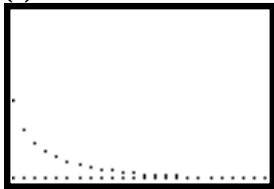
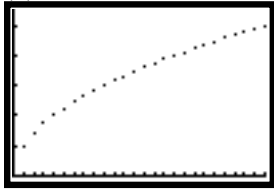


(c) i.



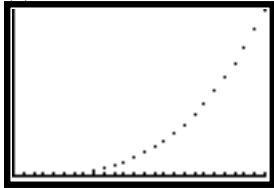
ii. $t_{10} = 0.1$, $t_{15} = 0.067$, $t_{20} = 0.05$

(d) i.



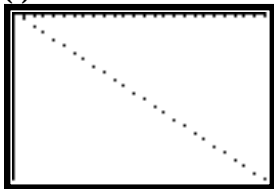
ii. $t_{10} = 3.162$, $t_{15} = 3.872$, $t_{20} = 4.472$

(e) i.



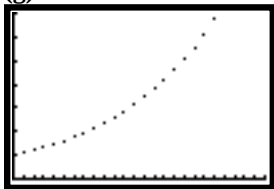
ii. $t_{10} = 1990$, $t_{15} = 6735$, $t_{20} = 15\,980$

(f) i.



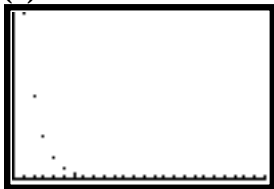
ii. $t_{10} = -10$, $t_{15} = -15$, $t_{20} = -20$

(g) i.



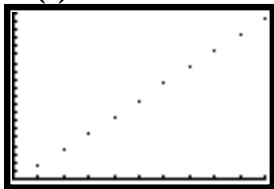
ii. $t_{10} = 25.937$, $t_{15} = 41.772$, $t_{20} = 67.274$

(h) i.



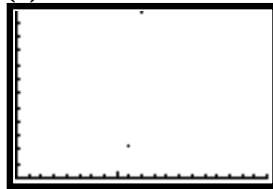
ii. $t_{10} = 0.009\,766$, $t_{15} = 0.000\,305\,2$, $t_{20} = 0.000\,009\,537$

2. (a) i.



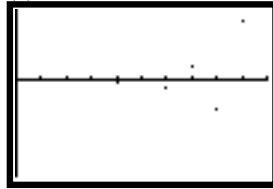
ii. $t_5 = 19$, $t_{10} = 39$

(b) i.



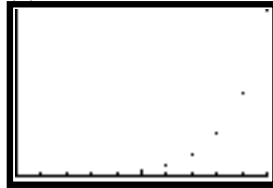
ii. $t_5 = 3750$, $t_{10} = 11\,718\,750$

(c) i.



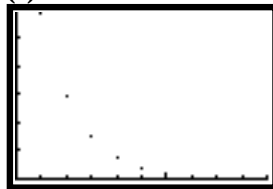
ii. $t_5 = 65$, $t_{10} = -2047$

(d) i.



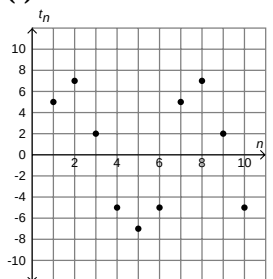
ii. $t_5 = 32$, $t_{10} = 1024$

(e) i.



ii. $t_5 = 6.25$, $t_{10} = 0.915\,312\,5$

(f) i.



ii. $t_5 = -7$, $t_{10} = -5$

Practise, Apply, Solve 1.6, page 47

1. (a) i. $d_1 = 11 - 4 = 7$, $d_2 = 15 - 11 = 4$, $d_1 \neq d_2$, not arithmetic

(b) i. arithmetic, ii. $d = 23 - 32 = -9$

(c) i. $d_1 = 6 - 5 = 1$, $d_2 = 5 - 6 = -1$, $d_1 \neq d_2$, not arithmetic

(d) i. $d_1 = 8 - 2 = 6$, $d_2 = 32 - 8 = 24$, $d_1 \neq d_2$, not arithmetic

(e) i. arithmetic, ii. $d = 33 - 22 = 11$

(f) i. arithmetic, ii. $d = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

2. (a) i. $t_n = 3n - 2 = 3n - 3 + 1 = 1 + (n - 1)(3)$, arithmetic, ii. $d = 3$

(b) i. $t_1 = 1$, $t_2 = 5$, $t_3 = 25$, $d_1 = 5 - 1 = 4$, $d_2 = 25 - 5 = 20$, $d_1 \neq d_2$, not arithmetic

(c) i. $t_n = -4n + 7 = -4n + 4 + 3 = 3 + (n-1)(-4)$, arithmetic,

ii. $d = -4$

(d) i. $t_1 = 0, t_2 = 3, t_3 = 8, d_1 = 3 - 0 = 3, d_2 = 8 - 3 = 5$, not arithmetic

(e) i. $t_n = \frac{1}{2}n - \frac{1}{2} + \frac{5}{2} = \frac{5}{2} + (n-1)\left(\frac{1}{2}\right)$, arithmetic,

ii. $d = \frac{1}{2}$

(f) i. $t_1 = 3, t_2 = 2, t_3 = \frac{5}{3}, d_1 = 2 - 3 = -1, d_2 = \frac{5}{3} - 2 = -\frac{1}{3}$,

$d_1 \neq d_2$, not arithmetic

3. (a) i. $d = 10 - 5 = 5$ ii. $t_n = 5 + (n-1)(5) = 5n$

iii. $t_{10} = 5(10) = 50$

(b) i. $d = -24 - (-30) = 6$ ii. $t_n = -30 + (n-1)(6) = 6n - 36$

iii. $t_{10} = 6(10) - 36 = 24$

(c) i. $d = 11 - 13 = -2$ ii. $t_n = 13 + (n-1)(-2) = -2n + 15$

iii. $t_{10} = -2(10) + 15 = -5$

(d) i. $d = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ ii. $t_n = \frac{1}{3} + (n-1)\left(\frac{1}{3}\right) = \frac{1}{3}n$

iii. $t_{10} = \frac{1}{3}(10) = \frac{10}{3}$

(e) i. $d = 0.35 - 0.2 = 0.15$

ii. $t_n = 0.2 + (n-1)(0.15) = 0.15n + 0.05$

iii. $t_{10} = 0.15(10) + 0.05 = 1.55$

(f) i. $d = -3 - (-3) = 0$ ii. $t_n = -3 + (n-1)0 = -3$

iii. $t_{10} = -3$

4. (a) $t_n = t_{n-1} + 5, t_1 = 5$

(b) $t_n = t_{n-1} + 6, t_1 = -30$

(c) $t_n = t_{n-1} - 2, t_1 = 13$

(d) $t_n = t_{n-1} + \frac{1}{3}, t_1 = \frac{1}{3}$

(e) $t_n = t_{n-1} + 0.15, t_1 = 0.2$

(f) $t_n = t_{n-1} - 3, t_1 = -3$

5. (a) $a = 7, d = 6, t_n = 7 + (n-1)(6) = 6n + 1$

(b) $a = -9, d = -2, t_n = -9 + (n-1)(-2) = -2n - 7$

(c) $a = 25, d = -6, t_n = 25 + (n-1)(-6) = -6n + 31$

(d) $a = 121, d = 12, t_n = 121 + (n-1)(12) = 12n + 109$

6. (a) -5, 4, 13, 22, **31, 40, 49**

(b) $t_n = -5 + (n-1)[4 - (-5)]$

$= -5 + (n-1)(9)$

$= 9n - 14$

(c) $t_{200} = 9(200) - 14 = 1800 - 14 = 1786$

7. (a) $a = 2, d = 2$ i. $t_n = 2 + (n-1)(2) = 2n$

ii. $t_{24} = 2(24) = 48$

(b) $a = -1, d = -3$ i. $t_n = -1 + (n-1)(-3) = -3n + 2$

ii. $t_{24} = -3(24) + 2 = -72 + 2 = -70$

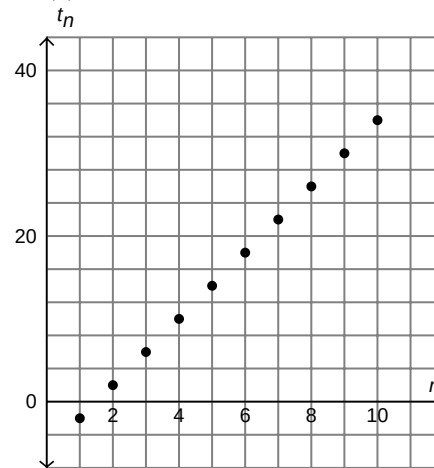
(c) $a = 10, d = 5$ i. $t_n = 10 + (n-1)(5) = 5n + 5$

ii. $t_{24} = 5(24) + 5 = 120 + 5 = 125$

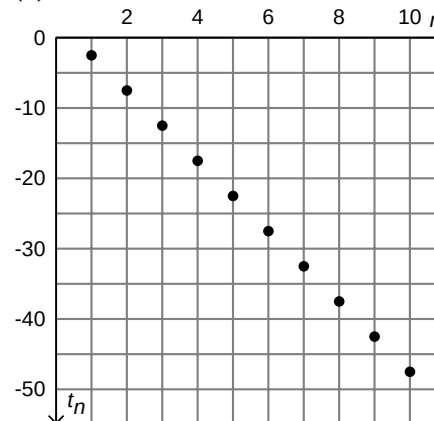
(d) $a = -30, d = -10$ i. $t_n = -30 + (n-1)(-10) = -10n - 20$

ii. $t_{24} = -10(24) - 20 = -240 - 20 = -260$

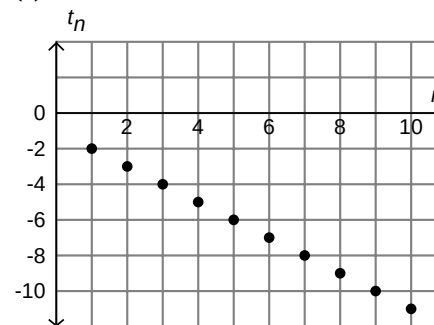
8. (a)



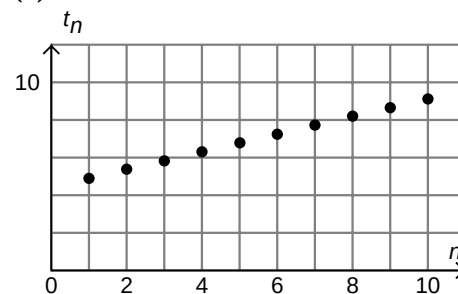
(b)

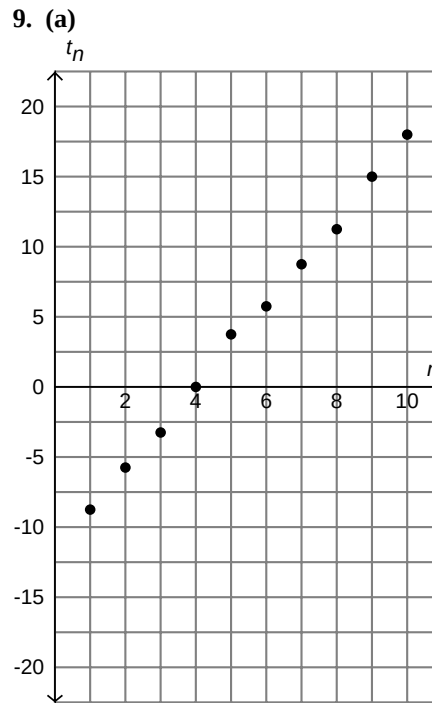
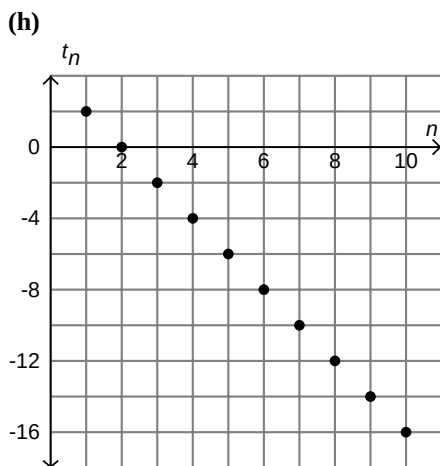
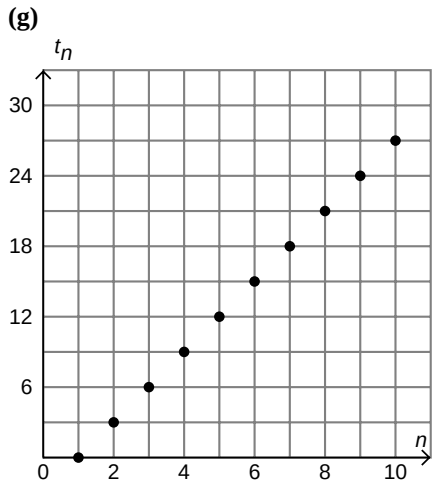
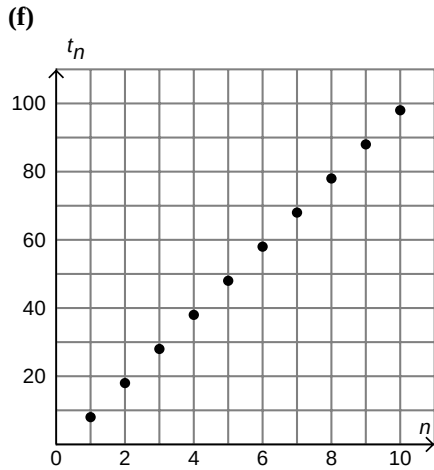
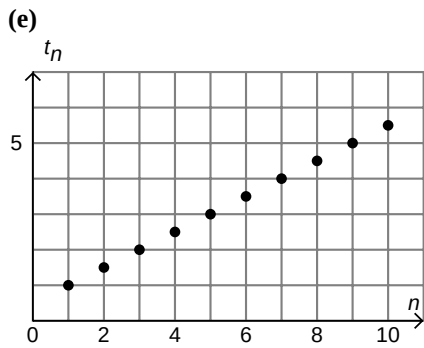


(c)



(d)





(b) From the graph, one can determine that term 11 has a value of 21. $\therefore t_{11} = 21$.

(c)

$$t_n = -9 + (n-1)(3)$$

$$t_n = -12 + 3n$$

$$21 = -12 + 3n$$

$$\frac{21 - (-12)}{3} = n$$

$$11 = n$$

(d) A graph only gives an approximation of the answer; the general term gives the most reliable answer.

10. (a) $P = \$500$, $r = 0.04$, $t = 2$, $I = \$500(0.04)(2) = \$40$

(b) $P = \$3000$, $r = 0.035$, $t = 5$, $I = \$3000(0.035)(5) = \$525$

(c) $P = \$4250$, $r = 0.06$, $t = \frac{38}{12}$,

$$I = \$4250(0.06)\left(\frac{38}{12}\right) = \$807.50$$

(d) $P = \$5000$, $r = 0.0575$, $t = \frac{45}{52}$,

$$I = \$5000(0.0575)\left(\frac{45}{52}\right) = \$248.80$$

(e) $P = \$900$, $r = 0.08$, $t = \frac{60}{365}$,

$$I = \$900(0.08)\left(\frac{60}{365}\right) = \$11.84$$

11. (a) $\{8, 11, 14, 17, \dots, 71\}$, $a = 8$, $d = 11 - 8 = 3$,

$$t_n = 8 + 3(n-1) = 3n + 5$$

How many terms are in the sequence? Solve for n .

$$3n + 5 = 71$$

$$n = \frac{1}{3}(71 - 5)$$

$$n = 22$$

$\therefore$ there are 22 terms in the sequence.

(b) $\{-10, -14, -18, -22, \dots, -138\}$, $a = -10$,
 $d = -14 - (-10) = -4$, $t_n = -10 - 4(n-1) = -4n - 6$
 How many terms are in the sequence? Solve for n .
 $-4n - 6 = -138$

$$n = \frac{-1}{-4}(-138 + 6)$$

$$n = 33$$

$\therefore$ there are 33 terms in the sequence.

(c) $\{6, 17, 28, 39, \dots, 435\}$, $a = 6$, $d = 17 - 6 = 11$,
 $t_n = 6 + 11(n-1) = 11n - 5$

How many terms are in the sequence? Solve for n .
 $11n - 5 = 435$

$$n = \frac{1}{11}(435 + 5)$$

$$n = 40$$

$\therefore$ there are 40 terms in the sequence.

(d) $\{21, 17, 13, 9, \dots, -35\}$, $a = 21$, $d = 17 - 21 = -4$
 $t_n = 21 - 4(n-1) = -4n + 25$

How many terms are in the sequence? Solve for n .
 $-4n + 25 = -35$

$$n = \frac{-1}{-4}(-35 - 25)$$

$$n = 15$$

$\therefore$ there are 15 terms in the sequence.

(e) $\{8, 15, 22, 29, \dots, 99\}$ $a = 8$, $d = 15 - 8 = 7$
 $t_n = 8 + 7(n-1) = 7n + 1$

How many terms are in the sequence? Solve for n .
 $7n + 1 = 99$

$$n = \frac{1}{7}(99 - 1)$$

$$n = 14$$

$\therefore$ there are 14 terms in the sequence.

(f) $\{-92, -84, -76, -68, \dots, 52\}$, $a = -92$, $d = -84 - (-92) = 8$,
 $t_n = -92 + 8(n-1) = 8n - 100$

How many terms are in the sequence? Solve for n .
 $8n - 100 = 52$

$$n = \frac{1}{8}(52 + 100)$$

$$n = 19$$

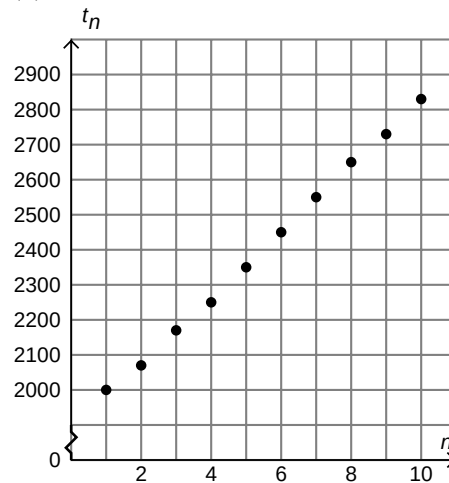
$\therefore$ there are 19 terms in the sequence.

12. (a) $I = Pr = (\$2000)(0.045) = \$90$

(b) $\$2000, \$2090, \$2180, \$2270, \$2360$

(c) $t_1 = \$2000$, $d = \$2090 - \$2000 = \$90$,
 $t_n = \$2000 + (n-1)(\$90) = \$90n + \$1910$

(d)



(e) $t_n = \$3350$, $t_n = \$90n + \$1910$

Solve for n to determine the number of years that the original deposit has been earning interest.

$$\$90n + \$1910 = \$3350$$

$$n = \frac{1}{\$90}(\$3350 - \$1910)$$

$$n = 16$$

$\therefore$ the original deposit has been earning interest for 16 years,

13. $a = \$175\,000$, $t_n = \$175\,000 + \$5500n$

To determine when Carlo can sell his house for at least twice the purchase price ($2 \times \$175\,000$), solve for n .

$$\$5500n + \$175\,000 = 2(\$175\,000)$$

$$n = \frac{\$350\,000 - \$175\,000}{\$5500}$$

$$n = 31.8$$

$\therefore$ Carlo can sell his house after approximately 32 years.

14. $\{85, 81, 77, \dots, 1\}$, $a = 85$, $d = 81 - 85 = -4$,

$$t_n = 85 + (n-1)(-4) = -4n + 89$$

(a) $t_{13} = -4(13) + 89 = -52 + 89 = 37$

$\therefore$ there are 37 bricks in the 13th row.

(b) To determine how many rows there are in the pile, solve for n .

$$-4n + 89 = 1$$

$$n = \frac{1 - 89}{-4}$$

$$n = 22$$

$\therefore$ there are 22 rows in the pile.

15. (a) $a = 1531$, $t_n = 1531 + 76n$, $n \in \mathbb{Z}$

(b)

$$1531 + 76n \geq 2001$$

$$1531 + 76n \leq 2100$$

$$n \geq \frac{2001 - 1531}{76}$$

$$n \leq \frac{2100 - 1531}{76}$$

$$n \geq 6.18$$

$$n \leq 7.49$$

$$\Rightarrow n \geq 7, n \in \mathbb{Z}$$

$$\Rightarrow n \leq 7, n \in \mathbb{Z}$$

$\therefore n = 7$. $t_n = 1531 + 7(76) = 2063$

$\therefore$ Halley's comet will be visible from earth in the year 2063.

(c) Two centuries after the 21st century (the 23rd century) will have Halley's comet appear at $2063 + 2(76) = 2215$. It will be back again in $2215 + 76 = 2291$.

(d) No, this model cannot be used to make precise predictions because the comet may not return.

$$16. t_1 = a, t_n = t_{n-1} + d$$

$$17. (a) t_6 = 23, t_{11} = 38$$

$$a + 5d = 23$$

$$a + 10d = 38$$

$$5d = 15$$

$$d = 3$$

$$a = 23 - 5(3) = 8$$

$$t_n = 8 + (n - 1)3 = 3n + 5$$

$$(b) t_8 = -49, t_{15} = -84$$

$$a + 7d = -49$$

$$a + 14d = -84$$

$$7d = -35$$

$$d = -5$$

$$a = -49 - 7(-5) = -14$$

$$t_n = -14 + (n - 1)(-5) = -5n - 9$$

$$(c) t_{12} = 52, t_{16} = 80$$

$$a + 11d = 52$$

$$a + 15d = 80$$

$$4d = 28$$

$$d = 7$$

$$a = 52 - 11(7) = -25$$

$$t_n = -25 + (n - 1)(7) = 7n - 32$$

$$(d) t_4 = 12, t_{21} = 20.5$$

$$a + 3d = 12$$

$$a + 20d = 20.5$$

$$17d = 8.5$$

$$d = 0.5$$

$$a = 12 - 3(0.5) = 10.5$$

$$t_n = 10.5 + (n - 1)(0.5) = 0.5n + 10$$

$$(e) t_5 = 52, t_{17} = 124$$

$$a + 4d = 52$$

$$a + 16d = 124$$

$$12d = 72$$

$$d = 6$$

$$a = 52 - 4(6) = 28$$

$$t_n = 28 + (n - 1)(6) = 6n + 22$$

$$(f) t_3 = 24, t_{19} = -88$$

$$a + 2d = 24$$

$$a + 18d = -88$$

$$16d = -112$$

$$d = -7$$

$$a = 24 - 2(-7) = 38$$

$$t_n = 38 + (n - 1)(-7) = -7n + 45$$

$$18. a = \$0.25, d = \$0.25$$

$$t_n = \$0.25 + \$0.25(n - 1) = \$0.25n$$

$$t_1 + t_{30} = \$0.25 + \$0.25(30) = \$0.25(31)$$

$$t_2 + t_{29} = \$0.25(2) + \$0.25(29) = \$0.25(31)$$

$$t_3 + t_{28} = \$0.25(3) + \$0.25(28) = \$0.25(31)$$

...

$$t_{15} + t_{16} = \$0.25(15) + \$0.25(16) = \$0.25(31)$$

$$\therefore \text{the total is } \$0.25(31)(15) = \$116.25.$$

$$19. (a) a = 17, d = 23 - 17 = 6,$$

$$t_n = 17 + (n - 1)(6) = 6n + 11, t_{25} = 6(25) + 11 = 161$$

(b) The properties that make a sequence arithmetic are: (1) a common difference exists between consecutive terms, (2) the sequence can be written as $t_n = a + (n - 1)d$.

$$20. a = \$40.00, d = \$44.50 - \$40.00 = \$4.50,$$

$$t_n = \$40.00 + (\$4.50)(n - 1) = \$4.50n + \$35.50$$

$$t_4 = \$4.50(4) + \$35.50 = \$53.50,$$

$$t_5 = \$4.50(5) + \$35.50 = \$58.00, \dots$$

$$t_{10} = \$4.50(10) + \$35.50 = \$80.50$$

$$t_1 + t_2 + t_3 + \dots + t_{10} = \$602.50$$

$\therefore$ the company would charge \$602.50 to drill a 10-m well.

21. For $c, \frac{c+d}{2}, d$ to be an arithmetic sequence, there must

be a common difference. Hence, for it to be arithmetic,

$$d - \frac{c+d}{2} = \frac{c+d}{2} - c$$

$$\begin{aligned} \text{L.S.} &= d - \frac{c+d}{2} \\ &= \frac{2d - (c+d)}{2} \end{aligned}$$

$$= \frac{d - c}{2}$$

$$\begin{aligned} \text{R.S.} &= \frac{c+d}{2} - c \\ &= \frac{(c+d) - 2c}{2} \\ &= \frac{d - c}{2} \\ &= \text{L.S.} \end{aligned}$$

Therefore, the sequence is arithmetic.

Alternate Solution

Since $\frac{c+d}{2}$ is the median of c and d , $d - \frac{c+d}{2} = \frac{c+d}{2} - c$.

By definition, the sequence is arithmetic.

$$22. a = 5x - 3, d = 3x - 4 - (5x - 3) = -2x - 1$$

$$t_n = 5x - 3 + (n - 1)(-2x - 1) = (-2x - 1)n + (7x - 2)$$

$$23. t_n = 3n,$$

$$3n \geq 40 \quad 3n \leq 1000$$

$$n \geq 13 \quad n \leq 333$$

$$\text{Total: } 333 - 13 = 320.$$

There are 320 multiples of 3 between 40 and 1000.