

(c)

$$S_{240} = \frac{R((1.00525)^{240} - 1)}{1.00525 - 1}$$

$$80\,000 = \frac{R(1.00525^{240} - 1)}{0.00525}$$

$$420 = R(1.00525^{240} - 1)$$

$$R = \$167.08$$

His deposits must be \$167.08.

(d) $n = 15 \times 12 = 180$

$$80\,000 = \frac{R(1.00525^{180} - 1)}{0.00525}$$

$$420 = R(1.00525^{180} - 1)$$

$$R = \$268.12$$

His new payment would be \$268.12.

(e) Total investment over 25 years = $240 \times \$167.08$
 $= \$40\,099.20$
 Total investment by waiting 5 years = $180 \times \$268.12$
 $= \$48\,261.60$

By waiting he had to invest an extra
 $\$48\,261.60 - \$40\,099.20 = \$8162.40$.

20. We find the amount of the initial \$4320 in 5 years and the future value of her deposits. Then, add the two sums.

$$A = 4320(1.015)^{20} = \$5818.41$$

$$FV = \frac{R((1+i)^n - 1)}{i}$$

$$= \frac{800(1.015^{20} - 1)}{0.015}$$

$$= \$18\,498.93$$

Her balance will be $\$5818.41 + \$18\,498.93$ or $\$24\,317.34$.

21. Answers will vary. However, in most cases, it is more advantageous to double the interest.

22. (a) By starting her deposits earlier, her deposits would be reduced substantially for two reasons. She would be making an extra 60 payments and these 60 payments would accumulate interest over an extra 5 years.

(b) We use

$$S_{300} = \frac{r^{300} - 1}{r - 1}$$

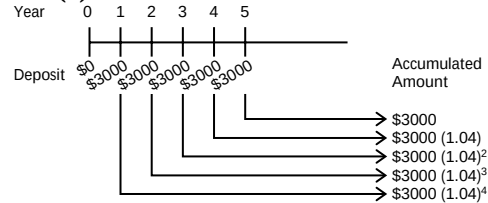
$$300\,000 = \frac{R(1.0075^{300} - 1)}{1.0075 - 1}$$

$$300\,000 = R(1121.121\,937)$$

$$R = 267.59$$

Her deposits for 300 months would be \$267.59/month for a total value of \$80 277. Her deposits over 240 months would be $\$449.18 \times 240$ or \$107 803.20. She would have to pay out \$27 526.20 less by starting 5 years earlier.
 $\$449.18 - \$267.59 = \$181.59$. She would have to pay \$181.59 less per payment.

23. (a)



(b) $S_5 = 3000 + 3000(1.04)^2 + 3000(1.04)^4 + 3000(1.04)^6 + 3000(1.04)^8$

Solve S_5 with $a = 3000$, $r = (1.04)^2$, and $n = 5$.

$$S_5 = \frac{a(r^5 - 1)}{r - 1}$$

$$= \frac{3000((1.04)^2)^5 - 1}{1.04^2 - 1}$$

$$= \frac{3000(1.04^{10} - 1)}{0.0816}$$

$$= \$17\,656.04$$

24. We first determine what is the equivalent future amount in 20 years to \$50 000 with $n = 20 \times 12 = 240$ and

$$i = \frac{0.06}{12} = 0.005.$$

$$A = 50\,000(1.005)^{240} = \$165\,510.22$$

We now find what monthly deposits will amount to \$165 510.22 in 20 years.

$$FV = \frac{R(1.005^{240} - 1)}{1.005 - 1}$$

$$165\,510.22 = \frac{R(1.005^{240} - 1)}{0.005}$$

$$165\,510.22 = R(462.040\,895\,2)$$

$$R = \$358.22$$

Therefore, his son must receive \$358.22 per month for 20 years to be equivalent to the \$50 000 his daughter got.

Practise, Apply, Solve 2.8, page 163

1. (a) 0.463 193

(b) 0.506 631

(c) 0.491 934

(d) 0.741 372

(e) 0.235 413

(f) 0.096 142

2. (a) 898.47

(b) 11 997.96

(c) 23 165.20

(d) 87 057.00

3. (a) $a = 100(1.075)^{-24}$, $r = 1.075$, $n = 24$

$$S_n = \frac{100(1.075)^{-24}(1.075^{24} - 1)}{1.075 - 1} = 1098.30$$

(b) $a = 750(1.001)^{-240}$, $r = 1.001$, $n = 240$

$$S_n = \frac{750(1.001)^{-240} \left((1.001)^{240} - 1 \right)}{1.001 - 1} = 159\,958.35$$

(c) $a = 1500(1.035)^{-10}$, $r = 1.035$, $n = 10$

$$S_n = \frac{1500(1.035)^{-10} \left((1.035)^{10} - 1 \right)}{1.035 - 1} = 12\,474.91$$

4. (a) i. $r = 1.09$, $n = 7$, $a = \frac{8000}{(1.09)^7}$

$8000(1.09)^{-1} = \$7339.45$, $8000(1.09)^{-2} = \$6733.44$,
 $8000(1.09)^{-3} = \$6177.47$, $8000(1.09)^{-4} = \$5667.40$,
 $8000(1.09)^{-5} = \$5199.45$, $8000(1.09)^{-6} = \$4770.14$,
 $8000(1.09)^{-7} = \$4376.27$

ii. $P = 8000(1.09)^{-7} + 8000(1.09)^{-6} + 8000(1.09)^{-5}$
 $+ 8000(1.09)^{-4} + 8000(1.09)^{-3} + 8000(1.09)^{-2} + 8000(1.09)^{-1}$

iii. $P = \frac{8000(1.09^7 - 1)}{(1.09)^7(0.09)} = \$40\,263.62$

(b) i. $r = 1.04$, $n = 7$, $a = \frac{300}{(1.04)^7}$

$300(1.04)^{-1} = \$288.46$, $300(1.04)^{-2} = \$277.37$,
 $300(1.04)^{-3} = \$266.70$, $300(1.04)^{-4} = \$256.44$,
 $300(1.04)^{-5} = \$246.58$, $300(1.04)^{-6} = \$237.09$,
 $300(1.04)^{-7} = \$227.98$

ii. $P = 300(1.04)^{-7} + 300(1.04)^{-6} + 300(1.04)^{-5} + 300(1.04)^{-4}$
 $+ 300(1.04)^{-3} + 300(1.04)^{-2} + 300(1.04)^{-1}$

iii. $P = \frac{300(1.04^7 - 1)}{(1.04)^7(0.04)} = \$1800.62$

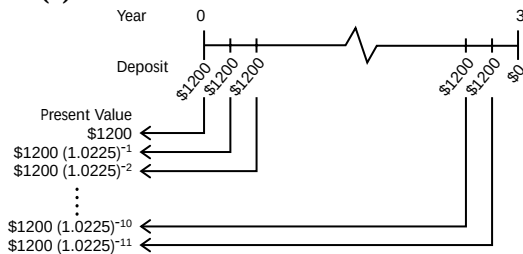
(c) i. $r = 1.02$, $n = 8$, $a = \frac{750}{(1.02)^8}$

$750(1.02)^{-1} = \$735.29$, $750(1.02)^{-2} = \$720.88$,
 $750(1.02)^{-3} = \$706.74$, $750(1.02)^{-4} = \$692.88$,
 $750(1.02)^{-5} = \$679.30$, $750(1.02)^{-6} = \$665.98$,
 $750(1.02)^{-7} = \$652.92$, $750(1.02)^{-8} = \$640.12$

ii. $P = 750(1.02)^{-8} + 750(1.02)^{-7} + 750(1.02)^{-6} + 750(1.02)^{-5}$
 $+ 750(1.02)^{-4} + 750(1.02)^{-3} + 750(1.02)^{-2} + 750(1.02)^{-1}$

iii. $P = \frac{750(1.02^8 - 1)}{(1.02)^8(0.02)} = \$5494.11$

5. (a)



(b) annuity due

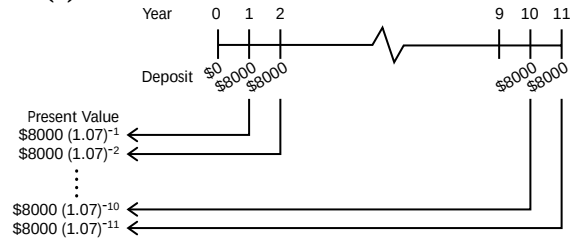
(c) $r = 1.0225$, $n = 12$, and $a = 1200(1.0225)^{-11}$
 $P = 1200(1.0225)^{-11} + 1200(1.0225)^{-10} + \dots + 1200(1.0225)^{-2}$
 $+ 1200(1.0225)^{-1} + 1200$

(d) $r = 1.0225$, $n = 12$, and $a = 1200(1.0225)^{-11}$

$$S_n = \frac{1200(1.0225)^{-11} \left((1.0225)^{12} - 1 \right)}{1.0225 - 1}$$

 $= \$12\,778.93$

6. (a) i.



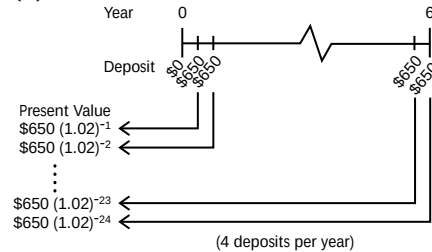
ii. $P = 8000(1.07)^{-11} + 8000(1.07)^{-10} + \dots + 8000(1.07)^{-2}$
 $+ 8000(1.07)^{-1}$

iii.

$$P = \frac{8000(1.07^{11} - 1)}{(1.07^{11})(0.07)}$$

 $= \$59\,989.39$

(b) i.



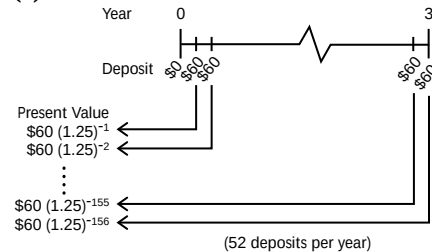
ii. $P = 650(1.02)^{-24} + 650(1.02)^{-23} + \dots + 650(1.02)^{-2}$
 $+ 650(1.02)^{-1}$

iii.

$$P = \frac{650(1.02^{24} - 1)}{(1.02)^{24}(0.02)}$$

 $= \$12\,294.05$

(c) i.



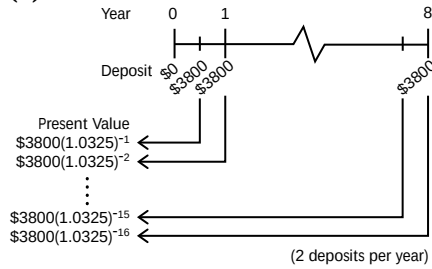
ii. $P = 60(1.25)^{-156} + 60(1.25)^{-155} + \dots + 60(1.25)^{-2}$
 $+ 60(1.25)^{-1}$

iii.

$$P = \frac{60(1.25^{156} - 1)}{(1.25)^{156}(0.25)}$$

 $= \$240$

(d) i.



$$\text{ii. } P = 3800(1.0325)^{-16} + 3800(1.0325)^{-15} + \dots + 3800(1.0325)^{-2} + 3800(1.0325)^{-1}$$

iii.

$$P = \frac{3800(1.0325^{16} - 1)}{(1.0325)^{16}(0.0325)} = \$46\,832.56$$

7. (a) $p = 0.075, R = 2000, n = 9$

$$PV = \frac{2000(1 - (1 + 0.075)^{-10})}{0.075} = \$13\,728.16$$

(b) $p = 0.075, R = 2000, n = 10$

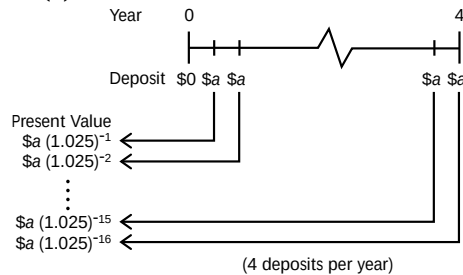
$$PV = \frac{2000(1 - (1 + 0.075)^{-9})}{0.075} = \$12\,757.77$$

$$\$12\,757.77 + \$2000 = \$14\,757.77$$

(c)

$$PV = \frac{13\,728.16}{(0.075)^3} = \$11\,050.63$$

8. (a)



(b) $P = R(1.025)^{-15} + R(1.025)^{-14} + \dots + R(1.025)^{-2} + R(1.025)^{-1}$

(c) $\$11\,133.79 = \frac{a(1.025^{16} - 1)}{0.025}$
 $a = \$574.49$

9. $a = \$280, n = 3 \times 12 = 36, r = 1.00817$

$$P = \frac{280(1.00817^{36} - 1)}{(1.00817)^{36}(0.00817)} = \$8702.34$$

10. $a = \$1000, r = 1.03, \text{ and } n = 20$

$$P = \frac{1000(1.03^{20} - 1)}{(1.03)^{20}(0.03)} = \$14\,877.47$$

11. (a) $a = \$40, r = 1.015, n = 10$

$$P = \frac{40(1.015^{10} - 1)}{(1.015)^{10}(0.015)} = \$368.89$$

$$\text{Selling price} = \$368.89 + \$50 = \$418.89$$

(b) Interest charged = Total monthly payments – Present value
 $= (\$40 \times 10) - \$368.89 = \$400 - \$368.89 = \$31.11$

12. (a) $P = \$2200, r = 1.002, \text{ and } n = 60$

$$2200 = \frac{a(1.002^{60} - 1)}{(1.002)^{60}(0.002)}$$

$$a = \$389.47$$

(b) $\$389.47 \times 60 = \$23\,368.36$

(c) $P = \$20\,000, r = 1.005, \text{ and } n = 6$

$$20\,000 = \frac{a(1.005^{60} - 1)}{(1.005)^{60}(0.005)}$$

$$a = \$386.68$$

The monthly bank payment is \$386.68. He should choose the bank option because the monthly payments are less.

13. (a) $a = \$55, r = 1.0125, \text{ and } n = 18$

$$P = \frac{55(1.0125^{18} - 1)}{(1.0125)^{18}(0.0125)} = \$881.63$$

$$\$881.63 + \$80 = \$961.63$$

(b) $\$80 + \$55(18) = \$1070$
 $\$1070 - \$961.63 = \$108.37$

14. (a) $P = \$64\,000, r = 1.0075, \text{ and } n = 84(7\text{-year term})$

$$A = 64\,000(1.0075)^{84}$$

$$= \$119\,884.93$$

$$119\,884.93 = \frac{a(1.0075^{84} - 1)}{0.0075}$$

$$a = \$1029.70$$

$P = \$64\,000, r = 1.0075, \text{ and } n = 120(10\text{-year term})$

$$A = 64\,000(1.0075)^{120}$$

$$= \$156\,886.85$$

$$156\,886.85 = \frac{a(1.0075^{120} - 1)}{0.0075}$$

$$a = \$810.72$$

(b) $84(\$1029.70) - \$64\,000 = \$22\,494.80$
 $120(\$810.72) - \$64\,000 = \$33\,286.40$
Interest saved = $\$33\,286.40 - \$22\,494.80 = \$10\,791.60$

(c) Answers may vary. Examples: other expense, affordability, down payment, taxes

15. $P = \$32\ 000$, $r = 1.002$, $n = 60$

$$32\ 000 = \frac{a(1.002^{60} - 1)}{(1.002)^{60}(0.002)}$$

$$a = \$566.51$$

$P = \$29\ 000$, $r = 1.0045$, $n = 60$

$$29\ 000 = \frac{a(1.0045^{60} - 1)}{(1.0045)^{60}(0.0045)}$$

$$a = \$552.60$$

Therefore, the best way to buy the car is the bank loan because the monthly payments are less.

16. $a = \$3000$, $r = 1.09$, and $n = 20$

$$A = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{3000(1.09^{20} - 1)}{1.09 - 1}$$

$$= \frac{3000(1.09^{20} - 1)}{0.09}$$

$$= \$153\ 480.36$$

$P = \$153\ 480.36$, $n = 60$, and $r = 1.01875$

$$P = \frac{R(r^n - 1)}{r^n(r - 1)}$$

$$\$153\ 480.36 = \frac{R(1.01875^{60} - 1)}{(1.01875)^{60}(0.01875)}$$

$$R = \$4282.71$$

17. The variable, i , appears twice in the present value formulas, so it is not possible to isolate and solve for i .

18. (a) $a = \$2600$, $r = 1.072$, and $n = 20$

$$A = \frac{2600(1.072^{20} - 1)}{1.072 - 1}$$

$$= \frac{2600(1.072^{20} - 1)}{0.072}$$

$$= \$108\ 945.18$$

$P = A$, $r = 1.021$, and $n = 60$

$$108\ 945.18 = \frac{a(1.021^{60} - 1)}{(1.021)^{60}(0.021)}$$

$$a = \$3210.47$$

(b) $a = \$2600$, $r = 1.072$, and $n = 25$

$$A = \frac{2600(1.072^{25} - 1)}{1.072 - 1}$$

$$= \frac{2600(1.072^{25} - 1)}{0.072}$$

$$= \$169\ 246.35$$

$P = A$, $r = 1.021$, and $n = 60$

$$169\ 246.35 = \frac{a(1.021^{60} - 1)}{(1.021)^{60}(0.021)}$$

$$a = \$4987.47$$

19. (a) $a = 1000$, $r = 1.0013$, $n = 1300$

$$P = \frac{1000(1.0013^{1300} - 1)}{(0.0013)(1.0013)^{1300}}$$

$$= \$627\ 136.80$$

Winner should take the cash option.

(b) $a = 1000$, $r = 1.0013$, $P = 660\ 000$

$$660\ 000 = \frac{1000(1.0013^n - 1)}{(0.0013)(1.0013)^n}$$

$$660\ 000 = \frac{1000(1.0013)^n - 1000}{(0.0013)(1.0013)^n}$$

$$858 = \frac{1000(1.0013)^n}{(1.0013)^n} - \frac{1000}{(1.0013)^n}$$

$$\frac{1000}{(1.0013)^n} = 1000 - 858$$

$$\frac{1000}{142} = (1.0013)^n$$

$$\log\left(\frac{500}{71}\right) = n \log(1.0013)$$

$$n = \frac{\log\left(\frac{500}{71}\right)}{\log(1.0013)}$$

$$n = 1502$$

About 1502 weeks or about $(1502 \div 52)$ 28.9 years.

20. Answers will vary.

21. (a) $a = \$750$, $r = 1.02$, $n = 40$

$$A = \frac{750(1.02^{40} - 1)}{1.02 - 1}$$

$$= \frac{750(1.02^{40} - 1)}{0.02}$$

$$= \$45\ 301.49$$

$$P = \frac{A}{r^n}$$

$$= \frac{45\ 301.49}{1.02^{40}}$$

$$= \$20\ 516.61$$

(b) Answers will vary.

(c) (annuities) $PV = \frac{R(1 - (1 + i)^{-n})}{i}$

$$FV = \frac{R((1 + i)^n - 1)}{i} \quad \textcircled{1}$$

(lump sum) $S = P(1 + i)^n$

$$P = \frac{S}{(1+i)^n} = S(1+i)^{-n} \quad \textcircled{2}$$

So, let $S = FV$. Substitute $\textcircled{1}$ into $\textcircled{2}$ to get

$$\begin{aligned} PV = P &= \frac{R((1+i)^n - 1)}{i(1+i)^n} \\ &= \frac{R(1+i)^n}{i(1+i)^n} - \frac{R}{i(1+i)^n} \\ &= \frac{R}{i} - \frac{R}{i(1+i)^n} \\ &= \frac{R}{i} - \frac{R(1+i)^{-n}}{i} \\ &= \frac{R(1 - (1+i)^{-n})}{i} \end{aligned}$$

22. $P = 17\,000$, $a = 325$, $r = 1.006$

$$17\,000 = \frac{325(1.006^n - 1)}{(1.006)^n(0.006)}$$

$$17\,000 = \frac{325(1.006)^n - 325}{(1.006)^n(0.006)}$$

$$102 = \frac{325(1.006)^n}{(1.006)^n} - \frac{325}{(1.006)^n}$$

$$\frac{325}{(1.006)^n} = 325 - 102$$

$$\frac{325}{223} = (1.006)^n$$

$$\log\left(\frac{325}{223}\right) = n \log(1.006)$$

$$n = \frac{\log\left(\frac{325}{223}\right)}{\log(1.006)}$$

$$n = 63$$

23. $P = 7500$, $a = 280$, $r = 1.0095$

$$7500 = \frac{280(1.0095^n - 1)}{(1.0095)^n(0.0095)}$$

$$7500 = \frac{280(1.0095)^n - 280}{(1.0095)^n(0.0095)}$$

$$71.25 = \frac{280(1.0095)^n}{(1.0095)^n} - \frac{280}{(1.0095)^n}$$

$$\frac{280}{(1.0095)^n} = 280 - 71.25$$

$$\frac{280}{208.75} = (1.0095)^n$$

$$\log\left(\frac{280}{208.75}\right) = n \log(1.0095)$$

$$n = \frac{\log\left(\frac{280}{208.75}\right)}{\log(1.0095)}$$

$$n = 31$$

Practice 2.9, page 171

(a) Guy's GIC will be worth \$14 774.55.

```
N=8.00
I%=5.00
PV=-10000.00
PMT=0.00
FV=14774.55
P/Y=1.00
C/Y=1.00
PMT:BEGIN
```

(b) You have to invest \$6315.27 today.

```
N=8.00
I%=6.00
PV=-6315.27
PMT=0.00
FV=8000.00
P/Y=2.00
C/Y=2.00
PMT:BEGIN
```

(c) The future value or amount of the annuity is \$45 301.49.

```
N=40.00
I%=8.00
PV=0.00
PMT=-750.00
FV=45301.49
P/Y=4.00
C/Y=4.00
PMT:BEGIN
```

(d) Gurtrude should deposit \$3305.57 today.

```
N=16.00
I%=3.50
PV=-3305.57
PMT=0.00
FV=3800.00
P/Y=4.00
C/Y=4.00
PMT:BEGIN
```

(e) Trudy would need \$37 936.57 to establish the Retirement Income Fund.

```
N=180.00
I%=5.00
PV=-37936.57
PMT=300.00
FV=0.00
P/Y=12.00
C/Y=12.00
PMT:BEGIN
```