

12. Since the vertex is at A(1, 2), the equation is of the form:

$$f(x) = a(x - 1)^2 + 2$$

$$f(5) = a(5 - 1)^2 + 2 = 16a + 2$$

We want (5, $f(5)$) to fall between B(5, 10) and C(5, 34):

$$10 < f(5) < 34$$

$$10 < 16a + 2 < 34$$

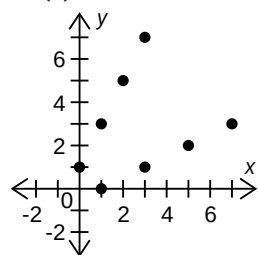
$$8 < 16a < 32$$

$$\frac{1}{2} < a < 2$$

Thus, the coefficient of x^2 in the equation of the parabola must lie between $\frac{1}{2}$ and 2.

Practise, Apply, Solve 3.4, page 255

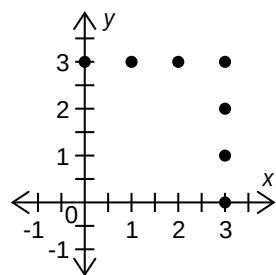
1. (a) i.



inverse: $\{(1, 0), (3, 1), (5, 2), (7, 3)\}$

ii. Both the relationship and its inverse are functions because each element of the domain corresponds to a unique value of the range.

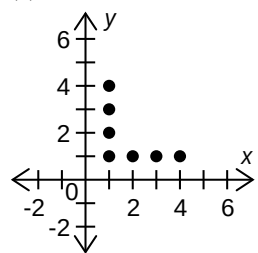
(b) i.



inverse: $\{(3, 0), (3, 1), (3, 2), (3, 3)\}$

ii. The original relationship is a function because it passes the vertical line test, but the inverse is not a function. 3 is mapped onto more than one value.

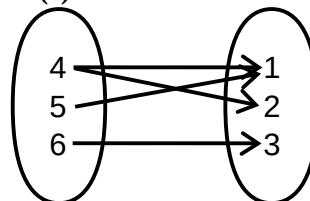
(c) i.



inverse: $\{(1, 1), (2, 1), (3, 1), (4, 1)\}$

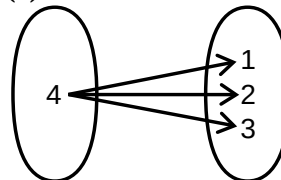
ii. The original relationship is not a function because 1 is mapped onto more than one value. The inverse is a function because each x -coordinate corresponds to a unique y -coordinate.

2. (a) i.



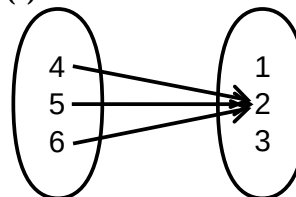
ii. The inverse is not a function because 4 maps onto 1 and 2.

(b) i.



ii. The inverse is not a function because 4 maps onto 1, 2, and 3.

(c) i.



ii. The inverse is a function because each element of the domain corresponds to one value in the range.

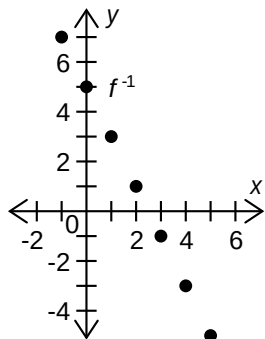
3. (a) f:

x	y	first differences
-5	5	
		-1
-3	4	
		-1
-1	3	
		-1
1	2	
		-1
3	1	
		-1
5	0	
		-1
7	-1	

(b) f^{-1} :

x	y	first differences
-1	7	
		-2
0	5	
		-2
1	3	
		-2
2	1	
		-2
3	-1	
		-2
4	-3	
		-2
5	-5	

(c) f^{-1} :



(d) $\text{slope} = \frac{\Delta y}{\Delta x} = -\frac{1}{2}$

(e) $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = -2$

(f) The y-intercept of the line containing the points in f is 2.5.

Thus, the equation of the line is: $y = -\frac{1}{2}x + 2.5$.

$f(x) = -0.5x + 2.5, x \in \{-5, -3, -1, 1, 3, 5, 7\}$.

(g) The y-intercept of the line containing the points in f^{-1} is 5.

Thus, the equation of the line is: $y = -2x + 5$.

$f^{-1}(x) = -2x + 5, \{x \mid -1 \leq x \leq 5, x \in \mathbf{I}\}$.

(h) The slope of the line through the points in f is the reciprocal of the slope of the line through the points in f^{-1} .

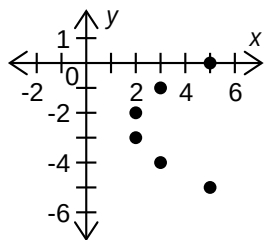
The 1st differences would normally also be reciprocals of each other, but the x -coordinates in f increase by 2 each time. To

find the inverse of $f(x) = -0.5x + 2.5$, we could use inverse

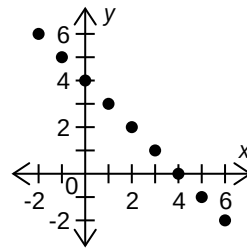
operations in the opposite order, so:

$f^{-1}(x) = (x - 2.5)(-2) = -2x + 5$.

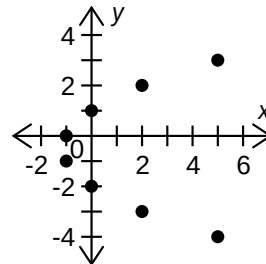
4. (a)



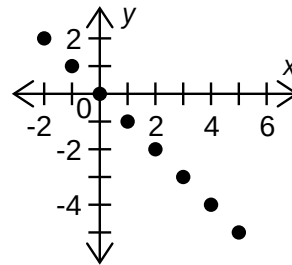
(b)



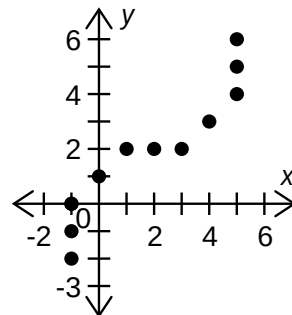
(c)



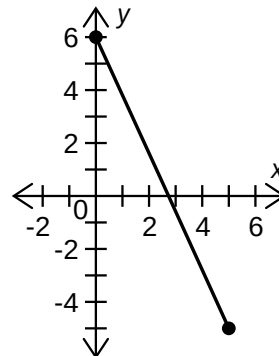
(d)



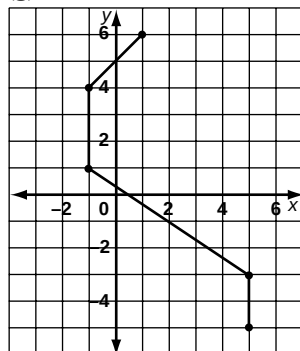
(e)



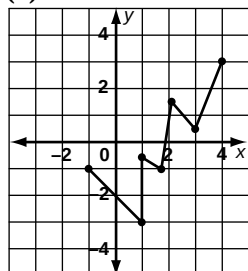
(f)



(g)



(h)



5. (a) no common points

(b) All of the points are common because the inverse is the same as the original relation.

(c) $(-1, -1)$ and $(2, 2)$ are common points. When the point is on the line $y = x$, it does not change when the coordinates are interchanged for the inverse relation.

(d) $(-2, 2)$, $(-1, 1)$, $(0, 0)$, $(1, -1)$, and $(2, -2)$ are the common points. When $(-2, 2)$ is reflected in the line $y = x$, to find the inverse relation, the result is $(2, -2)$ which is a point of the original function. Similarly for $(-1, 1)$ and $(0, 0)$.

(e) $(-1, -1)$, $(2, 2)$, and $(5, 5)$ are common points. When the point is on the line $y = x$, it does not change when the coordinates are interchanged for the inverse relation.

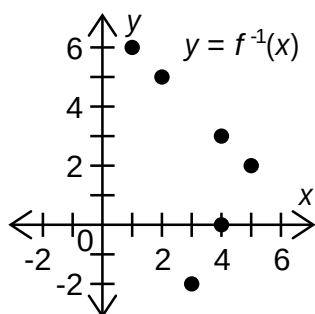
(f) $\left(\frac{15}{8}, \frac{15}{8}\right)$ is the common point. To find this point

accurately, determine the equation of the given line segment and find its intersection with the line $y = x$.

(g) The relation and its inverse intersect at $(2, -1)$, $(-1, 2)$ and $\left(\frac{1}{3}, 0\right)$.

(h) $(-1, -1)$ is a common point. When the point is on the line $y = x$, it does not change when the coordinates are interchanged for the inverse relation.

6. i.



ii. $D = \{1, 2, 3, 4, 5\}$, $R = \{-2, 0, 2, 3, 5, 6\}$

iii. (a) 4 (b) -2 (c) 5 (d) 5 (e) 2 (f) 0 or 3

(iv) f^{-1} is not a function because 4 maps onto 0 and 3. It fails the vertical line test.

(v) Since $f^{-1}(1) = 6$, $(1, 6)$ is the corresponding point to $f^{-1}(6)$ on the graph of f^{-1} . The number 1 is the x-coordinate of that point and $f^{-1}(1)$ represents the y-coordinate of that point.

7. (a) $g(0) = 3(0) - 2 = -2$

(b) $g^{-1}(-2) = 0$ (since $g(0) = -2$)

(c) $g(5) = 3(5) - 2 = 15 - 2 = 13$

(d) $g^{-1}(13) = 5$ (since $g(5) = 13$)

(e) $g: t \rightarrow \boxed{\times 3} \rightarrow (3t) \rightarrow \boxed{\text{subtract } 2} \rightarrow (3t - 2) \rightarrow g(t)$

$g^{-1}: g^{-1}(t) < \left(\frac{t+2}{3}\right) < \boxed{\div 3} < (t+2) < \boxed{\text{Add } 2} < t$

$$g^{-1}(t) = \frac{t+2}{3}$$

$$(f) \quad g^{-1}(0) = \frac{0+2}{3} = \frac{2}{3}$$

$$(g) \quad g^{-1}(5) = \frac{5+2}{3} = \frac{7}{3}$$

$$(h) \quad g^{-1}(a) = \frac{a+2}{3}$$

$$(i) \quad g^{-1}(x-2) = \frac{(x-2)+2}{3} = \frac{x}{3}$$

$$(j) \quad g^{-1}(3a-2) = \frac{(3a-2)+2}{3} = \frac{3a}{3} = a$$

$$(k) \quad 3g^{-1}(t) - 2 = 3\left(\frac{t+2}{3}\right) = t+2$$

$$(l) \quad 3g^{-1}(t) - 2 = 3\left(\frac{t+2}{3}\right) - 2 = t+2-2 = t$$

(m)

$$\begin{aligned} g^{-1}(8) - g^{-1}(7) &= \left(\frac{8+2}{3}\right) - \left(\frac{7+2}{3}\right) \\ &= \frac{10}{3} - \frac{9}{3} \\ &= \frac{1}{3} \end{aligned}$$

(n)

$$\begin{aligned} g^{-1}(14) - g^{-1}(13) &= \left(\frac{14+2}{3}\right) - \left(\frac{13+2}{3}\right) \\ &= \frac{16}{3} - \frac{15}{3} \\ &= \frac{1}{3} \end{aligned}$$

(o)

$$\begin{aligned} g^{-1}(a+1) - g^{-1}(a) &= \left(\frac{a+1+2}{3}\right) - \left(\frac{a+2}{3}\right) \\ &= \frac{a+3}{3} - \frac{a+2}{3} \\ &= \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{(p)} \quad g(13) &= 3(13) - 2 = 39 - 2 = 37 \\ g(7) &= 3(7) - 2 = 21 - 2 = 19 \end{aligned}$$

$$\begin{aligned} \frac{g(13) - g(7)}{13 - 7} &= \frac{37 - 19}{6} \\ &= \frac{18}{6} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{(q)} \quad g^{-1}(13) &= \frac{13 + 2}{3} = \frac{15}{3} = 5 \\ g^{-1}(7) &= \frac{7 + 2}{3} = \frac{9}{3} = 3 \end{aligned}$$

$$\begin{aligned} \frac{g^{-1}(13) - g^{-1}(7)}{13 - 7} &= \frac{5 - 3}{6} \\ &= \frac{2}{6} \\ &= \frac{1}{3} \end{aligned}$$

$$\text{8. } f(x) = \frac{2}{3}(x - 5)$$

$$f: x \rightarrow \boxed{\text{subtract 5}} \rightarrow (x - 5) \rightarrow \boxed{\times 2/3} \rightarrow \left(\frac{2}{3}\right) \rightarrow$$

$$f(x) = \frac{2}{3}(x - 5)$$

$$f^{-1}(x) = \frac{3}{2}x + 5$$

$$f^{-1}: \leftarrow \text{Add 5} \leftarrow \left(\frac{3}{2}x\right) \leftarrow \boxed{\times 3/2} \leftarrow x$$

$$\text{(a)} \quad f(-4) = \frac{2}{3}(-4 - 5) = \frac{2}{3}(-9) = -6$$

$$\text{(b)} \quad f^{-1}(-2) = \frac{3}{2}(-2) + 5 = -3 + 5 = 2$$

$$\text{(c)} \quad f^{-1}(6) = \frac{3}{2}(6) + 5 = 9 + 5 = 14$$

$$\text{(d)} \quad f^{-1}(x) = \frac{3}{2}x + 5$$

$$\text{(e)} \quad f(x) = \frac{2}{3}(x - 5)$$

$$\text{(f)} \quad f^{-1}(-6) = \frac{3}{2}(-6) + 5 = -9 + 5 = -4$$

$$\begin{aligned} \frac{2}{3}(f^{-1}(-6) - 5) &= \frac{2}{3}(-4 - 5) \\ &= \frac{2}{3}(-9) \\ &= -6 \end{aligned}$$

$$\text{(g)} \quad f^{-1}\left(\frac{8}{3}\right) = \frac{3}{2}\left(\frac{8}{3}\right) + 5 = 4 + 5 = 9$$

$$\text{(h)} \quad f^{-1}(a) = \frac{3}{2}a + 5$$

$$\text{(i)} \quad f^{-1}(a + 1) = \frac{3}{2}(a + 1) + 5$$

(j)

$$\begin{aligned} f^{-1}(a + 1) - f^{-1}(a) &= \frac{3}{2}(a + 1) + 5 - \left[\frac{3}{2}a + 5\right] \\ &= \frac{3}{2}a + \frac{3}{2} + 5 - \frac{3}{2}a - 5 \\ &= \frac{3}{2} \end{aligned}$$

(k)

$$\begin{aligned} \frac{2(f^{-1}(t) - 5)}{3} &= \frac{2\left(\frac{3}{2}t + 5 - 5\right)}{3} \\ &= \frac{2\left(\frac{3}{2}t\right)}{3} \\ &= \frac{3t}{3} \\ &= t \end{aligned}$$

$$\text{(l)} \quad f^{-1}(12) = \frac{3}{2}(12) + 5 = 18 + 5 = 23$$

$$f^{-1}(4) = \frac{3}{2}(4) + 5 = 6 + 5 = 11$$

$$\begin{aligned} \frac{f^{-1}(12) - f^{-1}(4)}{12 - 4} &= \frac{23 - 11}{8} \\ &= \frac{12}{8} \\ &= \frac{3}{2} \end{aligned}$$

$$\text{(m)} \quad f(17) = \frac{2}{3}(17 - 5) = \frac{2}{3}(12) = 8$$

$$f(8) = \frac{2}{3}(8 - 5) = \frac{2}{3}(3) = 2$$

$$\begin{aligned} \frac{f(17) - f(8)}{17 - 8} &= \frac{8 - 2}{17 - 8} \\ &= \frac{6}{9} \\ &= \frac{2}{3} \end{aligned}$$

(n)

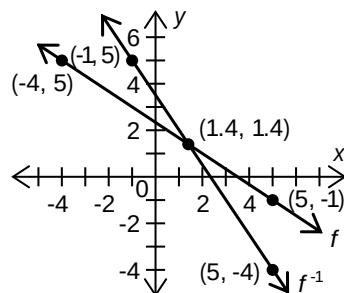
$$\begin{aligned} \frac{f^{-1}(a) - f^{-1}(b)}{a - b} &= \frac{\left[\frac{3}{2}a + 5\right] - \left[\frac{3}{2}b + 5\right]}{a - b} \\ &= \frac{\frac{3}{2}a + 5 - \frac{3}{2}b - 5}{a - b} \\ &= \frac{\frac{3}{2}a - \frac{3}{2}b}{a - b} \\ &= \frac{\frac{3}{2}(a - b)}{a - b} \\ &= \frac{3}{2} \text{ if } a \neq b \end{aligned}$$

If $a = b$, then the expression is undefined.

9. $2x + 3y = 7$

x	$y = f(x)$
2	1
5	-1
-1	3

x	$y = f^{-1}(x)$
1	2
-1	5
3	-1



(a)

$$2x + 3y = 7$$

$$3y = -2x + 7$$

$$y = -\frac{2}{3}x + \frac{7}{3}$$

$$f(x) = -\frac{2}{3}x + \frac{7}{3}$$

(b)

$$2y + 3x = 7$$

$$2y = -3x + 7$$

$$y = -\frac{3}{2}x + \frac{7}{2}$$

$$f^{-1}(x) = -\frac{3}{2}x + \frac{7}{2}$$

(c)

$$f(x) = 5$$

$$-\frac{2}{3}x + \frac{7}{3} = 5$$

$$3\left(-\frac{2}{3}x + \frac{7}{3}\right) = 3(5)$$

$$-2x + 7 = 15$$

$$-2x = 8$$

$$x = -4$$

(d) Part (c) tells us that $f(-4) = 5$. Thus, $(-4, 5)$ is the corresponding point on the graph of f .

(e) $(-4, 5)$ is the corresponding point on the graph of f , thus $(5, -4)$ is the corresponding point on the graph of f^{-1} .

(f) see graph

(g)

$$f^{-1}(x) = 5$$

$$-\frac{3}{2}x + \frac{7}{2} = 5$$

$$2\left(-\frac{3}{2}x + \frac{7}{2}\right) = 2(5)$$

$$-3x + 7 = 10$$

$$-3x = 3$$

$$x = -1$$

(h) Part (g) tells us that $f^{-1}(-1) = 5$, so $(-1, 5)$ is the corresponding point on the graph of f^{-1} . Thus, $(5, -1)$ is the corresponding point on the graph of f .

(i) $(-1, 5)$ is the corresponding point on the graph of f^{-1} .

(j) see graph

(k)

$$f(x) = f^{-1}(x)$$

$$-\frac{2}{3}x + \frac{7}{3} = -\frac{3}{2}x + \frac{7}{2}$$

$$6\left[-\frac{2}{3}x + \frac{7}{3}\right] = 6\left[-\frac{3}{2}x + \frac{7}{2}\right]$$

$$-4x + 14 = -9x + 21$$

$$5x = 7$$

$$x = \frac{7}{5}$$

$$x = 1.4$$

(l)

$$f\left(\frac{7}{5}\right) = -\frac{2}{3}\left(\frac{7}{5}\right) + \frac{7}{3}$$

$$= -\frac{14}{15} + \frac{35}{15}$$

$$= \frac{21}{15}$$

$$= \frac{7}{5}$$

$$= 1.4$$

Thus, $(1.4, 1.4)$ is the corresponding point on the graph of f .

(m)

$$f^{-1}\left(\frac{7}{5}\right) = -\frac{3}{2}\left(\frac{7}{5}\right) + \frac{7}{2}$$

$$= -\frac{21}{10} + \frac{35}{10}$$

$$= \frac{14}{10}$$

$$= 1.4$$

Thus, $(1.4, 1.4)$ is the corresponding point on the graph of f^{-1} .

(n) see graph

10. (a) f^{-1} : multiply by 10 and then divide by 4 to convert inches into centimetres.

(b) Most people know their height in feet and inches. This rule could be used to convert their height into centimetres.

$$(c) f: x \rightarrow \boxed{\times 4} \rightarrow (4x) \rightarrow \boxed{\div 10} \rightarrow f(x) = \frac{4x}{10}$$

$$(d) f^{-1}(x) = \frac{10x}{4} \leftarrow \boxed{\div 4} \leftarrow (10x) \leftarrow \boxed{\times 10} \leftarrow x$$

(e) $f(15)$ represents the approximate depth of snow that fell in centimetres.

(f) 5 feet 6 inches = 66 inches $(5 \times 12 + 6)$. $f^{-1}(66)$ represents Kelly's height in centimetres.

$$11. g(x) = -x + 5$$

$$(a) g(7) = -7 + 5 = -2$$

(b) To find $g^{-1}(7)$ solve:

$$7 = -x + 5$$

$$x = -2$$

$$\text{Thus, } g^{-1}(7) = -2$$

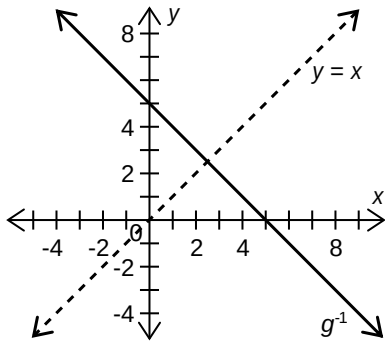
(c) $g^{-1}(x): x = -y + 5$

$$x + y = 5$$

$$y = -x + 5$$

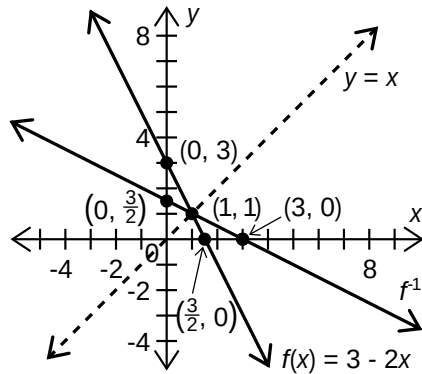
$$g^{-1}(x) = -x + 5$$

(d)



(e) g and g^{-1} both represent the same line. This is because of its symmetry in the line $y = x$. If you reflect $g(x) = -x + 5$ in the line $y = x$, it is projected back onto itself.

12. i.



ii. (a)

$$f(x) = 0$$

$$3 - 2x = 0$$

$$3 = 2x$$

$$x = \frac{3}{2}$$

(b)

$$f^{-1}(x) = \frac{x-3}{-2}$$

$$f^{-1}(x) = 0$$

$$0 = \frac{x-3}{-2}$$

$$0 = x - 3$$

$$x = 3$$

(c) $f(x) = x$

$$3 - 2x = x$$

$$3 = 3x$$

$$x = 1$$

(d)

$$f^{-1}(x) = x$$

$$\frac{x-3}{-2} = x$$

$$x - 3 = -2x$$

$$3x = 3$$

$$x = 1$$

(e)

$$f^{-1}(x) = f(x)$$

$$\frac{x-3}{-2} = 3 - 2x$$

$$-2\left(\frac{x-3}{-2}\right) = -2(3-2x)$$

$$x - 3 = -6 + 4x$$

$$3 = 3x$$

$$x = 1$$

(iii) (a) If $f(x) = 0$, then the y -coordinate is 0. This tells us

that the x -intercept on the graph of f is $\frac{3}{2}$. This also tells us

that the y -intercept on the graph of f^{-1} is $\frac{3}{2}$.

(b) If $f^{-1}(x) = 0$, then the y -coordinate on the graph of f^{-1} is 0. This tells us that the x -intercept on the graph of f^{-1} is 3. It also tells us that the y -intercept on the graph of f is 3.

(c) If $f(x) = x$, then the y -coordinate is the same as the x -coordinate on the graph of f . This tells us that $(1, 1)$ is the point on the graph of f where the x and y coordinates are equal. It also tells us that $(1, 1)$ is the point on the graph of f^{-1} where the x and y coordinates are equal. $(1, 1)$ is also the point of intersection of each of the graphs with the line $y = x$.

(d) If $f^{-1}(x) = x$, then the y -coordinate and x -coordinate are the same on the graph of f^{-1} . This is exactly the same as part **(c)**, since this point is also on f .

(e) If $f^{-1}(x) = f(x)$, then we are finding a value of x which makes both y -coordinates equal. This tells us that $x = 1$ at the point of intersection of the graphs of f and f^{-1} .

13. (a) $p(s) = 100 + 0.05s$

The employee's weekly pay is determined by multiplying the sales by 0.05 (take 5% of the sales) and then add \$100.

(b) The inverse is:

$$\begin{aligned} p^{-1}(s) &= \frac{s-100}{0.05} \\ &= \frac{20(s-100)}{20(0.05)} \\ &= \frac{20(s-100)}{1} \\ &= 20(s-100) \end{aligned}$$

(c) The inverse: Take the weekly pay, subtract \$100 and then multiply by 20 to get the employee's weekly sales.

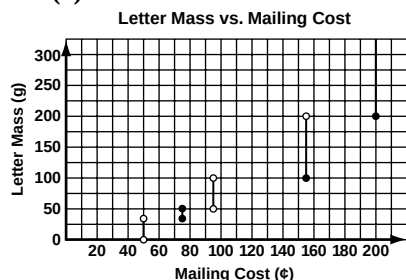
(d) If an employee received the pay cheque, he/she could use this rule to determine his/her weekly sales to see if the pay is correct.

(e) In one week, an employee might sell \$20 000 worth of equipment at best. In this case, his/her pay would be:

$$p(20\,000) = 100 + 0.05(20\,000) = 1100$$

If the employee sells nothing, his/her pay would still be \$100. Thus, a reasonable domain would be $\{s \mid 0 \leq s \leq 20\,000, s \in \mathbf{R}\}$ and a reasonable range would be $\{p \mid 100 \leq p \leq 1100, p \in \mathbf{R}\}$.

14. (a)



(b) This is not a function. Each mailing cost does not uniquely determine the mass of the letter.

(c) With a 94¢ stamp, we can assume that the letter weighs at least 50 g but less than 100 g: $50 \text{ g} < \text{mass} < 100 \text{ g}$.

15. Ali's equation was: $y = 2.63x - 1.29$. If the variables are interchanged, the equation becomes:

$$x = 2.63y - 1.29$$

$$x + 1.29 = 2.63y$$

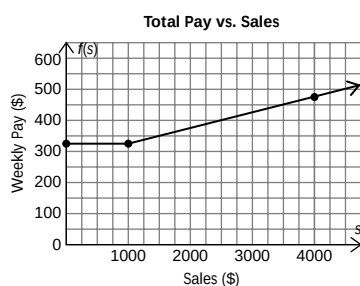
$$y = \frac{1}{2.63}x + \frac{1.29}{2.63}$$

$$y \doteq 0.380x + 0.490$$

Thus, the correct equation is: $y \doteq 0.380x + 0.490$.

16. (a)

Sales in \$	Weekly pay in \$
0	$8.05(40) = 322$
1000	$322 + 0.05(0) = 322$
2000	$322 + 0.05(1000) = 372$
3000	$322 + 0.05(2000) = 422$
4000	$322 + 0.05(3000) = 472$



(b) Let s represent her sales for the week, and $f(s)$ represent her total pay in dollars.

If $0 \leq s \leq 1000$, $f(s) = 322$

If $s > 1000$:

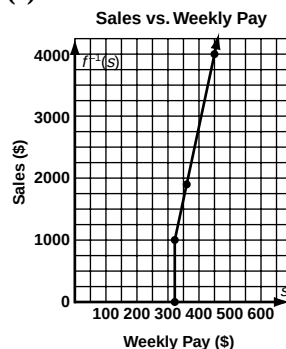
$$f(s) = 322 + (0.05)(s - 1000)$$

$$= 322 + 0.05s - 50$$

$$= 272 + 0.05s$$

Thus, $f(s) = 272 + 0.05s$.

(c)



(d) If $s > 322$:

$$f^{-1}(s) = \frac{s - 272}{0.05}$$

$$= 20s - 5440$$

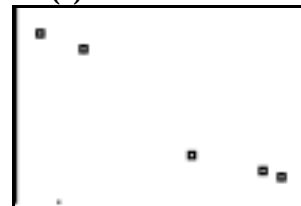
If $s = 322$, then $f^{-1}(s)$ can be any value between 0 and 1000.

(e) If weekly pay = \$420, then evaluate:

$$f^{-1}(420) = 20(420) - 5440 = 8400 - 5440 = 2960$$

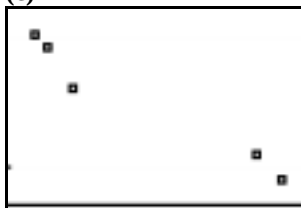
Thus, she had sales of \$2960 if she earned \$420 in pay.

17. (a)



(b) If p represents the price charged for the ground beef in \$/kg, and $f(p)$ represents the mass we expect will be sold in kg, then the formula is: $f(p) = -3040p + 14\,700$

(c)



(d) If m represents the mass of ground beef we expect to sell in kg, then $f^{-1}(m)$ represents the price to be charged for the ground beef in \$/kg.

$$f^{-1}(m) = -0.000\,314m + 4.81$$

(e) The equation for f is: $m = -3040p + 14\,700$

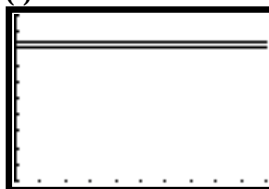
$$3040p = -m + 14\,700$$

$$p = -\frac{1}{3040}m + \frac{14\,700}{3040}$$

$$\doteq -0.000\,329m + 4.84$$

$$f^{-1}(m) = -0.000\,329m + 4.84$$

(f)



The equations for the two graphs are almost exactly the same. Their domains and ranges are nearly identical. The process for determining the line of best fit causes slight differences.

(g) If price = \$4.75/kg, then

$$f(4.75) = -3040(4.75) + 14\,700 = 260$$

Thus, if the price rises to \$4.75/kg, he can expect to sell only 260 kg of ground beef.

(h) If price = \$5/kg, then

$$f(5) = -3040(5) + 14\,700 = -500$$

Thus, if the price rises to \$5/kg, he will not sell any ground beef.

(i) If mass = 4000 kg, then

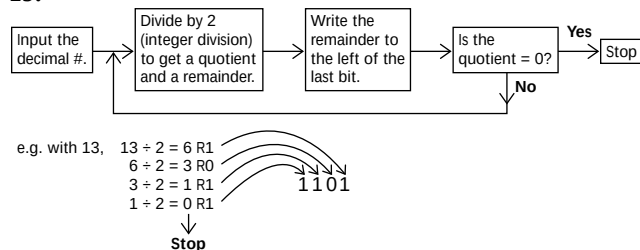
$$f^{-1}(4000) = -0.000\,314(4000) + 4.81 = 3.554$$

Using the second model, we get $f^{-1}(4000) = 3.554$. Thus, the price would have to be approximately \$3.55 in order to sell 4000 kg of ground beef.

18. (a) A postal code must be a function because each address in Canada must have a reliable, consistent postal code. One address cannot have more than one postal code.

(b) The inverse converts a postal code into its corresponding address. Several neighboring houses may have the same postal code, so one postal code produces several possible addresses. It is not a function.

19.



To convert binary to decimal, the number (at each stage) is multiplied by 2 and then a digit is added (1 or 0). To reverse this process, divide by 2, to get a quotient and a remainder. Write the remainder down as the next bit (moving left). Repeat using the quotient. Continue until the quotient is 0.

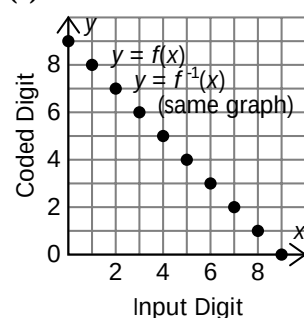
20. (a) 5 667 821 743

(b) The inverse is the same as the original code, so if 207 456 127 is received, then 792 543 872 was sent.

(c) $f(x) = 9 - x$, $\{x \mid x \leq 9, x \in \mathbf{W}\}$

(d) $f^{-1}(x) = 9 - x$, $\{x \mid x \leq 9, x \in \mathbf{W}\}$

(e)



21. (a) B --> 2, 0 --> 15, Y --> 25

BOY becomes 21525

(b) This is a function because each word can only be coded in one way.

(c) 21520201513 might be:

2	15	20	20	15	13
B	O	T	T	O	M

or

2	15	2	0	20	15	13
B	O	B	—	T	O	M

or many other combinations that are not necessarily English words.

(d) This is not a good encryption scheme because its inverse is not a function. Thus, if you know the encoded digits this will not necessarily determine the correct original message.

22. In the original soft drink machine example, a combination of coins and a button selection were the input, and the soft drink was the output.

(a) For the inverse relation, the independent variable is the soft drink received.

(b) For the inverse relation, the dependent variable is the combination of coins and a button selection.

(c) The domain of the inverse is the set of all the different drinks that the machine holds.

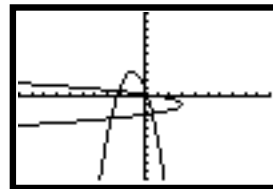
(d) The range of the inverse is the set of all possible combinations of coins and button selections that would yield a soft drink from the machine.

(e) The inverse is not a function because the size and type of soft drink received does not correspond to one unique combination of coins and button selections.

Practice 3.5, page 264

1. (a) $f(x) = -2(x+1)^2 + 3$, $f^{-1}(x) = \pm \sqrt{\frac{x-3}{-2}} - 1$

i.



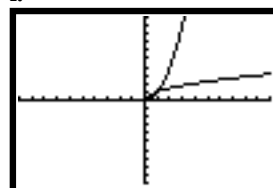
ii. $D = \{x \mid x \in \mathbf{R}\}$ and $R = \{y \mid y \leq 3, y \in \mathbf{R}\}$

iii. $D = \{x \mid x \leq 3, x \in \mathbf{R}\}$ and $R = \{y \mid y \in \mathbf{R}\}$

iv. The inverse is not a function. It fails the vertical line test.

(b) $g(x) = \sqrt{x}$, $g^{-1}(x) = x^2, x \geq 0$

i.



ii. $D = \{x \mid x \geq 0, x \in \mathbf{R}\}$ and $R = \{y \mid y \geq 0, y \in \mathbf{R}\}$