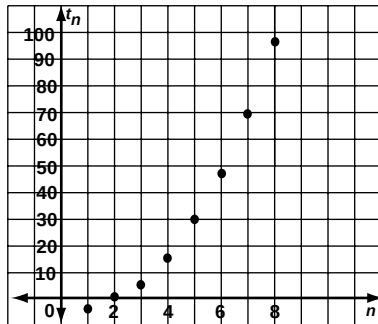


Cumulative Review Test 1

Financial Applications of Sequences and Series, page 205

1. (a) $t_1 = 2(1)^2 - 4(1) = -2$, $t_2 = 2(2)^2 - 4(2) = 8 - 8 = 0$,
 $t_3 = 2(3)^2 - 4(3) = 18 - 12 = 6$,
 $t_4 = 2(4)^2 - 4(4) = 32 - 16 = 16$
 (b) $t_{15} = 2(15)^2 - 4(15) = 450 - 60 = 390$
 (c) $t_5 = 2(5)^2 - 4(5) = 50 - 20 = 30$,
 $t_6 = 2(6)^2 - 4(6) = 72 - 24 = 48$,
 $t_7 = 2(7)^2 - 4(7) = 98 - 28 = 70$,
 $t_8 = 2(8)^2 - 4(8) = 128 - 32 = 96$



- (d) $880 = 2n^2 - 4n$
 $0 = 2n^2 - 4n - 880$
 $0 = n^2 - 2n - 440$
 $0 = (n + 20)(n - 22)$
 $n = 22$ ($n = -20$ is admissible)
2. (a) $t_0 = 50\,000$, $t_1 = 52\,000$, $t_2 = 54\,000$, $t_3 = 56\,000$,
 $t_4 = 58\,000$
 50 000, 52 000, 54 000, 56 000, 58 000
 (b) $t_n = a + (n - 1)d$
 $t_n = 50\,000 + 2000n$
 (c) $t_{10} = 50\,000 + 2000(9) = 68\,000$
 (d) $S_{20} = \frac{20}{2} [2(50\,000) + 19(2000)]$
 $= 10(100\,000 + 38\,000)$
 $= 10(138\,000)$
 $= 1\,380\,000$
3. (a) $-18, -54, -162, \dots$
 i. $t_n = -18(3)^{n-1}$
 ii. $t_{15} = -18(3)^{14} = -86\,093\,442$
 iii. $S_{25} = \frac{-18(3^{25} - 1)}{2} = -7.626 \times 10^{12}$
- (b) $30, 26, 22, \dots$
 i. $t_n = 30 + (n - 1)(-4) = 34 - 4n$
 ii. $t_{15} = 34 - 4(15) = -26$
 iii. $S_{25} = \frac{25}{2} [2(30) + (24)(-4)]$
 $= \frac{25}{2} [60 - 96]$
 $= 25(-18)$
 $= -450$

4. (a) $364 = -2 + (n - 1)6$
 $364 = -2 + 6n - 6$
 $364 = -8 + 6n$
 $372 = 6n$
 $n = 62$

(b) $\frac{3}{1024} = 3\left(\frac{1}{2}\right)^{n-1}$
 $\frac{1}{1024} = \left(\frac{1}{2}\right)^{n-1}$
 $\ln\left(\frac{1}{1024}\right) = \ln\left(\frac{1}{2}\right)^{n-1}$
 $\ln\left(\frac{1}{1024}\right) = (n - 1)\ln\left(\frac{1}{2}\right)$
 $(n - 1) = \frac{\ln\left(\frac{1}{1024}\right)}{\ln\left(\frac{1}{2}\right)}$
 $n = \frac{\ln\left(\frac{1}{1024}\right)}{\ln\left(\frac{1}{2}\right)} + 1$
 $n = 11$

5. (a) $i = \frac{0.06}{4} = 0.015$, $n = 4 \times 4 = 16$
 $A = P(1 + i)^n = 12\,000(1.015)^{16} = 15\,227.83$
 (b) $i = \frac{0.05}{2} = 0.025$, $n = 5 \times 2 = 10$, $P = 8000$
 $A = P(1 + i)^n$
 $8000 = P(1 + 0.025)^{10}$
 $P = \frac{8000}{(1.025)^{10}}$
 $= \$6249.59$

6. (a) $25^{\frac{3}{2}} = 125$
 (b) $-27^{\frac{-2}{3}} = \frac{1}{9}$
 (c) $20^{\frac{4}{5}} = 10.986$
 (d) $200(1.08)^{-10} = 92.64$

7. (a) $\frac{(25x^6y^4)^{\frac{1}{2}}(9x^4y^2)^{\frac{1}{2}}}{\left(-x^{\frac{1}{2}}y^{\frac{1}{2}}\right)^6}$
 $= \frac{(5x^3y^2)(3x^2y)}{-x^3y^3}$

$$\begin{aligned}
 &= \frac{15x^5y^3}{-x^3y^3} \\
 &= -15x^2 \\
 \text{(b)} \quad &\frac{(49x^{-8}y^2)^{-\frac{1}{2}}(27x^{-3}y^{12})^{\frac{2}{3}}}{\sqrt[4]{16x^8y^{-4}}} \\
 &= \frac{(7x^{-4}y)^{-1}(3x^{-1}y^4)^2}{2x^2y^{-1}} \\
 &= \frac{9x^{-2}y^8}{(7x^{-4}y)(2x^2y^{-1})} \\
 &= \frac{9y^8}{14}
 \end{aligned}$$

$$\begin{aligned}
 \text{8. (a)} \quad &2^{3x+2} = \frac{1}{32} \\
 &2^{3x+2} = 2^{-5} \\
 &3x+2 = -5 \\
 &3x = -7 \\
 &x = -\frac{7}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad &3^{2x^2+9x} = 243 \\
 &3^{2x^2+9x} = 3^5 \\
 &2x^2+9x = 5 \\
 &2x^2+9x-5 = 0 \\
 &(2x-1)(x+5) = 0 \\
 &x = \frac{1}{2}, x = -5
 \end{aligned}$$

$$\begin{aligned}
 \text{9. } i &= \frac{0.06}{12} = 0.005 \\
 A &= P(1+i)^n \\
 6545.42 &= 5000(1+0.005)^n \\
 \frac{6545.42}{5000} &= (1+0.005)^n \\
 1.309\,084 &= (1.005)^n \\
 \ln(1.309\,084) &= n\ln(1.005) \\
 n &= \frac{\ln(1.309\,084)}{\ln(1.005)} \\
 n &= 54 \text{ months} \\
 \therefore \frac{54}{12} &= 4.5 \text{ years}
 \end{aligned}$$

10. (a) Amortization Table:

A	B	C	D	E
Period	Amount Paid	Interest Paid	Principal Payment	New Balance
0	\$0.00	\$0.00	\$0.00	\$4800.00
1	\$220	\$60.00	\$160.00	\$4640.00
2	\$220	\$58.00	\$162.00	\$4478.00
3	\$220	\$55.98	\$164.02	\$4313.98
4	\$220	\$53.92	\$166.08	\$4147.90
5	\$220	\$51.85	\$168.15	\$3979.75
6	\$220	\$49.75	\$170.25	\$3809.50

(b)

	A	B	C	D	E
1	Pay.	Amount	Interest	Previously	Outstanding
2	#	Paid	Paid	Paid	Balance
3	0	0	0	0	\$4800
4	=A3+1	\$220.00	=E3*0.0125	=B4-C4	=E3+C4-B4

The values in the spreadsheet are the same as in part (a).

$$\text{(c)} \quad i = \frac{0.15}{12} = 0.0125, R = 220, PV = 4800$$

$$4800 = \frac{220(1 - (1 + 0.0125)^{-n})}{0.0125}$$

$$60 = 220(1 - (1.0125)^{-n})$$

$$1 - (1.0125)^{-n} = \frac{60}{220}$$

$$(1.0125)^{-n} = 1 - \frac{60}{220}$$

$$n = \frac{-\log\left(\frac{8}{11}\right)}{\log(1.0125)}$$

$$= 25.6$$

$$\approx 26 \text{ months}$$

Final Payment:

Amount owing after 25th payment:

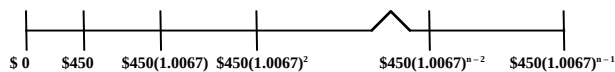
$$= 4800 - 220 \left(\frac{1 - (1 + 0.0125)^{-25}}{0.0125} \right) = \$101.40$$

$$\text{final payment} = 101.40(1.0125)^{26} = 140.06$$

$$\text{(d)} \quad 220 \times 25 + 140.06 = 5640.06$$

$$\text{(e)} \quad \text{Payment} = \frac{4800(1.0125)}{1 - (1 + 0.0125)^{-12}} = 433.24$$

11. (a) Time line diagram to represent annuity:



$$\text{(b)} \quad 10\,000 = 450(1.0067) + 450(1.0067)^2 + 450(1.0067)^3 + \dots + 450(1.0067)^n$$

$$\text{(c)} \quad S_n = \frac{a(r^n - 1)}{r - 1} \text{ where } S_n = 10\,000, a = 450, \text{ and}$$

$$r = 1 + \frac{0.08}{12} = 1.0067.$$

$$10\,000 = \frac{450(1.0067^n - 1)}{0.0067}$$

$$67 = 450(1.0067^n - 1)$$

$$0.1489 = 1.0067^n - 1$$

$$1.1489 = 1.0067^n$$

$$\ln(1.1489) = n\ln(1.0067)$$

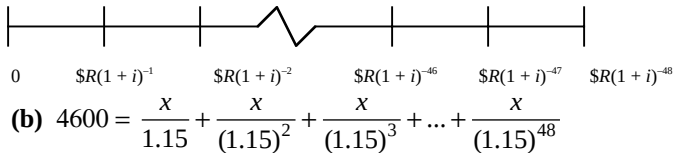
$$n = \frac{\ln(1.1489)}{\ln(1.0067)}$$

$$n = 21 \text{ months}$$

(d) Answers will vary. Depends on the length of the term.

12. (a) Time Line representing repayment of loan:

$$n = 4 \times 12 = 48 \text{ months}$$



$$(b) \quad 4600 = \frac{X}{1.15} + \frac{X}{(1.15)^2} + \frac{X}{(1.15)^3} + \dots + \frac{X}{(1.15)^{48}}$$

$$(c) \quad i = (1.15)^{\frac{1}{12}} - 1 = 0.0117$$

$$\text{payment amount} = \frac{4600(0.0117)}{1 - (1 + 0.0117)^{-48}} = 125.84$$

(d)



$$13. (a) \quad p = (1.038)^{\frac{2}{12}} - 1 = 0.006 \, 235$$

$$n = \frac{-\log \left(1 - \frac{120 \, 900(0.006 \, 235)}{1200} \right)}{\log (1 + 0.006 \, 235)} = 159.2$$

Therefore, approximately 160 months or 13 years, 4 months.

(b) 160 payments of \$1200 + final payment

final payment =

$$120 \, 900 - 1200 \times \frac{1 - (1 + 0.006 \, 235)^{-159}}{0.006 \, 235} = \$77.65$$

$$\text{Adjustment for time} = (77.65)(1 + p)^{160} = \$209.93$$

$$\text{Total} = \$1200(160) + \$209.93 = \$192 \, 209.93$$

(c) Payments = \$1800, $n = 88$ months

$$\begin{aligned} \text{final payment} &= 120 \, 900 - 1800 \times \frac{1 - (1 + 0.006 \, 235)^{-87}}{0.006 \, 235} \\ &= \$315.98 \end{aligned}$$

$$\text{Adjusted} = 315.98(1.006 \, 235)^{88} = \$546.05$$

$$\text{Total} = 87(1800) + 546.05 = \$157 \, 146.05$$

$$\text{Savings} = \$192 \, 209.93 - 157 \, 146.05 = \$35 \, 063.88$$

$$14U. \quad t_n = -3(t_{n-1}) + 8, \quad t_1 = -5,$$

$$t_2 = -3(t_1) + 8 = -3(-5) + 8 = 23$$

$$t_3 = -3(t_2) + 8 = -3(23) + 8 = -61$$

$$t_4 = -3(t_3) + 8 = -3(-61) + 8 = 191$$

$$t_5 = -3(t_4) + 8 = -3(191) + 8 = -565$$

$$t_6 = -3(t_5) + 8 = -3(-565) + 8 = 1703$$

15U. (a) t_n = medication remaining at each dose

$$t_n = 350 + 0.32(t_{n-1}), \quad t_1 = 350$$

(b) This is an infinite geometric series.

$$|r| = 0.32 < 1$$

$$S_{\infty} = \frac{t_1}{1 - r}$$

$$= \frac{350}{1 - 0.32}$$

$$= 514.7 \text{ mg}$$

(c) An infinite length of time.