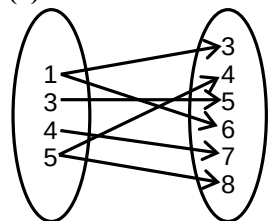


Chapter 3

Practise, Apply, Solve 3.2, page 234

1. (a) $D = \{1, 3, 4, 5\}$, $R = \{3, 4, 5, 6, 7, 8\}$

(b)



(c) This is not a function. In this relationship 1 maps onto 3 and 4. Also, 5 maps onto two different values of the range.

2. (a) i. $D = \{1, 3, 4, 7\}$, $R = \{1, 2\}$

ii. This is a function because each value of the domain is mapped onto one value of the range.

- (b) i. $D: \{1, 4, 6\}$, $R = \{1, 2, 3, 5\}$

ii. This is not a function because 1 maps onto both 2 and 3.

- (c) i. $D = \{0, 1, 2, 3\}$, $R = \{0, 1, 2, 3\}$

ii. This is a function because each value of the domain maps onto one value of the range.

- (d) i. $D = \{1, 2, 3\}$, $R = \{4, 5, 6\}$

ii. This is not a function because 1 maps onto both 4 and 5.

- (e) i. $D = \{1, 2, 3\}$, $R = \{4\}$

ii. This is a function because each value of the domain maps onto one value of the range.

- (f) i. $D = \{2\}$, $R = \{4, 5, 6\}$

ii. This is not a function because 2 maps onto 3 different values of the range.

3. (a) $\sqrt{25} = 5$

(b) No. Every input value that produces an output value produces only one.

(c) When we use the square root button on the calculator, the output must be consistent. For example, if we enter $\sqrt{25}$ in the calculator, we expect to get the same answer every time. It must be a function.

(d) No negative number can be used as input on the calculator.

- (e) $D = \{x \mid x \geq 0, x \in \mathbf{R}\}$

4. (a) $25 \rightarrow \pm 5$. Both 5 and -5 are valid square roots of 25.

(b) Every positive real number produces 2 different output values.

(c) This is not a function because there is at least one input value that has more than one output value.

(d) No negative numbers may be used as input.

5. i. $D = \{-2, 0, 2, 3, 5, 6\}$, $R = \{1, 2, 3, 4, 5\}$

ii. (a) $f(3) = 4$ (b) $f(5) = 2$ (c) $f(5 - 3) = f(2) = 5$

(d) $f(5) - f(3) = 2 - 4 = -2$

(iii) The function in (d) is not the same as (c) because of order of operations. $f(5 - 3)$ is not the same as $f(5) - f(3)$.

(iv) $f(2) = 5$ corresponds to (2, 5). 2 is the x -coordinate of the point and $f(2)$ is the y -coordinate of the point.

6. (a) $g(3) = 3 - 2(3) = 3 - 6 = -3$

(b) $g(0) = 3 - 2(0) = 3$

(c) $g(-2) = 3 - 2(-2) = 3 + 4 = 7$

(d) $2g(1) = 2[3 - 2(1)] = 2[3 - 2] = 2[1] = 2$

(e) $g(-2) - 3 = [3 - 2(-2)] - 3 = [3 + 4] - 3 = 7 - 3 = 4$

(f) $g(-2 - 3) = g(-5) = 3 - 2(-5) = 3 + 10 = 13$

(g) $g(a) = 3 - 2(a) = 3 - 2a$

7. (a) not a function: fails the vertical line test

(b) function: passes the vertical line test

(c) function: passes the vertical line test

(d) not a function: fails the vertical line test

(e) not a function: fails the vertical line test

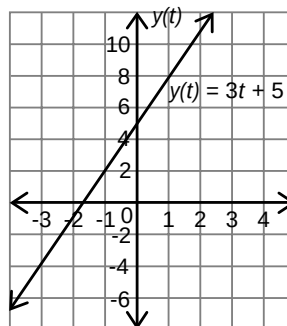
(f) not a function: fails the vertical line test

(g) function: passes the vertical line test

(h) not a function: fails the vertical line test

(i) not a function: fails the vertical line test (In a region, there are an infinite number of points above each other.)

8. i.



- ii. (a) $g(0) = 3(0) + 5 = 5$

(b) $g(1) = 3(1) + 5 = 3 + 5 = 8$

(c) $g(2) = 3(2) + 5 = 6 + 5 = 11$

(d) $g(3) = 3(3) + 5 = 9 + 5 = 14$

(e) $g(1) - g(0) = 8 - 5 = 3$

(f) $g(2) - g(1) = 11 - 8 = 3$

(g) $g(3) - g(2) = 14 - 11 = 3$

(h) $g(1001) - g(1000)$
 $= [3(1001) + 5] - [3(1000) + 5]$
 $= 3008 - 3005$
 $= 3$

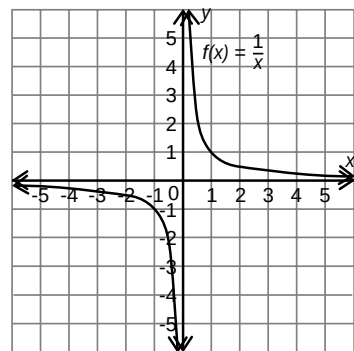
(i) $g(a + 1) - g(a)$
 $= [3(a + 1) + 5] - [3(a) + 5]$
 $= (3a + 8) - (3a + 5)$
 $= 3a + 8 - 3a - 5$
 $= 3$

(j) $g(4) = 3(4) + 5 = 17$
 $\frac{g(4) - g(0)}{4 - 0} = \frac{17 - 5}{4 - 0}$
 $= \frac{12}{4}$
 $= 3$

(iii) The answers for (e), (f) and (g) are all first differences. Part (j) is the slope of the straight line between $(0, g(0))$ and $(4, g(4))$. Since it is a linear relation, the slope is equal to the first differences (answers for parts (e), (f), (g), (h), and (i)).

9. (a) i. $y = \frac{1}{x}$

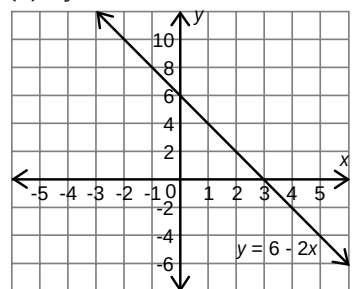
x	y
5	0.2
4	0.25
2	0.5
1	1
0.5	2
0.25	4
0.2	5
-5	-0.2



ii. $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$, $R = \{y \mid y \neq 0, y \in \mathbf{R}\}$

iii. It is a function because it passes the vertical line test. Every value of x corresponds to one value of y .

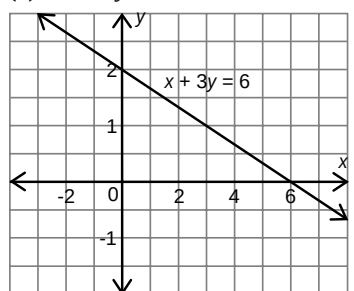
(b) i. $y = 6 - 2x$



ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \in \mathbf{R}\}$

iii. It is a function because it passes the vertical line test. Every value of x corresponds to one value of y .

(c) i. $x + 3y = 6$

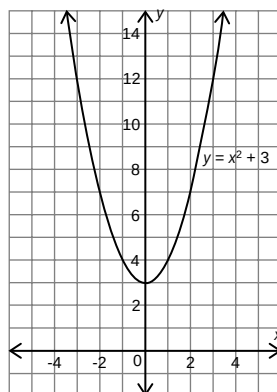


ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \in \mathbf{R}\}$

iii. It is a function because it passes the vertical line test. Every value of x corresponds to one value of y .

(d) i. $y = x^2 + 3$

x	y
0	3
± 1	4
± 2	7
± 3	12



ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \geq 3, y \in \mathbf{R}\}$

iii. It is a function because it passes the vertical line test. Every x value corresponds to one value of y .

10. There may be several different brands of tires available in the size requested, so it is quite possible to get different prices if Bill only gave the clerk the size of the tire he wanted. This is not a function because one tire size can generate more than one price. In order to make this process a function, the input would have to be more specific. For example, the stock number of the tire size, the brand of the tire and so on could be identified. To avoid the possibility of a price change during the day, the input may also have to include the date and time.

11.

x	y	1 st differences
1	5	
		2
2	7	
		2
3	9	
		2
4	11	
		2
5	13	

i. Since the first differences are constant, this is a linear relationship. The first difference is the slope of the line. Thus, the slope of the line is 2. Extrapolating, we see that when $x = 0$, $y = 3$, so the y -intercept is 3. Thus, the equation is: $y = 2x + 3$.

ii. $g(x) = 2x + 3$

iii. (a) $g(5) = 2(5) + 3 = 10 + 3 = 13$

(b) $g(5 - 3) = g(2) = 2(2) + 3 = 4 + 3 = 7$

(c) $g(5) - g(3) = 13 - [2(3) + 3] = 13 - [6 + 3] = 13 - 9 = 4$

(d) $2g(3) - 5 = 2(9) - 5 = 18 - 5 = 13$

12. i. $f(s) = s^2 - 6s + 9$

S	f(s)
0	9
1	4
2	1
3	0
4	1

ii. From the table:

(a) $f(0) = 9$

(b) $f(1) = 4$

(c) $f(2) = 1$

(d) $f(3) = 0$

(e) $f(4) = 1$

(f) $f(1) - f(0) = 4 - 9 = -5$

(g) $f(2) - f(1) = 1 - 4 = -3$

(h) $f(3) - f(2) = 0 - 1 = -1$

(i) $f(4) - f(3) = 1 - 0 = 1$

(j) $[f(2) - f(1)] - [f(1) - f(0)]$

$= -3 - (-5)$

$= 2$

(k) $[f(3) - f(2)] - [f(2) - f(1)]$

$= -1 - (-3)$

$= 2$

(l) $[f(4) - f(3)] - [f(3) - f(2)]$

$= 1 - (-1)$

$= 2$

(m) $[f(1002) - f(1001)] - [f(1001) - f(1000)]$

$= [998\ 001] - 996\ 004] - [996\ 004 - 994\ 009]$

$= 1997 - 1995$

$= 2$

(iii) The answers to (f), (g), (h) and (i) are called first differences. The answers to (j), (k), and (l) are called second differences.

13. (a) $f(-2) = 2(x - 3)^2 - 1$

$= 2(-2 - 3)^2 - 1$

$= 2(-5)^2 - 1$

$= 49$

(b) $f(-2)$ is the y-coordinate of the point on the graph with x-coordinate -2.

(c) $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \geq -1, y \in \mathbf{R}\}$, since the vertex is at $(3, -1)$ and the parabola opens upward.

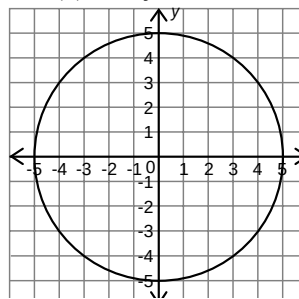
(d) From the graph of f , we can see that it passes the vertical line test, so it must be a function.

(e) From its equation, we can see that every value of x will produce only one possible value for $f(x)$ so it must be a function.

14. (a) No. Every value of the independent variable, x , produces a unique value for the dependent variable, y .

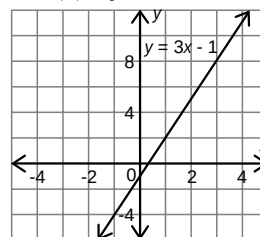
(b) This relation must be a function because of the answer to part (a). Its graph is a parabola which opens upward, so it will also pass the vertical line test.

15. (a) $x^2 + y^2 = 25$



(b) This relation is not a function. It fails the vertical line test for all values of x between -5 and 5.

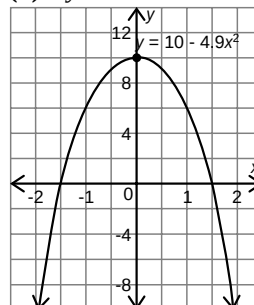
16. (a) i. $y = 3x - 1$



ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \in \mathbf{R}\}$

iii. It is a function because it passes the vertical line test. Every value of x corresponds to one value of y .

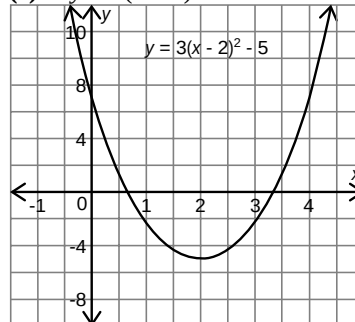
(b) i. $y = 10 - 4.9x^2$



ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \leq 10, y \in \mathbf{R}\}$

iii. It is a function because it passes the vertical line test. Every value of x corresponds to one value of y .

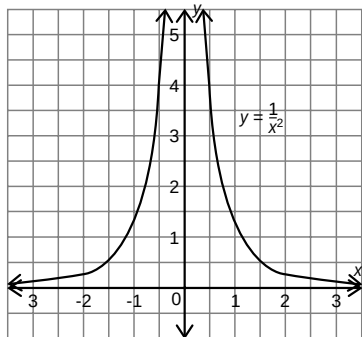
(c) i. $y = 3(x - 2)^2 - 5$



ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \geq -5, y \in \mathbf{R}\}$

iii. It is a function because it passes the vertical line test. Every value of x corresponds to one value of y .

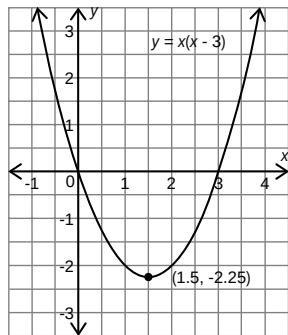
(d) i. $y = \frac{1}{x^2}$



ii. $D = \{x \mid x \neq 0, x \in \mathbf{R}\}$, $R = \{y \mid y > 0, y \in \mathbf{R}\}$

iii. It is a function because it passes the vertical line test. Every value of x corresponds to one value of y .

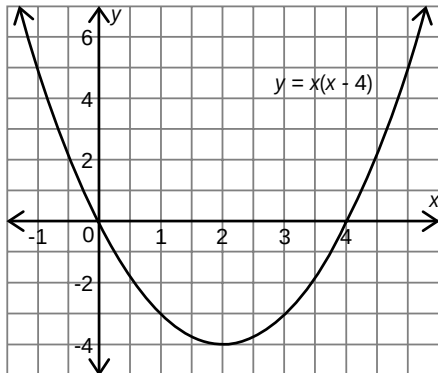
(e) i. $x^2 - y = 3x$



ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \geq -2.25, y \in \mathbf{R}\}$

iii. It is a function because it passes the vertical line test. Every value of x corresponds to one value of y .

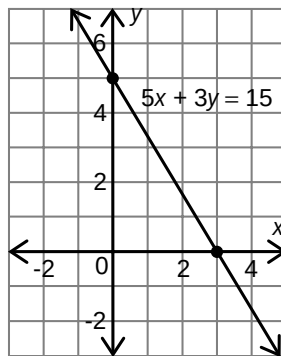
(f) i. $y = x(x - 4)$



ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \geq -4, y \in \mathbf{R}\}$

iii. It is a function because it passes the vertical line test. Every value of x corresponds to one value of y .

(g) i. $5x + 3y = 15$



ii. $D = \{x \mid x \in \mathbf{R}\}$, $R = \{y \mid y \in \mathbf{R}\}$

iii. It is a function because it passes the vertical line test. Every value of x corresponds to one value of y .

17. $y = \sqrt{x-1} + 2$: Since we cannot take the square root of a negative number in $\mathbf{R}$, we must have $x - 1 \geq 0$ or $x \geq 1$. Thus, the domain of the function is: $\{x \mid x \geq 1, x \in \mathbf{R}\}$. Also, since $\sqrt{x-1}$ will always be at least 0, if $\sqrt{x-1} \geq 0$ then $\sqrt{x-1} + 2 \geq 2$, and thus the range of the function is: $\{y \mid y \geq 2, y \in \mathbf{R}\}$.

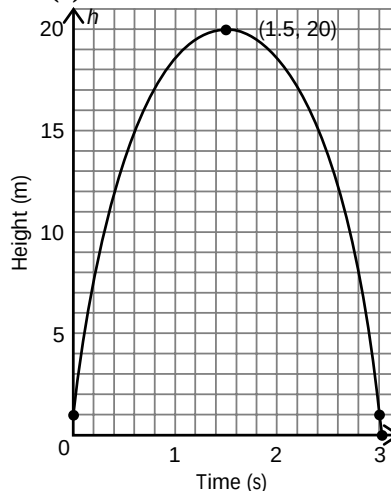
18. (a) If this relationship was not a function, the customers would become upset if they found out that someone else received a better price for exactly the same number of meals served.

(b) The domain appears to be: $\{x \mid 0 \leq x \leq 200, x \in \mathbf{R}\}$. The range appears to be: $\{y \mid 1250 \leq y \leq 3700, y \in \mathbf{R}\}$.

(c) The domain has an upper limit because the room has a maximum seating capacity.

(d) The graph is reasonable because the owners of the hall would want to receive a minimum total payment. If fewer than 50 people attend, they will still charge \$1250 because they could rent the room to a large group and make more money. They do not want to reserve a room and then discover only 10 people are coming and only receive payment for those 10 people.

19. (a)

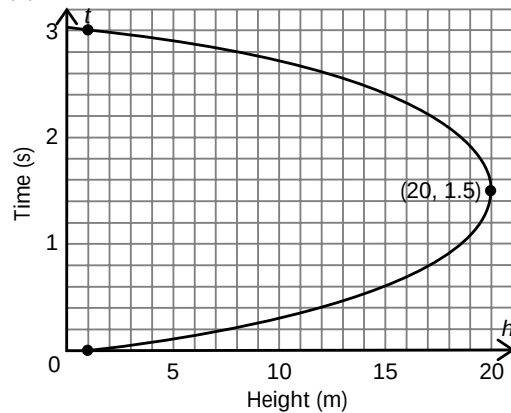


(b) The ball is probably thrown from about 1.6 m above the ground. We know that the shape will approximate a parabola, and because of its symmetry, we expect the ball to be 1.6 m

above the ground after 3 s. Shortly thereafter, it will hit the ground. Thus, a reasonable domain is: $\{t \mid 0 \leq t \leq 3.1, t \in \mathbf{R}\}$. The ball reaches a maximum height of 20 m, and eventually hits the ground, so the range is $\{h \mid 0 \leq h \leq 20, h \in \mathbf{R}\}$.

(c) This relation is a function because every value of t corresponds to a unique value of h .

(d)

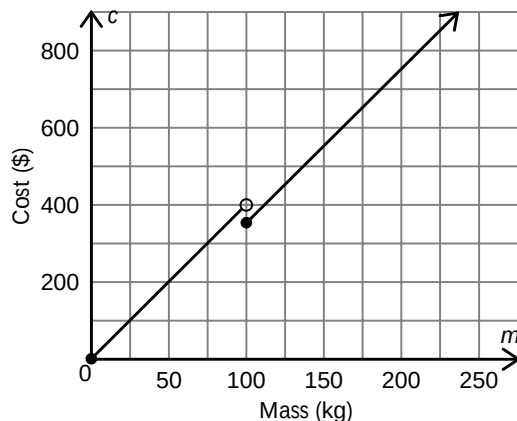


(e) This relationship is not a function because each height between 1.6 m and 20 m corresponds to two different times. Also, its graph fails the vertical line test.

20. (a) This relationship must be a function to ensure that every quantity ordered has a unique and reliable price.

(b) From the information given, the size of the order is not limited, but it makes no sense to have an order of 0 kg (or less), so $D = \{x \mid x \geq 0, x \in \mathbf{R}\}$. Similarly, the price will be any value greater than 0, so $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$.

(c)



(d) The problem with their pricing structure is that someone who orders 99 kg pays almost \$50 more than someone who orders 100 kg. It might be better to change \$4/kg for the first 100 kg and then \$3.50/kg for the part of the order over 100 kg.

21. (a) If we graph all 3 plans, we can see that plans B and C have the same weekly payment of \$600 when \$8000 worth of clothing is sold. It is reasonable to expect that the owner expects some of her employees to sell more than \$8000 in order to take advantage of plan C. We could investigate sales at stores to find out what is reasonable, but based only on the information given, a reasonable estimate for the domain is:

$\{s \mid 0 \leq s \leq 16000, s \in \mathbf{R}\}$. If we assume that the owner will pay each employee a minimum of \$300/week, then a reasonable estimate for the range of this relation is: $\{p \mid 300 \leq p \leq 1200, p \in \mathbf{R}\}$. (Since the pay would be \$1200 on plan C if \$16 000 worth of clothing is sold.)

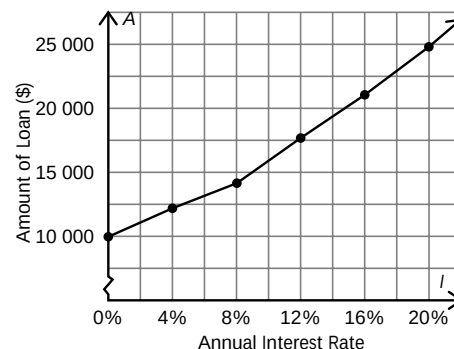
(b) This relation should be a function to ensure fairness. Two employees with the same sales should receive the same pay.

(c) A reasonable combination of the plans would be as follows: If sales are less than \$2000, then the weekly pay should be \$300. If sales are between \$2000 and \$8000, then the weekly pay should be based on plan B: a payment of \$200 plus 5% commission. If sales exceed \$8000, then a 7.5% sales commission should be paid. This combination allows the employees to earn the most possible under each plan, provides an incentive for the employees to sell more, and provides security for the employees when sales are not good. By choosing the point of intersection on the graph, the owner ensures that there is no “jump” or discrepancy. Once a minimum sales target has been reached it is important that employees see a steady increase in pay with greater sales to encourage more sales.

22. (a) $A = \$10000(1 + i)^5$

To graph make a table of values:

i	A
0	\$10 000.00
0.04	\$12 166.53
0.08	\$14 693.28
0.12	\$17 623.42
0.16	\$21 003.42
0.2	\$24 883.20



(b) A reasonable domain would be: $\{i \mid 0 \leq i \leq 0.2, i \in \mathbf{R}\}$.

This is reasonable because even most credit cards do not charge over 20% annual interest rate. A reasonable rate would be: $\{A \mid 10\,000 \leq A \leq 25\,000, A \in \mathbf{R}\}$.

23. (a) Since age is the independent variable, if two members of the family have the same age and different foot lengths, then the relationship will not be a function. However, if they are all of differing ages, the relationship between foot length and the age of family members will be a function.

(b) Since some of Sara’s friends are likely to be her age, and they are likely to have feet of differing lengths, then this relationship will not be a function.

(c) The line of best fit for the data will be a function. The model predicts a unique foot length for each age.

(d) In general, foot length is not a function of age. A person's age does not allow us to predict a unique foot length. For example, a 14 year old may have feet that grow considerably during the year that they are 14.

24.

x	y
0	3
1	4
2	5

x	y
0	3
0	4
2	5

The given tables of values show a function on the left and a non-function on the right. In the first relation, each value of x corresponds to a unique value of y . In the second relation, 0 corresponds to both 3 and 4, so it is not a function.

25. (a) $1 \rightarrow \boxed{\times 2} \rightarrow 2 \rightarrow (1) \rightarrow \boxed{\text{Add}} \rightarrow 3 \rightarrow \boxed{\times 2} \rightarrow 6 \rightarrow (0) \rightarrow \boxed{\text{Add}} \rightarrow 6 \rightarrow \boxed{\times 2} \rightarrow 12 \rightarrow (1) \rightarrow \boxed{\text{Add}} \rightarrow 13$
Therefore, 1101 converts to 13.

(b) This is a function because each binary number will be converted to a unique decimal number.

(c) i. $1 \rightarrow \boxed{\text{last?}} \rightarrow \text{no} \rightarrow \boxed{\times 2} \rightarrow 2 \rightarrow (0) \rightarrow \boxed{\text{Add}} \rightarrow 2 \rightarrow \boxed{\text{last?}} \rightarrow \text{no} \rightarrow \boxed{\times 2} \rightarrow 4 \rightarrow (1) \rightarrow \boxed{\text{Add}} \rightarrow 5 \rightarrow \boxed{\text{last?}} \rightarrow \text{no} \rightarrow \boxed{\times 2} \rightarrow 10 \rightarrow (0) \rightarrow \boxed{\text{Add}} \rightarrow 10 \rightarrow \boxed{\text{last?}} \rightarrow \text{no} \rightarrow \boxed{\times 2} \rightarrow 20 \rightarrow (1) \rightarrow \boxed{\text{Add}} \rightarrow 21 \rightarrow \boxed{\text{last?}} \rightarrow \text{yes} \rightarrow 21$

Therefore, 10101 converts to 21 in decimal.

ii. $1 \rightarrow \boxed{\text{last?}} \rightarrow \text{no} \rightarrow \boxed{\times 2} \rightarrow 2 \rightarrow (1) \rightarrow \boxed{\text{Add}} \rightarrow 3 \rightarrow \boxed{\text{last?}} \rightarrow \text{yes} \rightarrow 3$

Therefore, 11 converts to 3 in decimal.

iii. $1 \rightarrow \boxed{\text{last?}} \rightarrow \text{no} \rightarrow \boxed{\times 2} \rightarrow 2 \rightarrow (0) \rightarrow \boxed{\text{Add}} \rightarrow 2 \rightarrow \boxed{\text{last?}} \rightarrow \text{no} \rightarrow \boxed{\times 2} \rightarrow 4 \rightarrow (0) \rightarrow \boxed{\text{Add}} \rightarrow 4 \rightarrow \boxed{\text{last?}} \rightarrow \text{no} \rightarrow \boxed{\times 2} \rightarrow 8 \rightarrow (0) \rightarrow \boxed{\text{Add}} \rightarrow 8 \rightarrow \boxed{\text{last?}} \rightarrow \text{yes} \rightarrow 8$

Therefore, 1000 converts to 8 in decimal.

(d) The largest binary number we can get is 1111 with 4 digits. This converts to 15 in decimal. The smallest would be 0. Thus, the range would be: $\{y \mid y \leq 15, y \in \mathbf{W}\}$.

(e) Keyboards have a different number of keys.

Combinations of keys (e.g. shift + another key) produce even more symbols. It is reasonable that there is approximately 200 symbols to represent with binary numbers. $1111 \rightarrow 15$, $11111 \rightarrow 31$, $111111 \rightarrow 63$, $1111111 \rightarrow 127$, and $11111111 \rightarrow 255$. We probably need 8 bits to represent all the letters and symbols on a keyboard with binary numbers.

26. (a) Answers will vary. Possible numbers include:

0.405 807 641 8

0.733 812 311 2

0.043 991 987 5

0.339 362 525

0.995 466 341 1

(b) It appears that **rand** does not produce a unique output each time it is pressed. If you want “random” numbers, perhaps to play a game, we would not want a predictable output.

(c) 0.943 559 740 25

0.908 318 861

0.146 687 829 2

0.514 701 950 5

0.405 809 641 8

These values are the same for everyone.

(d) 0.943 597 402 5

0.908 318 861

0.146 687 829 2

0.514 701 950 5

0.405 809 641 8

This list is exactly the same as before.

(e)

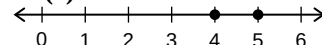
0.4	0.8	1.2
0.943 597 402 5	0.943 597 402 5	0.745 560 772 8
0.908 318 861	0.908 318 861	0.855 900 597 1
0.146 687 829 2	0.146 687 829 2	0.225 360 061 7
1.6	2.0	2.4
0.745 560 772 8	0.491 121 545 1	0.491 121 545 1
0.855 900 597 1	0.711 801 117 9	0.711 801 117 9
0.225 360 061 7	0.450 720 047 5	0.450 720 047 5
2.8		
0.491 121 545 1		
0.711 801 117 9		
0.450 720 047 5		

(f) It appears that the calculator truncates the number we store to **rand**, and then generates a list of numbers using that seed number. The “random numbers” outputted depends on the seed number and the number of times you have chosen **rand**.

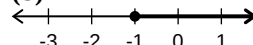
(g) **rand** is a function of the seed and the number of times you have selected **rand**. For example, knowing that 0 was stored in **rand**, and **rand** was selected 3 times always produces the answer: 0.146 687 829 2. These are not really random numbers - they just appear to be random. If we perform an experiment or design a game using random numbers from the calculator, the results will not be random, but actually quite predictable.

Practise, Apply, Solve 3.3, page 245

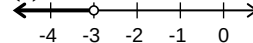
1. (a)



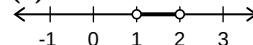
(b)



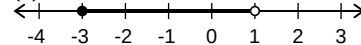
(c)



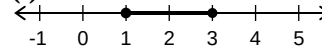
(d)



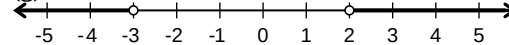
(e)



(f)



(g)



(h)

