

The amortization of the loan is shown below.

X	Y ₁	Y ₂
1.00	-46.38	-126.5
2.00	-44.16	-128.7
3.00	-41.91	-131.0
4.00	-39.62	-133.3
5.00	-37.28	-135.6
6.00	-34.91	-138.0
7.00	-32.50	-140.4

X=7

X	Y ₂	Y ₃
1.00	-126.5	2523.5
2.00	-128.7	2384.7
3.00	-131.0	2263.7
4.00	-133.3	2150.5
5.00	-135.6	1994.8
6.00	-138.0	1856.8
7.00	-140.4	1744.3

Y₃=1716.453897

X	Y ₁	Y ₂
8.00	-30.04	-142.9
9.00	-27.54	-145.4
10.00	-24.99	-147.9
11.00	-22.41	-150.5
12.00	-19.77	-153.1
13.00	-17.09	-155.8
14.00	-14.37	-158.5

Y₂=-158.533307

X	Y ₂	Y ₃
8.00	-142.9	1573.6
9.00	-145.4	1428.2
10.00	-147.9	1280.3
11.00	-150.5	1129.8
12.00	-153.1	976.71
13.00	-155.8	820.90
14.00	-158.5	662.37

Y₃=662.366319

X	Y ₁	Y ₂
12.00	-19.77	-153.1
13.00	-17.09	-155.8
14.00	-14.37	-158.5
15.00	-11.59	-161.3
16.00	-8.77	-164.1
17.00	-5.90	-167.0
18.00	-2.97	-169.9

Y₂=-169.925356

X	Y ₂	Y ₃
12.00	-153.1	976.71
13.00	-155.8	820.90
14.00	-158.5	662.37
15.00	-161.3	501.06
16.00	-164.1	336.93
17.00	-167.0	169.93
18.00	-169.9	-7.77

Y₃=-6E-6

Practise, Apply, Solve 2.12 page 191

1. (a) $(1+i)^{12} = (1.05)^2$

$$\sqrt[12]{(1+i)^{12}} = \sqrt[12]{1.05^2}$$

$$1+i = 1.05^{\frac{2}{12}}$$

$$i = 1.05^{\frac{1}{6}} - 1$$

$$i = 0.008\ 164\ 85$$

(b) $(1+i)^{12} = (1.035)^2$

$$\sqrt[12]{(1+i)^{12}} = \sqrt[12]{1.035^2}$$

$$1+i = 1.035^{\frac{2}{12}}$$

$$i = 1.035^{\frac{1}{6}} - 1$$

$$i = 0.005\ 750\ 04$$

(c) $(1+i)^{24} = (1.04)^2$

$$\sqrt[24]{(1+i)^{24}} = \sqrt[24]{1.04^2}$$

$$1+i = 1.04^{\frac{2}{24}}$$

$$i = 1.04^{\frac{1}{12}} - 1$$

$$i = 0.003\ 273\ 74$$

(d) $(1+i)^{52} = (1.06)^2$

$$\sqrt[52]{(1+i)^{52}} = \sqrt[52]{1.06^2}$$

$$1+i = 1.06^{\frac{2}{52}}$$

$$i = 1.06^{\frac{1}{26}} - 1$$

$$i = 0.002\ 243\ 63$$

(e) $(1+i)^{365} = (1.055)^2$

$$\sqrt[365]{(1+i)^{365}} = \sqrt[365]{1.055^2}$$

$$1+i = 1.055^{\frac{2}{365}}$$

$$i = 0.000\ 293\ 42$$

(f) $(1+i)^{52} = (1.055)^2$

$$\sqrt[52]{(1+i)^{52}} = \sqrt[52]{1.055^2}$$

$$1+i = 1.055^{\frac{2}{52}}$$

$$i = 1.055^{\frac{1}{26}} - 1$$

$$i = 0.002\ 061\ 38$$

2. (a) $(1+i)^{12} = (1.035)^2$

$$i = 1.035^{\frac{2}{12}} - 1$$

$$i = 0.005\ 750\ 04 \text{ or } i = 0.575\ 004\%/a$$

(b) $(1+i)^{12} = (1.0425)^2$

$$i = 1.0425^{\frac{2}{12}} - 1$$

$$i = 0.006\ 961\ 06 \text{ or } i = 0.696\ 106\%/a$$

(c) $(1+i)^{12} = (1.0495)^2$

$$i = 1.0495^{\frac{2}{12}} - 1$$

$$i = 0.008\ 084\ 82 \text{ or } i = 0.808\ 482\%/a$$

(d) $(1+i)^{12} = (1.06)^2$

$$i = 1.06^{\frac{2}{12}} - 1$$

$$i = 0.009\ 758\ 79 \text{ or } i = 0.975\ 879\%/a$$

3. Answers will vary.

4. (a) \$646.18/month is equivalent to \$646.18 × 12 or \$7754.16 per year, while \$161.55/week is equivalent to \$161.55 × 52 or \$8400 per year.

(b) If \$646.18/month is equal to \$7754.16 per year, then the weekly equivalent payment is \$7754.16 ÷ 52 or \$149.12.

(c) By making weekly payments of \$149.12, the loan is repaid in 1040.5 weeks. By making monthly payments of \$646.18, the loan is repaid in 20 years or 1040 weeks.

Changing the payment frequency has little or no effect on the time needed to repay the loan.

N=1040.496526
I%=8.44
PV=75800
PMT=-149.12
FV=0
P/Y=52
C/Y=2
PMT:BEGIN

(d) The payment amount has a much greater effect on the repayment of a loan.

5. Answers will vary. *Example:* Interest on Canadian mortgages can be compounded at most semiannually.

However, payments are generally made monthly, weekly, or biweekly, which does not coincide with the interest paid.

6. (a) (i)

N=300
I%=8.8
PV=80000
PMT=-651.90235...
FV=0
P/Y=12
C/Y=2
PMT:BEGIN

Monthly mortgage payment = \$651.90

(ii)

```

N=240
I%=8.8
PV=80000
PMT=-701.50967...
FV=0
P/Y=12
C/Y=2
PMT: [ ] [ ] [ ] BEGIN

```

Monthly mortgage payment = \$701.51

(b) Savings = $\$651.90 \times 300 \text{ months} - \$701.51 \times 240 \text{ months}$
 $= \$195\,570 - \$168\,362.40$
 $= \$27\,207.60$

(c) Shorten the amortization period of your loan by increasing the payment amount.

7. (a) (i)

```

N=152.35
I%=8.00
PV=60000.00
PMT=-624.00
FV=0.00
P/Y=12.00
C/Y=2.00
PMT: [ ] [ ] [ ] BEGIN

```

Term of the mortgage = 152.35 months

(ii)

```

N=303.84
I%=8.00
PV=60000.00
PMT=-312.00
FV=0.00
P/Y=24.00
C/Y=2.00
PMT: [ ] [ ] [ ] BEGIN

```

Term of the mortgage = 303.84 payment periods or 151.92 months

(iii)

```

N=657.34
I%=8.00
PV=60000.00
PMT=-144.00
FV=0.00
P/Y=52.00
C/Y=2.00
PMT: [ ] [ ] [ ] BEGIN

```

Term of the mortgage = 657.34 payment periods or 151.69 months

(b) The frequency of his payments has very little effect on the time required to repay the mortgage.

8. (a)

```

N=576.07
I%=8.00
PV=60000.00
PMT=-156.00
FV=0.00
P/Y=52.00
C/Y=2.00
PMT: [ ] [ ] [ ] BEGIN

```

$576.07 \div 52 \times 12 = 132.94 \text{ months}$

(b) The answers will range between \$4788.60 to \$4789.81 depending on the method and accuracy used.

9. (a)

```

N=300
I%=8.6
PV=160000
PMT=-1282.9601...
FV=0
P/Y=12
C/Y=2
PMT: [ ] [ ] [ ] BEGIN

```

Monthly payment = \$1282.96

(b)

bal(12,2)	158049.86	bal(36,2)	155928.39
bal(24,2)	155928.39	bal(48,2)	153620.55
bal(36,2)	153620.55	bal(60,2)	151109.99
			148378.88

Balance at the end of year 1 = \$158 049.86, end of year 2 = \$155 928.39, end of year 3 = \$153 620.55, end of year 4 = \$151 109.99, end of year 5 = \$148 378.88

(c)

```

ΣPrn(1,60)
-11621.12
ΣInt(1,60)
-65356.48
ΣInt(1,300)
-224888.23

```

\$11 621.12 in principal has been repaid after 5 years.

\$65 356.48 in interest has been repaid after 5 years.

(d) Refer to screen capture in part (c). Over 25 years, the Tangs can expect to spend \$224 888.23. Their home will have cost them a total of \$427 888.23.

10. (a)

<pre> N=300 I%=7.8 PV=160000 PMT=-1200.7831... FV=0 P/Y=12 C/Y=2 PMT: [] [] [] BEGIN </pre>	<pre> ΣInt(1,300) -200236.75 </pre>
--	-------------------------------------

Monthly payments = \$1200.78

The total interest cost is \$200 236.75. By getting a lower rate of 7.8%/a they would save a total of \$24 651.48 ($\$224\,888.23 - \$200\,236.75$) in interest.

(b)

```

N=250.6552815
I%=7.8
PV=160000
PMT=-1282.9601...
FV=0
P/Y=12
C/Y=2
PMT: [ ] [ ] [ ] BEGIN

```

Their mortgage would be paid off in 251 months.

$300 - 251 = 49 \text{ months}$

They would repay the loan 49 months sooner or over 4 years sooner.

11. (a) Year 2 payment = $\$1282.96 \times 1.10 = \$1411.26$

Years 3 to 5 payments = $\$1411.26 \times 1.10 = \$1552.38$

(b)

N=12	N=171.5946533
I%=8.6	I%=8.6
PV=158049.86	PV=154327.75
PMT=-1411.26	PMT=-1552.38
FV=-154327.7499	FV=0
P/Y=12	P/Y=12
C/Y=2	C/Y=2
PMT: BEGIN	PMT: BEGIN

```
bal(12,6)
148518.0678
bal(24,6)
142198.0109
bal(36,6)
135322.7433
```

Balance for the end of year 1 = \$158 049.86. For the year 2 balance, we use $PV = \$158\,049.86$ and change the payment to \$1411.26 and calculate the year end balance after 12 months. Balance for the end of year 2 = \$154 327.75. For the year 3 balance, we use $PV = \$154\,327.75$ and change the payment to \$1552.38. Using these new values, we find the balance for year 3 in 12 months to be \$148 518.07, year 4's balance in 24 months to be \$142 198.01, and year 5's balance in 36 months to be \$135 322.74.

(c) Refer to the second screen capture from part (b). With the new payment of \$1552.38 after year 2, the Tangs will have paid off the remaining mortgage in 172 months. The house would be paid off in $(172 + 24)$ 196 months or 104 months sooner.

12. (a)

N=84
I%=6.8
PV=49200
PMT=-735.49981...
FV=0
P/Y=12
C/Y=2
PMT: BEGIN

Monthly payment = \$735.50

(b)

Pay. #	Pay.	Int. Paid (0.56%/a)	Princ. Paid	Out. Bal.
0				\$49 200.00
1	\$735.50	\$275.52	\$459.98	\$48 740.02
2	\$735.50	\$272.94	\$462.56	\$48 277.46
...	...	...	...	...
83	\$735.50	\$8.37	\$727.13	\$767.94
84	\$735.50	\$4.30	\$731.20	\$36.74

(c)

```
ΣPrn(12,12,6)
-489.684109
ΣInt(12,12,6)
-245.815703
```

(i) \$489.68 went to paying the principal.

(ii) \$245.82 went to paying the interest.

(d)

```
ΣPrn(50,50,6)
-605.172832
ΣInt(50,50,6)
-130.32698
```

(i) \$605.17 went to paying the principal.

(ii) \$130.33 went to paying the interest.

(e)

```
ΣInt(1,84,6)
-12581.98423
ΣPrn(1,84,6)
-49199.99998
49200+12581.98+1
2300
74081.98
```

Total cost = Purchase price + Total interest
= \$61 500 + \$12 581.98
= \$74 081.98

(f)

N=84	ΣPrn(1,84,6)
I%=6.8	-46125.00001
PV=46125	ΣInt(1,84,6)
PMT=-689.531074	-11795.61021
FV=0	46125+11795.61+1
P/Y=12	5375
C/Y=2	73295.61
PMT: BEGIN	

By borrowing \$46 125 she would pay \$689.53/month and \$11 795.61 in interest. The total cost of the cottage would be \$73 295.61 (\$61 500 + \$11 795.61).

13. Answers will vary. The following should be included: maximize down payment, maximize payments and increase them over the term if possible, get lowest interest rate, make lump sum payments, and increase the payment frequency.

14.

N=48	N=532.960999
I%=6	I%=8
PV=0	PV=132273.52
PMT=-440	PMT=-525
FV=23726.48437	FV=0
P/Y=12	P/Y=24
C/Y=1	C/Y=2
PMT: BEGIN	PMT: BEGIN

```
ΣInt(1,533,6)
-147531.0366
```

At the end of 4 years he will have saved \$23 726.48 for a down payment. He must finance \$132 273.52. It will take him about 533 payments or 266.5 months (22 years and 2.5 months) to repay the loan.

15.

N=66	N=402.8070312
I%=6	I%=8
PV=0	PV=121850.07
PMT=-440	PMT=-545
FV=34149.93308	FV=0
P/Y=12	P/Y=24
C/Y=1	C/Y=2
PMT: BEGIN	PMT: BEGIN

```
ΣInt(1,403,6)
-97679.90067
```

At the end of 5.5 years he will have saved \$34 149.93 for a down payment. He must finance \$121 850.07. It will take him 403 payments or 201.5 months, which is 65 fewer months, to repay the loan. It will cost him \$49 851.14 less in interest costs for the house.

16. (a)

```
N=240
I%=10
PV=80000
PMT=-761.33154...
FV=0
P/Y=12
C/Y=2
PMT: [ ] BEGIN
```

```
bal(36,3)
75495.392
```

The monthly payment of the term is \$761.33. The balance owing after the 3 year term is \$75 495.39.

(b)

```
N=204
I%=22
PV=75495.39
PMT=-1363.8479...
FV=0
P/Y=12
C/Y=2
PMT: [ ] BEGIN
```

The remaining term is 204 months. The new payment required is \$1363.85.

(c)

$$\frac{1363.85}{761.33} = 1.7914$$

The new payment is 1.7914 times the previous payment. The original payment had to be increased by 79.14% in order to maintain the 20-year amortization.

(d) Interest rates increased very dramatically. When people came to renew their mortgages, the new payment required to maintain the mortgage was beyond what they could afford. Therefore, they had to declare bankruptcy.

17. (a) Payback Option

```
N=300
I%=7.4
PV=116400
PMT=-844.23132...
FV=0
P/Y=12
C/Y=2
PMT: [ ] BEGIN
```

```
ΣInt(1,60,6)
-40753.78975
bal(60,6)
106499.9103
106499.91+40753.79
147253.7
```

Other option

```
N=300
I%=7
PV=120000
PMT=-840.4989...
FV=0
P/Y=12
C/Y=2
PMT: [ ] BEGIN
```

```
ΣInt(1,60,6)
-39683.44851
bal(60,6)
109253.5142
109253.51+39683.45
148936.96
```

We find the payments needed to amortize the loan over 25 years and then use these payments to find the balance outstanding and interest paid under each option after 5 years.
Cash back option: payments \$844.23

Interest paid after 5 years: \$40 753.79
Balance after 5 years: \$106 499.91
Total costs after 5 years: \$147 253.70

Select the cash-back option if your mortgage is \$120 000.

(b) competitor's option

18. Answers will vary.

19. The interest rate changed, the amount borrowed, the term (length) of the mortgage.

Chapter 2, Review and Practice, page 197

1. $t_n = a + (n - 1)d$; $t_n = n^{\text{th}}$ term, a = first term, d = common difference, n = number of terms in the sequence

2. A series is the sum of all of the terms in a sequence. To add the first n terms of an arithmetic series, use

$S_n = \frac{n}{2}[2a + (n - 1)d]$ where n is the position of the term in the series, a is the first term, and d is the common difference between each term in the series.

3. (a)

$$\begin{aligned} S_{17} &= \frac{17}{2}[2(32) + 16(11)] \\ &= \frac{17}{2}(240) \\ &= 2040 \end{aligned}$$

(b)

$$\begin{aligned} S_{23} &= \frac{23}{2}[2(19) + 22(-5)] \\ &= \frac{23}{2}(-72) \\ &= -828 \end{aligned}$$

(c) $t_n = a + (n - 1)d$

$$58 = -68 + (n - 1)(7)$$

$$58 = -68 + 7n - 7$$

$$133 = 7n$$

$$n = 19$$

$$\begin{aligned} S_{19} &= \frac{19}{2}[2(-68) + 18(7)] \\ &= \frac{19}{2}(-10) \\ &= -95 \end{aligned}$$

(d) $t_n = a + (n - 1)d$

$$\frac{9}{2} = \frac{1}{3} + (n - 1)\left(\frac{1}{6}\right)$$

$$\frac{9}{2} = \frac{1}{3} + \frac{1}{6}n - \frac{1}{6}$$

$$27 = 2 + n - 1$$

$$n = 26$$