

(f) Ramona's bank balance will be \$58 874.59 after 3 years. It therefore earns much more than \$240 in interest every 3 months.

N=12.00
I%=5.00
PV=-53380.00
PMT=240.00
FV=58874.59
P/Y=4.00
C/Y=4.00
PMT:1234 BEGIN

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1. (a)

$$\begin{aligned}\sqrt{(1+i)^2} &= \left((1+i)^2\right)^{\frac{1}{2}} \\ &= (1+i)^{\frac{2}{2}} \\ &= 1+i\end{aligned}$$

(b)

$$\begin{aligned}\sqrt[4]{(1+i)^4} &= \left((1+i)^4\right)^{\frac{1}{4}} \\ &= (1+i)^{\frac{4}{4}} \\ &= 1+i\end{aligned}$$

(c)

$$\begin{aligned}\sqrt[7]{(1+i)^7} &= \left((1+i)^7\right)^{\frac{1}{7}} \\ &= (1+i)^{\frac{7}{7}} \\ &= 1+i\end{aligned}$$

(d)

$$\begin{aligned}\sqrt[3]{(1+i)^3} &= \left((1+i)^3\right)^{\frac{1}{3}} \\ &= (1+i)^{\frac{3}{3}} \\ &= 1+i\end{aligned}$$

2. (a)

$$\begin{aligned}\sqrt{(1.06)^4} &= (1.06)^{\frac{4}{2}} \\ &= 1.06^2 \\ &= 1.1236\end{aligned}$$

(b)

$$\begin{aligned}\sqrt[3]{(1.08)^{12}} &= (1.08)^{\frac{12}{3}} \\ &= 1.08^4 \\ &= 1.3605\end{aligned}$$

(c)

$$\begin{aligned}\sqrt[4]{(1.0125)^{12}} &= (1.0125)^{\frac{12}{4}} \\ &= 1.0125^3 \\ &= 1.0380\end{aligned}$$

(d)

$$\begin{aligned}\sqrt[2]{(1.04)^{18}} &= (1.04)^{\frac{18}{2}} \\ &= 1.04^9 \\ &= 1.4233\end{aligned}$$

3. (a) $1+i = (1.025)^4$

$$i = (1.025)^4 - 1$$

$$i = 0.1038$$

(b) $1+i = (1.01)^{12}$

$$i = (1.01)^{12} - 1$$

$$i = 0.1268$$

(c) $(1+i)^2 = 1.10$

$$1+i = \sqrt{1.10}$$

$$i = \sqrt{1.10} - 1$$

$$i = 0.0488$$

(d) $(1+i)^{12} = 1.015$

$$1+i = \sqrt[12]{1.015}$$

$$i = \sqrt[12]{1.015} - 1$$

$$i = 0.0012$$

(e) $(1+i)^4 = (1.06)^2$

$$1+i = \sqrt[4]{(1.06)^2}$$

$$i = \sqrt[4]{(1.06)^2} - 1$$

$$i = 0.0296$$

(f) $(1+i)^{12} = (1.025)^4$

$$1+i = \sqrt[12]{(1.025)^4}$$

$$i = \sqrt[12]{(1.025)^4} - 1$$

$$i = 0.0083$$

(g) $(1+i)^{52} = (1.03)^4$

$$1+i = \sqrt[52]{(1.03)^4}$$

$$i = \sqrt[52]{(1.03)^4} - 1$$

$$i = 0.0023$$

(h) $(1+i)^2 = (1.0055)^{52}$

$$1+i = \sqrt{(1.0055)^{52}}$$

$$i = (1.0055)^{26} - 1$$

$$i = 0.1533$$

4. (a) $1+i = (1.04)^2$

$$i = 0.0816$$

$$i = 8.16\%/a$$

(b) $1+i = (1.02)^4$

$$i = 0.0824$$

$$i = 8.24\%/a$$

(c) $1+i = (1.002)^{52}$

$$i = 0.1095$$

$$i = 10.95\%/a$$

(d) $1+i = (1.000\ 24)^{365}$

$$i = 0.0915$$

$$i = 9.15\%/a$$

(e) $1+i = (1.045)^4$

$$i = 0.1925$$

$$i = 19.25\%/a$$

(f) $1+i = (1.0175)^{12}$

$$i = 0.2314$$

$$i = 23.14\%/a$$

5. (a) $1 + i = (1.045)^4$

$i = 0.1925$

$i = 19.25\%/a$

(b) $(1 + i)^2 = (1.045)^4$

$1 + i = \sqrt{(1.045)^4}$

$1 + i = (1.045)^2$

$i = 0.0920$ semiannually

$i = 18.40\%/a$

(c) $(1 + i)^{12} = (1.045)^4$

$12\sqrt[12]{(1 + i)^{12}} = 12\sqrt[12]{(1.045)^4}$

$1 + i = \sqrt[3]{1.045}$

$i = 0.01478/\text{month}$

$i = 0.1777/a$

$i = 17.77\%/a$

(d) $(1 + i)^{52} = (1.045)^4$

$52\sqrt[52]{(1 + i)^{52}} = 52\sqrt[52]{(1.045)^4}$

$1 + i = \sqrt[13]{1.045}$

$i = 0.003392/\text{week}$

$i = 0.1764/a$

$i = 17.64\%/a$

(e) $(1 + i)^{365} = (1.045)^4$

$365\sqrt[365]{(1 + i)^{365}} = 365\sqrt[365]{(1.045)^4}$

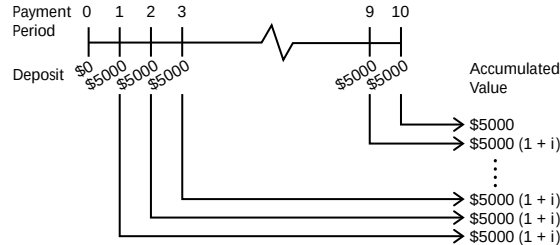
$i = \sqrt[365]{(1.045)^4} - 1$

$i = 0.00048249/\text{day}$

$i = 0.17611/a$

$i = 17.61\%/a$

6. (a) i.



ii. Find the yearly rate, i , equivalent to $8\%/a$ compounded semi-annually.

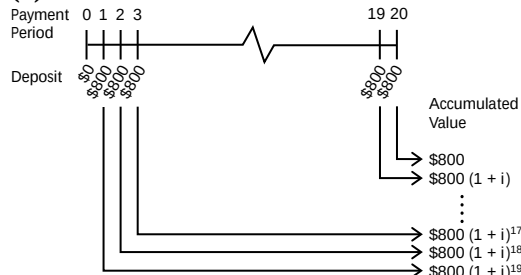
$1 + i = (1.04)^2$

$i = 0.0816$

$S_{10} = 5000 + 5000(1.0816) + 5000(1.0816)^2 + \dots + 5000(1.0816)^8 + 5000(1.0816)^9$

iii. $S_{10} = \frac{5000(1.0816^{10} - 1)}{1.0816 - 1} = \$72\,985.49$

(b) i.



ii. Find the quarterly rate, i , equivalent to $8\%/a$ compounded yearly.

$(1 + i)^4 = 1.08$

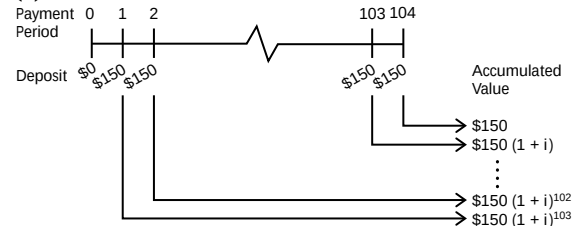
$i = \sqrt[4]{1.08} - 1$

$i = 0.019426547$

$S_{20} = 800 + 800(1.019426547) + 800(1.019426547)^2 + \dots + 800(1.019426547)^{18} + 800(1.019426547)^{19}$

iii. $S_{20} = \frac{800(1.019426547^{20} - 1)}{1.019426547 - 1} = \$19\,327.29$

(c) i.



ii. Find the weekly rate, i , equivalent to $11\%/a$ compounded semiannually.

$(1 + i)^{52} = (1.055)^2$

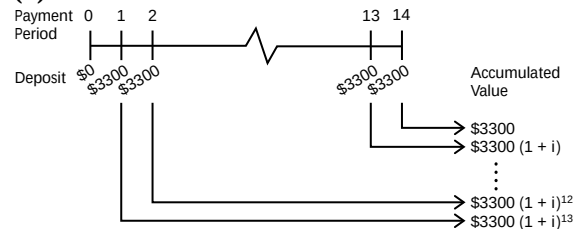
$1 + i = \sqrt[26]{1.055}$

$i = 0.001878304$

$S_{104} = 150 + 150(1.001878304) + 150(1.001878304)^2 + \dots + 150(1.001878304)^{102} + 150(1.001878304)^{103}$

iii. $S_{104} = \frac{150(1.001878304^{104} - 1)}{1.001878304 - 1} = \$17\,210.15$

(d) i.



ii. Find the semiannual rate, i , equivalent to $10\%/a$ compounded quarterly.

$(1 + i)^2 = (1.025)^4$

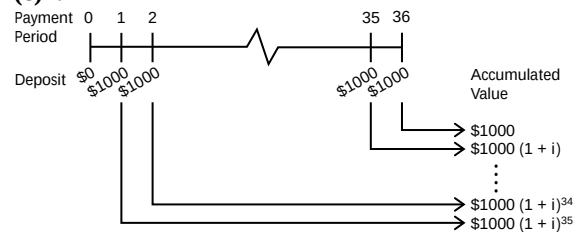
$1 + i = (1.025)^2$

$i = 0.050625$

$S_{14} = 3300 + 3300(1.050625) + 3300(1.050625)^2 + \dots + 3300(1.050625)^{12} + 3300(1.050625)^{13}$

iii. $S_{14} = \frac{3300(1.050625^{14} - 1)}{1.050625 - 1} = \$64\,956.71$

(e) i.



ii Find the monthly rate, i , equivalent to $9\%/a$.

$(1 + i)^{12} = 1.09$

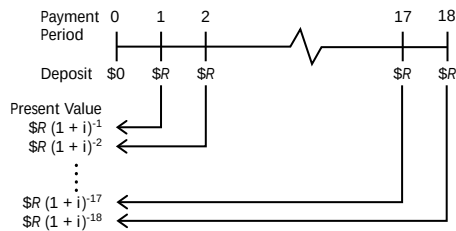
$i = \sqrt[12]{1.09} - 1$

$i = 0.007207323$

$$S_{36} = 1000 + 1000(1.007\ 207\ 323) + 1000(1.007\ 207\ 323)^2 + \dots + 1000(1.007\ 207\ 323)^{34} + 1000(1.007\ 207\ 323)^{35}$$

$$\text{iii. } S_{36} = \frac{1000(1.007\ 207\ 323^{36} - 1)}{1.007\ 207\ 323 - 1} = \$40\ 934.61$$

7.



Find the monthly rate, i , equivalent to 18%/a compounded yearly.

$$(1+i)^{12} = 1.18$$

$$i = \sqrt[12]{1.18} - 1$$

$$i = 0.013\ 888\ 43$$

We use the present value formula.

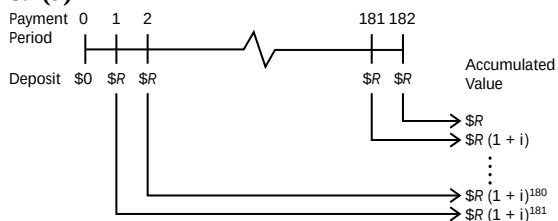
$$PV = \frac{R(1 - (1+i)^{-n})}{i}$$

$$PV = \frac{R(1 - 1.013\ 888\ 43^{-18})}{0.013\ 888\ 43}$$

$$3500 = R(15.829\ 867\ 97)$$

$$R = \$221.10$$

8. (a)



(b) Find the weekly rate, i , equivalent to 7.2%/a compounded monthly.

$$(1+i)^{52} = (1.006)^{12}$$

$$1+i = \sqrt[52]{(1.006)^{12}}$$

$$i = 0.001\ 381\ 431$$

$$S_{182} = R + R(1.001\ 381\ 431) + R(1.001\ 381\ 431)^2 + \dots + R(1.001\ 381\ 431)^{180} + R(1.001\ 381\ 431)^{181}$$

(c)

$$25\ 000 = \frac{R(1.001\ 381\ 431^{182} - 1)}{1.001\ 381\ 431 - 1}$$

$$25\ 000 = R(206.762\ 061)$$

$$R = \$120.91$$

9. (a) Find the monthly rate, i , equivalent to 9.8%/a compounded yearly.

$$(1+i)^{12} = 1.098$$

$$i = \sqrt[12]{1.098} - 1$$

$$i = 0.007\ 821\ 29$$

Solve for the present value of 36 monthly payments of \$280.

$$PV = \frac{R(1 - (1+i)^{-n})}{i}$$

$$PV = \frac{280(1 - 1.007\ 821\ 29^{-36})}{0.007\ 821\ 29}$$

$$PV = 280(31.270\ 056\ 7)$$

$$PV = \$8755.62$$

He can spend \$8755.62 on a car.

(b) The 0.782 129%/month interest rate is very close to the 9.8%/a compounded monthly he was charged in section 2.8. Also, because the loan was for such a short period of time (3 years), the difference in the two rates has little effect on the present value.

10. (a) Answers will vary. *Example:* Michael needs \$2800 to go travelling in one year. How much must he deposit at the end of every three months into an account that pays 8%/a compounded quarterly?

(b) Answers will vary. *Example:* Alisha buys a new car by agreeing to make equal monthly payments of \$350 for 5 years at a rate of 6%/a compounded semiannually. What did she pay for the car?

11. The problem is represented by a timeline diagram.

Solve for i . Use the present value of his payments, equate them to \$2900, and solve for the payment R .

$$(1+i)^{12} = 1.096$$

$$i = \sqrt[12]{1.096} - 1$$

$$i = 0.007\ 668\ 183$$

$$2900 = \frac{R(1 - (1.007\ 668\ 183)^{-36})}{0.007\ 668\ 183}$$

$$2900 = R(31.354)$$

$$R = \$92.49$$

Daniel's monthly payments will be \$92.49.

Verify using the graphing calculator.

12. (a) (i)

The future value will be \$74 118.37.

(ii)

The future value will be \$74 402.62.

(iii)

```
N=72
I%=7.2
PV=0
PMT=-832
FV=74650.93278
P/Y=12
C/Y=12
PMT: [ ] BEGIN
```

The future value will be \$74 650.93.

(iv)

```
N=72
I%=7.2
PV=0
PMT=-832
FV=74690.10947
P/Y=12
C/Y=52
PMT: [ ] BEGIN
```

The future value will be \$74 690.11.

(v)

```
N=72
I%=7.2
PV=0
PMT=-832
FV=74700.23136
P/Y=12
C/Y=365
PMT: [ ] BEGIN
```

The future value will be \$74 700.23.

(b) Considering they are saving for a fairly lengthy period of time (6 years), the compounding period does not have a significant effect on their total savings. However, compounding daily is preferred over compounding yearly if the annual rate is fixed.

13. (a) Year-end deposit = 12×832
= \$9984.00

```
N=6
I%=7.2
PV=0
PMT=-9984
FV=71779.38791
P/Y=1
C/Y=1
PMT: [ ] BEGIN
```

FV = \$71 779.39

(b) Semimonthly deposit = $9984 \div 2 = \$4992.00$

```
N=12
I%=7.2
PV=0
PMT=-4992
FV=73048.96173
P/Y=2
C/Y=1
PMT: [ ] BEGIN
```

FV = \$73 048.96

(c) Weekly deposit = $9984 \div 52 = \$192.00$

```
N=312
I%=7.2
PV=0
PMT=-192
FV=74283.82271
P/Y=52
C/Y=1
PMT: [ ] BEGIN
```

FV = \$74 283.82

(d) Daily deposit = $9984 \div 365 = \$27.35$

```
N=2190
I%=7.2
PV=0
PMT=-27.35
FV=74317.11998
P/Y=365
C/Y=1
PMT: [ ] BEGIN
```

FV = \$74 317.12

(e) The frequency of the deposits does have a significant effect on their savings plan. The greater the frequency of the payments, the greater the value of the savings. By going from yearly to daily deposits they can save an additional \$2537.73.

14. (a) (i)

```
N=72
I%=7.8
PV=0
PMT=-832
FV=75443.34767
P/Y=12
C/Y=1
PMT: [ ] BEGIN
```

FV = \$75 443.35

(ii)

```
N=72
I%=8.4
PV=0
PMT=-832
FV=76791.4981
P/Y=12
C/Y=1
PMT: [ ] BEGIN
```

FV = \$76 791.50

(iii)

```
N=72
I%=6
PV=0
PMT=-832
FV=71536.57609
P/Y=12
C/Y=1
PMT: [ ] BEGIN
```

FV = \$71 536.58

(b) The interest rate also has a significant effect on their savings. A change from 6%/a to 7.8%/a would enable them to save an additional \$3906.77.

15. (a) (i)

```
N=72
I%=7.2
PV=0
PMT=-915.2
FV=81530.21232
P/Y=12
C/Y=1
PMT: [ ] BEGIN
```

FV = \$81 530.21

(ii)

```
N=72
I%=7.2
PV=0
PMT=-748.8
FV=66706.53735
P/Y=12
C/Y=1
PMT: [ ] BEGIN
```

FV = \$66 706.54

(iii)

```
N=72
I%=7.2
PV=0
PMT=-800
FV=71267.66811
P/Y=12
C/Y=1
PMT: [ ] BEGIN
```

FV = \$71 267.67

(b) The amount of the deposit has a very significant effect on their savings plan. A decrease in their deposits by \$32 to \$800 would result in a decrease in savings of about \$2850.

16. (a)

```
N=72
I%=7.2
PV=0
PMT=-832
FV=74549.05116
P/Y=12
C/Y=1
PMT:END BEGIN
```

FV = \$74 549.05

(b)

```
N=60
I%=7.2
PV=0
PMT=-832
FV=59523.35678
P/Y=12
C/Y=1
PMT:END BEGIN
```

FV = \$59 523.36

(c) If they stop after 5 years, we can simply take the future value from (16b) and calculate the compounded interest for one year at 7.2%/a compounded yearly. The future value is therefore $59\,523.36 \times 1.072 = \$63\,809.04$.

(d) $n = 78$ since the term is 6.5×12

```
N=78
I%=7.2
PV=0
PMT=-832
FV=81805.34383
P/Y=12
C/Y=1
PMT:END BEGIN
```

FV = \$81 805.34

(e) The sooner they begin to save, the sooner their savings can accumulate compound interest. The longer time period they save, the more they can accumulate.

17. They should include the following key feature: Start saving now (do not wait). Make the deposits as large and as often as possible. Shop around for the greatest interest rates available. Save for as long as you can to accumulate a larger deposit. The compounding period is not as important, but look for the shortest compounding period if the interest rates are the same.

18. (a)

```
N=24
I%=20.99854691
PV=2390
PMT=-122.81
FV=0
P/Y=12
C/Y=1
PMT:END BEGIN
```

Annual rate of interest = 21%.

(b) Interest cost = Total cost – Amount borrowed
 $= \$122.81 \times 24 - \$2390$
 $= \$557.44$

19. 12%/a compounded monthly will produce 1%/month compounded monthly. This means that for the second and third months, interest will be earned on the 1% interest earned in the previous month. After 3 months, the initial deposit will grow by $(1.01)(1.01)(1.01)$, 1.030 301 times its value or 3.030 301% over 3 months while 12%/a compounded quarterly earns 3% over 3 months. Therefore, 12%/a

compounded monthly is a greater rate than 12%/a compounded quarterly.

20. (a) By trial and error, annual interest charge is 20.04% compounded monthly.

(b)

$$\left(\frac{0.2004}{12} + 1\right)^{12} - 1 = 1.21987 - 1$$

$$= 21.987\%$$

$$\approx 21.99\% \text{ compounded annually}$$

21. (a) No you should not. One possible example to illustrate that the payment does not double is shown below.

```
N=48
I%=10
PV=10000
PMT=-253.62583...
FV=0
P/Y=12
C/Y=1
PMT:END BEGIN
```

```
N=48
I%=20
PV=10000
PMT=-304.30362...
FV=0
P/Y=12
C/Y=1
PMT:END BEGIN
```

(b) Yes you should. One possible example to illustrate that the payment also doubles, when the amount borrowed doubles, is shown below. It is like taking two loans of equal value, therefore, the payment is twice as much.

```
N=48
I%=10
PV=8000
PMT=-202.9006...
FV=0
P/Y=12
C/Y=1
PMT:END BEGIN
```

```
N=48
I%=10
PV=16000
PMT=-405.801335
FV=0
P/Y=12
C/Y=1
PMT:END BEGIN
```

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(a)

```
N=18.00
I%=21.00
PV=2650.00
PMT=-172.90
FV=0.00
P/Y=12.00
C/Y=12.00
PMT:END BEGIN
```

Alysa's monthly payments are \$172.90.

(b) She will pay $\$172.90(18) - \$2650 = \$462.18$ in interest for the loan.

(c) She still owes \$1280.33 on the truck after her 10th payment.

(d) \$147.90 of this 10th payment goes towards the principal.

```
ΣInt(1,18,6)
-462.18
bal(10,6)
1280.33
ΣPrn(10,10,6)
-147.90
```

(e) Store the interest paid, principal paid, and outstanding balance in Y1, Y2, and Y3.

```
Plot1 Plot2 Plot3
Y1=ΣInt(X,X,6)
Y2=ΣPrn(X,X,6)
Y3=bal(X,6)
Y4=
Y5=
Y6=
Y7=
```