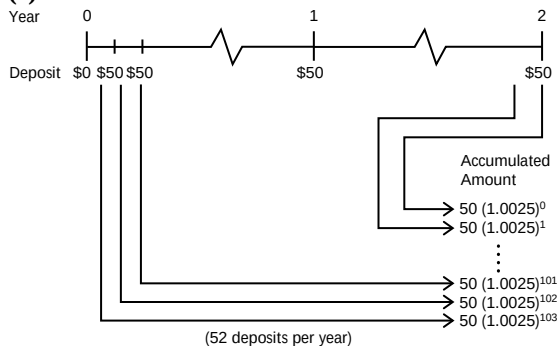
(b) i.



ii. $S = 750 + 750(1.02) + 750(1.02)^2 + \dots + 750(1.02)^{19}$

iii. $S_{20} = \frac{750(1.02^{20} - 1)}{1.02 - 1} = \$18\,223.03$

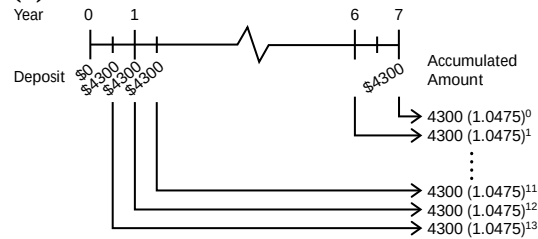
(c) i.



ii. $S_{104} = 50 + 50(1.0025) + 50(1.0025)^2 + \dots + 50(1.0025)^{103}$

iii. $S_{104} = \frac{50(1.0025^{104} - 1)}{1.0025 - 1} = \$5930.19$

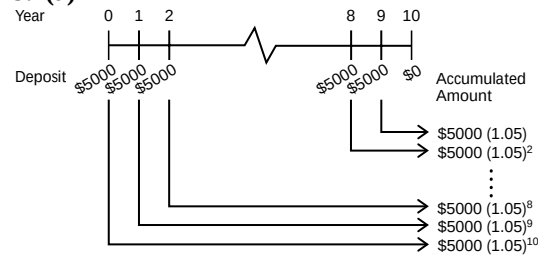
(d) i.



ii. $S_{14} = 4300 + 4300(1.0475) + 4300(1.0475)^2 + \dots + 4300(1.0475)^{13}$

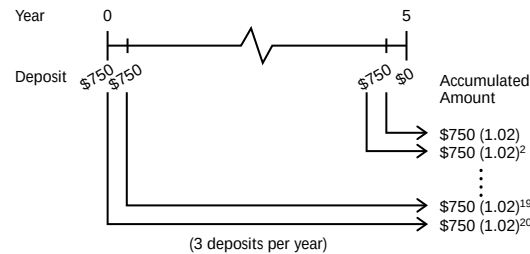
iii. $S_{14} = \frac{4300(1.0475^{14} - 1)}{1.0475 - 1} = \$82\,826.66$

9. (a)



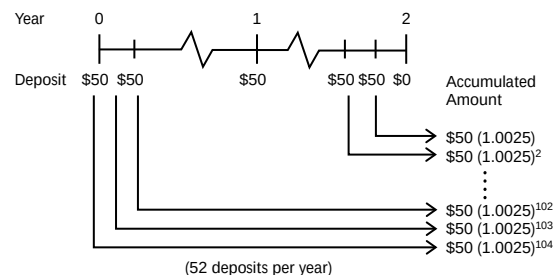
$S_{10} = 5000 + 5000(1.05) + 5000(1.05)^2 + \dots + 5000(1.05)^{10}$,

$S_{10} = \frac{5000(1.05)(1.05^{10} - 1)}{1.05 - 1} = \$66\,033.94$



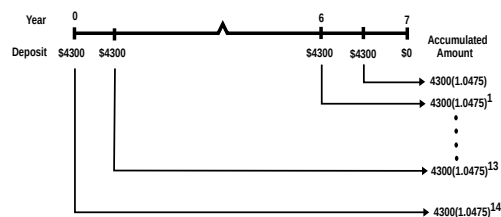
$S = 750 + 750(1.02) + 750(1.02)^2 + \dots + 750(1.02)^{20}$

$S_{20} = \frac{750(1.02)(1.02^{20} - 1)}{1.02 - 1} = \$18\,587.49$



$S_{104} = 50 + 50(1.0025) + 50(1.0025)^2 + \dots + 50(1.0025)^{104}$

$S_{104} = \frac{50(1.0025)(1.0025^{104} - 1)}{1.0025 - 1} = \$5945.01$



$$S_{14} = 4300 + 4300(1.0475) + 4300(1.0475)^2 + \dots + 4300(1.0475)^{14}$$

$$S_{14} = \frac{4300(1.0475)(1.0475^{14} - 1)}{1.0475 - 1} = \$86\,760.92$$

10. (a)



(b) $S_{36} = R + R(1.005) + R(1.005)^2 + \dots + R(1.005)^{35}$

(c)

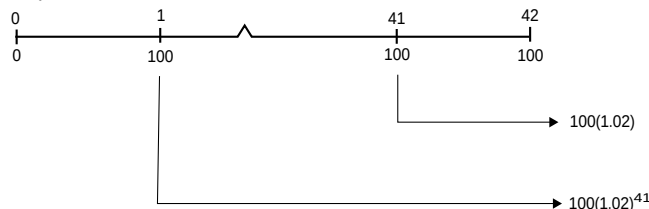
$$S_{36} = \frac{a(r^{36} - 1)}{r - 1}$$

$$30\,000 = \frac{R(1.005^{36} - 1)}{1.005 - 1}$$

$$30\,000(0.005) = R(1.005^{36} - 1)$$

$$R = \frac{30\,000(0.005)}{1.005^{36} - 1} = \frac{150}{0.196\,680\,524\,8} = \$762.66$$

11.



$$S_{42} = \frac{100(1.02^{42} - 1)}{1.02 - 1} = \$6486.22$$

On his 21st birthday, the account balance is \$6486.22. It collects interest for 4 years or 8 terms.

$$A = 6486.22(1.02)^8 = \$7599.64$$

12. We first find the balance after 25 years.

$$i = \frac{0.048}{12} = 0.004, n = 12 \times 25 = 300$$

$$R = 50$$

$$S_{300} = \frac{50(1.004^{300} - 1)}{1.004 - 1} = \$28\,902.24$$

We then find the amount after 5 more years or 60 months.

$$A = P(1 + i)^n$$

$$A = 28\,902.24(1.004)^{60}$$

$$A = \$36\,724.36$$

13. $R = 195, n = 8, i = \frac{0.09}{12} = 0.0075$

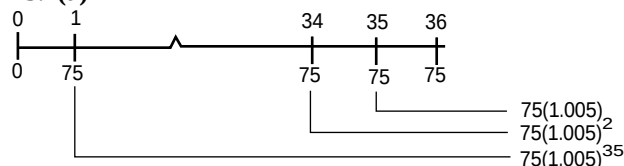
We find the balance at the end of 8 months.

$$S_8 = \frac{R((1 + i)^8 - 1)}{i} = \frac{195(1.0075^8 - 1)}{0.0075} = \$1601.57$$

Marcel will have \$1601.57 one month before his trip. Therefore, he will have enough money to go.

14. Answers will vary. *Example:* Both formulas are based on being able to express the future value of the payments as a geometric series. This can only be accomplished if the payment and interest interval coincide.

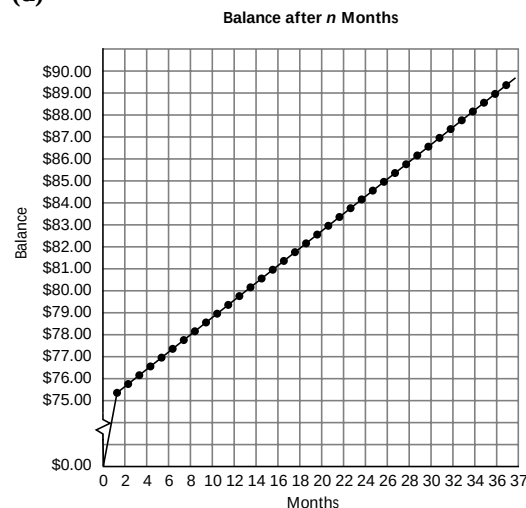
15. (a)



(b) $S_{36} = 75 + 75(1.005) + 75(1.005)^2 + 75(1.005)^3 + 75(1.005)^4 + \dots + 75(1.005)^{33} + 75(1.005)^{34} + 75(1.005)^{35}$

(c) $S_{36} = \frac{75(1.005^{36} - 1)}{1.005 - 1} = \$2950.21$

(d)



(e) We solve for n letting $S_n = 5200$.

$$5200 = \frac{75(1.005^n - 1)}{0.005}$$

$$26 = 75(1.005^n - 1)$$

$$0.34\bar{6} = 1.005^n - 1$$

$$1.34\bar{6} = 1.005^n$$

By trial and error, $n \approx 60$. It will take 60 payments to accumulate at least \$5200.

16. (a) $FV = \frac{R((1+i)^n - 1)}{i}$, $FV = 45\ 000$, $R = 2000$,

$$i = \frac{0.06}{4} = 0.015$$

$$45\ 000 = \frac{2000(1.015^n - 1)}{0.015}$$

$$675 = 2000(1.015^n - 1)$$

$$0.3375 = 1.015^n - 1$$

$$1.3375 = 1.015^n$$

$$\ln(1.3375) = \ln(1.015)^n$$

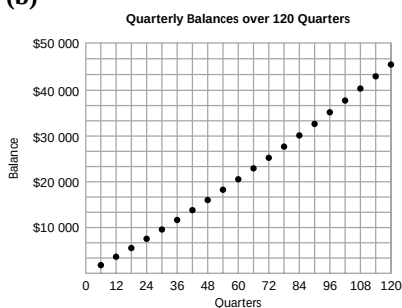
$$\ln(1.3375) = n \ln(1.015)$$

$$n = \frac{\ln(1.3375)}{\ln(1.015)}$$

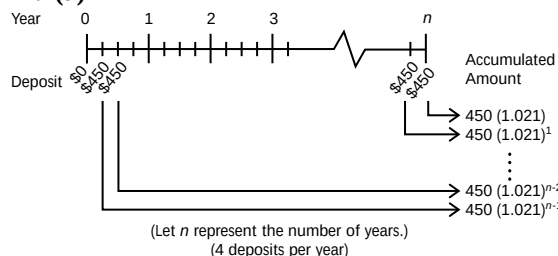
$$n = 19.53$$

It will take at least 20 payments to have a balance of at least \$45 000.

(b)



17. (a)



(b) $S_n = 24\ 000$

$$= 450 + 450(1.021) + 450(1.021)^2 + \dots + 450(1.021)^{n-1}$$

(c)

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$24\ 000 = \frac{450(1.021^n - 1)}{0.021}$$

$$504 = 450(1.021^n - 1)$$

$$1.12 = 1.021^n - 1$$

$$2.12 = 1.021^n$$

$$\ln(2.12) = \ln(1.021)^n$$

$$n = \frac{\ln(2.12)}{\ln(1.021)}$$

$$n = 36.156$$

It will take just over 36 quarters or just over 9 years for the fund to be worth \$24 000. Samantha will have just celebrated her 16th birthday.

(d) We replace $a = 450$ with $a = 530$ and solve for n .

$$24\ 000 = \frac{530(1.021^n - 1)}{0.021}$$

$$504 = 530(1.021^n - 1)$$

$$1.950\ 94 = 1.021^n$$

$$n \approx 32.157$$

Therefore, it will take 4 quarter or 1 year less.

18. We first find the balance after 4 years.

$$FV = \frac{R((1+i)^n - 1)}{i}$$

$$= \frac{25(1.008^{48} - 1)}{0.008}$$

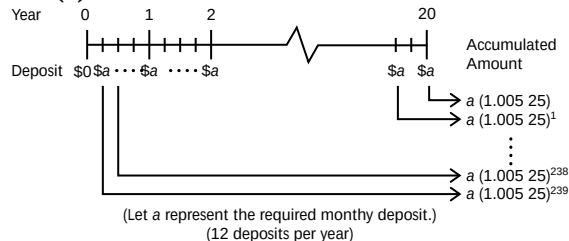
$$= \$1455.95$$

We now find the amount of \$1455.95 after accumulating interest for 6 years or 72 months.

$$A = P(1+i)^n = 1455.95(1.008)^{72} = \$2584.07$$

He will have \$2584.07 after 10 years.

19. (a)



$$(b) S_{240} = R + R(1.005\ 25) + R(1.005\ 25)^2 + R(1.005\ 25)^3 + \dots + R(1.005\ 25)^{238} + R(1.005\ 25)^{239}$$

(c)

$$S_{240} = \frac{R((1.00525)^{240} - 1)}{1.00525 - 1}$$

$$80\,000 = \frac{R(1.00525^{240} - 1)}{0.00525}$$

$$420 = R(1.00525^{240} - 1)$$

$$R = \$167.08$$

His deposits must be \$167.08.

(d) $n = 15 \times 12 = 180$

$$80\,000 = \frac{R(1.00525^{180} - 1)}{0.00525}$$

$$420 = R(1.00525^{180} - 1)$$

$$R = \$268.12$$

His new payment would be \$268.12.

(e) Total investment over 25 years = $240 \times \$167.08$
 $= \$40\,099.20$
 Total investment by waiting 5 years = $180 \times \$268.12$
 $= \$48\,261.60$

By waiting he had to invest an extra
 $\$48\,261.60 - \$40\,099.20 = \$8162.40$.

20. We find the amount of the initial \$4320 in 5 years and the future value of her deposits. Then, add the two sums.

$$A = 4320(1.015)^{20} = \$5818.41$$

$$FV = \frac{R((1+i)^n - 1)}{i}$$

$$= \frac{800(1.015^{20} - 1)}{0.015}$$

$$= \$18\,498.93$$

Her balance will be $\$5818.41 + \$18\,498.93$ or $\$24\,317.34$.

21. Answers will vary. However, in most cases, it is more advantageous to double the interest.

22. (a) By starting her deposits earlier, her deposits would be reduced substantially for two reasons. She would be making an extra 60 payments and these 60 payments would accumulate interest over an extra 5 years.

(b) We use

$$S_{300} = \frac{r^{300} - 1}{r - 1}$$

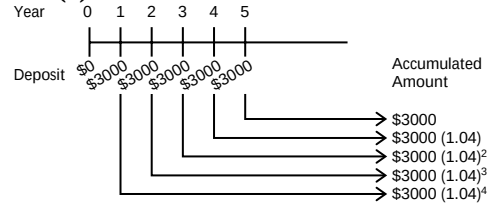
$$300\,000 = \frac{R(1.0075^{300} - 1)}{1.0075 - 1}$$

$$300\,000 = R(1121.121\,937)$$

$$R = 267.59$$

Her deposits for 300 months would be \$267.59/month for a total value of \$80 277. Her deposits over 240 months would be $\$449.18 \times 240$ or \$107 803.20. She would have to pay out \$27 526.20 less by starting 5 years earlier.
 $\$449.18 - \$267.59 = \$181.59$. She would have to pay \$181.59 less per payment.

23. (a)



(b) $S_5 = 3000 + 3000(1.04)^2 + 3000(1.04)^4 + 3000(1.04)^6 + 3000(1.04)^8$

Solve S_5 with $a = 3000$, $r = (1.04)^2$, and $n = 5$.

$$S_5 = \frac{a(r^5 - 1)}{r - 1}$$

$$= \frac{3000((1.04)^2)^5 - 1}{1.04^2 - 1}$$

$$= \frac{3000(1.04^{10} - 1)}{0.0816}$$

$$= \$17\,656.04$$

24. We first determine what is the equivalent future amount in 20 years to \$50 000 with $n = 20 \times 12 = 240$ and

$$i = \frac{0.06}{12} = 0.005.$$

$$A = 50\,000(1.005)^{240} = \$165\,510.22$$

We now find what monthly deposits will amount to \$165 510.22 in 20 years.

$$FV = \frac{R(1.005^{240} - 1)}{1.005 - 1}$$

$$165\,510.22 = \frac{R(1.005^{240} - 1)}{0.005}$$

$$165\,510.22 = R(462.040\,895\,2)$$

$$R = \$358.22$$

Therefore, his son must receive \$358.22 per month for 20 years to be equivalent to the \$50 000 his daughter got.

Practise, Apply, Solve 2.8, page 163

1. (a) 0.463 193

(b) 0.506 631

(c) 0.491 934

(d) 0.741 372

(e) 0.235 413

(f) 0.096 142

2. (a) 898.47

(b) 11 997.96

(c) 23 165.20

(d) 87 057.00

3. (a) $a = 100(1.075)^{-24}$, $r = 1.075$, $n = 24$

$$S_n = \frac{100(1.075)^{-24}(1.075^{24} - 1)}{1.075 - 1} = 1098.30$$